# Entropic Information Theory:
# Formulae and Quantum Gravity
## *Bits from Bit*

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Abstract

We show here that entropic information is capable of unifying all aspects of the universe at all scales in a coherent and global theoretical mathematical framework materialized by entropic information framework, theory and formulas, where dark matter, dark energy and gravity are truly informationals processes. Here, we reconcile general relativity and quantum mechanics by introducing quantum gravity for the Planckian scale. The formulas of entropic information are expressed in natural units, physical units of measurement based only on universal constants, constants, which refer to the basic structure of the laws of physics: c and G are part of the structure of space-time in general relativity, and h captures the relationship between energy and frequency that is the basis of quantum mechanics. Here we show that entropic information formulas are able to present entropic information in various unifying aspects and introduce gravity at the Planck scale. We prove that Entropic information theory is thus building the bridge between general relativity and quantum mechanics.

KEYWORDS: information, general relativity, quantum mechanics, quantum gravity, dark matter, dark energy, universal physics constants, entropic information, unification, Theory of Everything, Grand Unified theory

## 1.Introduction

The entropic information formulas are able to present entropic information in various unifying aspects where general relativity and quantum mechanics are reconciled by introducing quantum gravity for Planckian scale. Entropic information is capable of unifying all aspects of the universe at all scales in a coherent and global theoretical mathematical framework materialized by entropic information theory and its formulas, where dark matter, dark energy and gravity are truly informationals processes. The model and formulas of entropic information theory leads to a total paradigm shift in the understanding of concepts as space, time, gravity, dark matter and dark energy.

## 2 Methodology

### 2.1 Boltzmann and kinetic theory of gases

The relation between entropy and the number of ways the atoms or molecules of a certain kind of thermodynamic system can be arranged is showed by the Boltzmann–Planck equation**.**

The number of real microstates corresponding to the gas's macrostate: W or Ω

A macrostate is a state that can be experimentally observed, with at least a finite extent in spacetime.

The value of *W* was originally intended to be proportional to the *Wahrscheinlichkeit* (the German word for probability) of a macroscopic state for some probability distribution of possible microstate

A macrostate is characterized by a probability distribution of possible states across a certain statistical ensemble of all microstates. This distribution describes the probability of finding the system in a certain microstate.

To quote Planck: “the logarithmic connection between entropy and probability was first stated by L. Boltzmann in his kinetic theory of gases".

$$S = kB \log W$$

.(1) Boltzmann entropy

where $k_B$ is the Boltzmann constant (also written as simply k) and equal to 1.380649 × $10^{-23}$ J/K.

Entropy is almost universally called simply S or the statistical entropy or the thermodynamic entropy which have equally meaning

In statistical mechanics, a microstate is a specific microscopic configuration of a thermodynamic system that the system may occupy with a certain probability in the course of its thermal fluctuations.

In contrast, the macrostate of a system refers to its macroscopic properties, such as its temperature, pressure, volume and density.[1]

Microstates appear as different possible ways the system can achieve a particular macrostate. [2],[3]

The statistical entropy perspective was introduced by Boltzmann in 1870, who established a new field of physics providing based on the rigorous treatment of a large ensembles of microstates that constitute thermodynamic systems, the descriptive linkage between the macroscopic observation of nature and the microscopic view

In statistical mechanics, entropy is formulated as a statistical property using probability theory.

## 2.2 Gibb's entropy:

The macroscopic state of a system is characterized by a distribution on the microstates. The entropy of this distribution is given by the Gibbs entropy formula, named after J. Willard Gibbs.

The value p(i) is a probability in percentage and therefore dimensionless, and the logarithm is to the basis of the dimensionless mathematical constant e so, the entire summation is dimensionless

For a set of microstates, *i*, and p(i) the probability that it occurs during the system's fluctuations, then the entropy of the system is

$$S = - k_B \sum_i p(i) \ln(p(i))$$

.(2) Gibbs entropy

The set of microstates (with probability distribution) on which the sum is done is called a statistical ensemble.

## 2.3 Shannon Entropy

The concept of information entropy was introduced by Claude Shannon in his 1948 paper "A Mathematical Theory of Communication".[4][5]

Shannon's entropy is denoted by H and its equation is written like this with p (i), the probability of event i for the distribution P.

$$H(P) = - \sum_i p(i) \log_2(p(i))$$

.(3) Shannon entropy

Shannon's entropy is a measure of uncertainty calculated in bits.

This is the smallest amount of information needed to remove uncertainty.

The entropy increases as the uncertainty increases.

More exactly, the entropy is maximum when all the possible events are equally probable.

