Real-time Optimization and Implementation of Multi-