Entropy is thus a measure allowing to characterize a statistical distribution...

## 2.4 Entropic Equivalence:

*The entropy in statistical thermodynamics is directly analogous to Entropy in information theory*

*The analogy results when the values of the random variable designate energies of microstates, so Gibb's formula for the entropy is formally identical to Shannon's formula.*

Boltzmann 'entropy formula can be derived from Shannon entropy formula when all state are equiprobable

So W microstate equiprobable with probability pi= 1/W

$$S= \quad -K \sum pi \ln(p(i)) \; = \quad k \sum \frac{(\ln(W))}{w} \quad = \quad k \ln(W)$$

.(4) equivalence

## 2.5 Hidden thermodynamics

The hidden thermodynamics of isolated particles was De Broglie's final idea.

It is an attempt to bring together the three furthest principles of physics: the principles of Fermat, Maupertuis, and Carnot.

In this work, entropy becomes a sort of opposite to action with an equation that relates the only two universal dimensions of the form:

$$\frac{action}{h} = -\frac{entropy}{k}$$

.(5) hidden relation

With action = Energy * time

## 2.6 Planck-Einstein relation

Fundamental equation in quantum mechanics though the latter might also refer to Planck's law [6] which states that the energy of a photon, E, known as photon energy, is proportional to its frequency, v where Photons are viewed as the carriers of the electromagnetic interaction between electrically charged elementary particles.

$$E= h\nu$$

.(6) Planck-Einstein

## 2.7 Mass–energy equivalence

$$E=mc^2$$

.(7) mass-energy equivalences

Mass–energy equivalence formula was introduced in 1900 by Poincaré in an article on Lorentz’s theory and the principle of action and reaction.

Just as E = mc2 is not a one-person formula, it does not originate from the only theory of relativity: it is found at the confluence of the principles of mechanics, principle of relativity and electromagnetic theory. [7]

## 2.8 Bekenstein

Indeed, according to Bekenstein,

“The thermodynamic entropy and Shannon entropy are conceptually equivalent. The number of arrangements that are counted by Boltzmann entropy reflects the amount of Shannon information that would be needed to implement any particular arrangement ......of matter and energy [8].”

The only fundamental difference between the thermodynamic entropy of physics and the entropy of Shannon lies in the units of measurement; the first is expressed in units of energy divided by the temperature, the second in "bits" of information essentially dimensionless.

## 2.9 First equivalence

In, $\frac{action}{h} = - \frac{entropy}{k}$

replace k ln(W) and E= $h$v

with action = Energy* time

with Energy following (6), E= $h$v

so,

$$\frac{htv}{h} = \frac{k \ln(W)}{k}$$

We have:

$$\ln(W) = tv = \frac{action}{h} \quad .(8)$$

With (7) in (8)

with action = Energy* time

Energy = $mc^2$

We obtain:

$$\ln(W) = \frac{action}{h} = tv = \frac{mtc^2}{h}$$

.(9) first result

## 2.10 The mass bit of information

Following [9], the mass bit information is given by

$$m_{bit} = \frac{k\, T \ln(2)}{c^2} \quad .(10)$$

T is temperature at which the bit of information is stored

where kb = 1.38064 ×10-23 J/K is the Boltzmann constant

where c is the speed of light in vacuum 299792458 m·s−1 ((exact by definition)

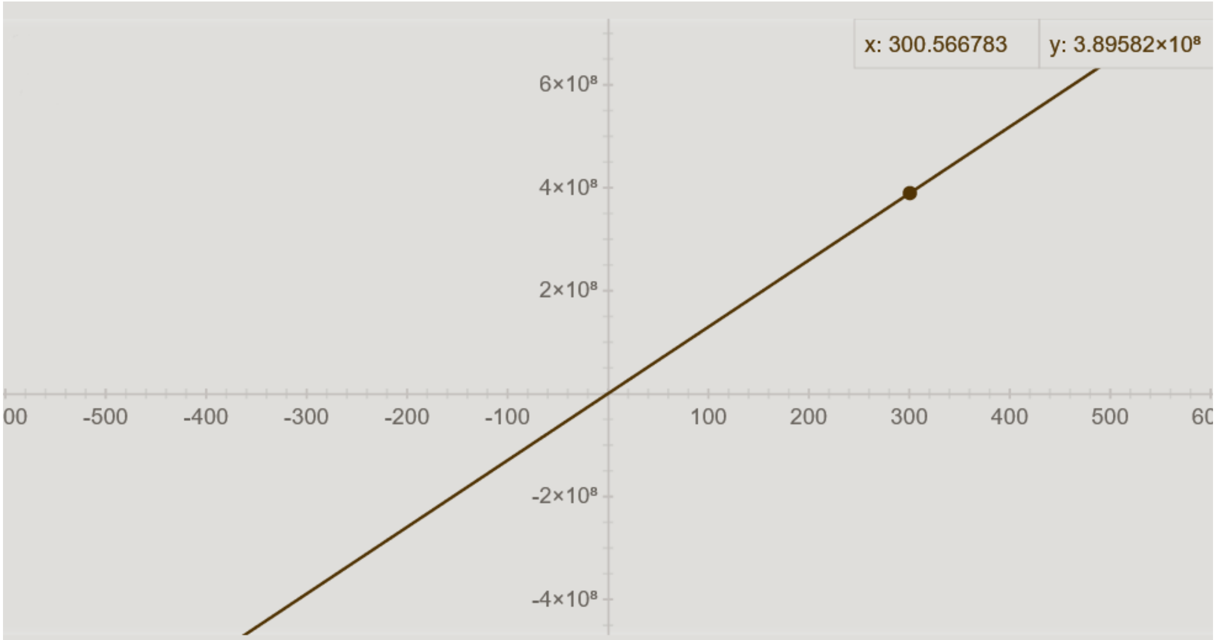


Figure 1: graph showing the mass bit information (kg) with temperature (kelvin) relation

## 2.11 Second equivalence

Replace in (9) with (10)

$$\frac{\mathrm{mtc}^2}{h}$$

With m$_{bit}$ = $\frac{k\,T\,\ln(2)}{c^2}$

We have:

$$\frac{k\,T\,\ln(2)t\,c^2}{c^2}$$

So,

$$\frac{k\,T\,\ln(2)t}{h}$$

We obtain:

$$\ln(W) = \frac{action}{h} = tv = \frac{mtc^2}{h} = \frac{k\,T\ln(2)t}{h}$$

.(11)

## 2.12 Mathematics proof with Landauer's principle

We have

$$\frac{K*T*\ln(2)*t}{h} = \frac{m*t*c^2}{h}$$

$$\Rightarrow K*T*\ln(2) = mc^2$$

There is a minimum possible amount of energy required to erase one bit of information, that is that Landauer's principle asserts and it's known as the *Landauer limit*:

$$E = K\,T\,ln(2)$$

.(12) Landauer

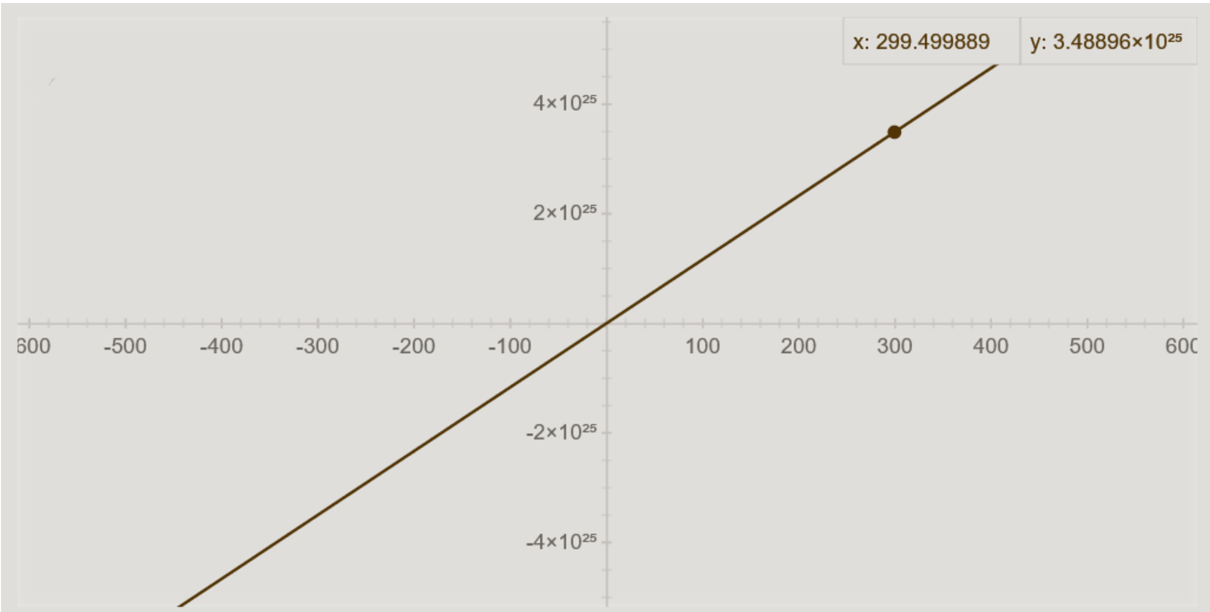


Figure 2 graph showing the energy mass bit of information (eV) with temperature (Kelvin)

## 2.13 Planck length

Replace in (9)

$$with\ c = \frac{lP}{tP} \rightarrow \frac{mt\frac{lP^2}{tP^2}}{h} \rightarrow \frac{m\,t\,lP^2}{h\,tP^2} \rightarrow \frac{m\,tP\,lP^2}{h\,tP^2} \rightarrow \frac{m\ \ lP^2}{h\,tP}$$ .(13)

From $\ell_P = \sqrt{\frac{\hbar G}{c^3}} \approx 1.616\ 255(18) \times 10^{-35}$ m,

We have: $l(P) = \sqrt{\frac{\hbar G}{c^3}} \rightarrow l(P) = \sqrt{\frac{hG}{2\pi c^3}}$ in (13) :

$$\frac{m\ l(P)^2}{h\ t(P)} = \frac{m\ hG}{2\pi c^3 h\ t(P)} = \frac{m\ G}{2\pi c^3 t(P)}\ .(14)$$

We have as Gravitational Relation:

$$\frac{m\ G}{2\pi c^3 tP}\ .(15)$$

In (15) with k = 2 π /λ as wavenumber

We have

$$\ln(W) = \frac{action}{h} = t\nu = \frac{mtc^2}{h} = \frac{k\ T \ln(2) t}{h} = \frac{m\ l(P)^2}{h\ t(P)} = \frac{m\ G}{2\pi c^3 tP} = \frac{m\ G}{kc^3}\ .(16)$$

## 2.14 Planck time

Replace in $\frac{mtc^2}{h}$

with $t_P = \sqrt{\frac{\hbar G}{c^5}}$

$$\frac{m\sqrt{\frac{\hbar G}{c^5}}\, c^2}{h} = \frac{mtc^2}{h}$$

We obtain:

$$\frac{m\sqrt{\frac{\hbar G}{2\pi c}}}{h} = \frac{m\sqrt{\frac{G}{2\pi c}}}{\sqrt{h}} = \frac{m\sqrt{G}}{\sqrt{2\pi ch}} = m\sqrt{\frac{G}{2\pi ch}} \quad .(17)$$

## 2.15 Bekenstein bound

An upper limit on the thermodynamic entropy S, or Shannon entropy H, that can be contained within a given finite region of space which has a finite amount of energy—or conversely, the maximal amount of information required to perfectly describe a given physical system down to the quantum level is given by the Bekenstein bound.[10]

The universal form of the bound was originally found by Jacob Bekenstein in 1981 as the inequality[11],[12]

$$S \leq \frac{2\pi kRE}{\hbar c} \quad .(18)$$

where S is the entropy,

k is Boltzmann's constant,

R is the radius of a sphere that can enclose the given system,

E is the total mass–energy including any rest masses,

ħ is the reduced Planck constant, and

c is the speed of light.

As a side note, it can also be shown that the Boltzmann entropy is *an upperbound to the entropy that a system can have for a fixed number of microstates* meaning:

$$S \leq k \ln W \quad .(19)$$

Boltzmann 'entropy formula can be derived from Shannon entropy formulae when all states are equally probable

So you have W microstate equiprobable with probability pi= 1/W

| | | | |
|---|---|---|---|
| S= | -K $\sum pi \ln(p(i))$ = | k $\sum \frac{(\ln(W))}{w}$ = | k ln(W) |

$$k \ln W = \frac{2\pi kRE}{\hbar c} \quad .(20)$$

with E, Energy = mc², we obtain :

$$\ln W = \frac{2\pi Rmc}{\hbar} \quad .(21)$$

## 2.16 Mathematics proof with Schwarzschild and Bekenstein–Hawking boundary entropy

This radius can be calculated using the equation:

$$R = \frac{2GM}{c^2} \quad (22)$$

where the gravitational constant G is $6.67430\times10^{-11}$ N·m²/kg², M is the mass of the object, and c is the speed of light which is 299,792,458 m/s.

so we have replacing R in $\frac{2\pi Rmc}{\hbar}$ :

$$\frac{2\pi \frac{2GM}{c^2} mc}{\hbar} \text{ so } \frac{4GM^2\pi}{\hbar c}$$

It happens that the Bekenstein–Hawking boundary entropy of three-dimensional black holes exactly saturates the bound

$$r_s = \frac{2GM}{c^2},$$

$$A = 4\pi r_s^2 = \frac{16\pi G^2 M^2}{c^4},$$

$$l_P^2 = \hbar G/c^3,$$

$$S = \frac{kA}{4\, l_P^2} = \frac{4\pi kGM^2}{\hbar c},$$

where k is Boltzmann's constant, *A* is the two-dimensional area of the black hole's event horizon and l(p) is the Planck length.

In regard to our result $\frac{4GM^2\pi}{\hbar c}$ at factor k near.

## 2.17 Black hole Entropy

Again, in regard to S= k ln(W) ,

$$k(b)\ln W = \frac{4\pi kGm^2}{\hbar c} = \frac{k(b)A}{4l(p)^2}$$

$$\ln W = \frac{4\pi Gm^2}{\hbar c} = \frac{A}{4l(p)^2}$$

## 2.18 Final Equations

# Finally, we obtain as final result:

$$\ln(W) = \frac{action}{h} = tv = \frac{\mathrm{mtc}^2}{h} = \frac{k(b)\,T\ln(2)t}{h} = \frac{\mathrm{m}\ \mathrm{l(P)}^2}{h\ \mathrm{t(P)}} = \frac{A}{4l(p)^2} = \frac{\mathrm{m}\ G}{2\pi c^3\mathrm{t(P)}} = \frac{\mathrm{m}\ G}{kc^3} = \frac{2\pi Rmc}{\hbar} = \frac{4\pi Gm^2}{\hbar c} = \mathrm{m}\sqrt{\frac{\mathrm{G}}{2\pi ch}}$$

Where:

I: information

W, Ω: number of ways the atoms or molecules of how a thermodynamic system can be arranged

the number of arrangements that are counted by Boltzmann entropy reflects the amount of Shannon information that would be needed to implement any particular arrangement of matter and energy

w, Ω number of complexions of the system or number of configurations

Action: E*t

t: time

$v$: frequency

m: mass

c: Speed of light in vacuum 299792458 m·s−1 ((exact by definition)

h: Planck constant defined as 6.62607015×10−34 J·s exactly)

Kb: Boltzmann constant 1.38064 ×10-23 J/K

T: temperature at which the bit of information is stored

lp: Planck length 1.616255(18)×10−35 m

$t_p$: Planck time $5.391247(60)\times10^{-44}$ s

G: gravitational constant 6,674 30(15) × $10^{-11}$ $m^3$ $kg^{-1}$ $s^{-2}$

k: wavenumber as 2 π /λ

R: is the radius (in meter) of the system.

A is the two-dimensional area of the black hole's event horizon.

## 3. Discussion

We pass in review here the evolution of the concept of information and implement quantum gravity at Planck scale into entropic information formulas. Information being apprehended, first, from the macroscopic approach and the statistical one. Indeed, the macrostate formulae from Boltzmann and his statistical entropy perspective has been followed by the statistical ensemble of distribution on the microstates approach of Gibbs. After what, Shannon perspective of information has arrived where entropy is a measure allowing to characterize a statistical distribution as the probability of event for a specific distribution. So, we understand better why Boltzmann 'entropy formula can be derived from Shannon entropy formulae when all states are equiprobable. After this statistical approach of the entropy, we arrive at the quantic perspective with the Planck-Einstein relation which make us dive in to quantum physics approach. Near this quantic way of description of the information concept we arrive at mass-energy equivalence relation from Einstein, here, we are at confluence of the principles of mechanics, principle of relativity and electromagnetic theory. We reach, now, the first fundamental equivalence with the hidden thermodynamics relation of De Broglie unifying the principles of Fermat, Maupertuis, and Carnot. After that, comes the second equivalence, here given by the mass bit equation introduction. At this level, we can introduce gravity concept for Planck dimensional order. Indeed, as dimensionless entropic information formulae express themselves with universals constants we can work with the Planck unit system with Planck length and Planck time introduction, so, we are able to work with gravity at a quantum level as so expressing the quantum gravity as an informational process as we do for dark matter and dark energy.

## 4. Conclusions

After passing in review the evolution of the concept of information at different organizational level, we are able to follow the entropic information formulas which express that dark matter, dark energy and gravity as being truly informationals processes. Entropic Information formulas are able to unifying all aspects of the universe following the common base element: the information. Entropic information is capable of unifying all aspects of the universe at all scales in a coherent and global theoretical mathematical framework materialized by entropic information theory and its formulas where general relativity and quantum mechanics are reconciled by introducing quantum gravity for Planckian scale. The model and formulas of entropic information theory leads to a total paradigm shift about the universe we live in

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## 6. Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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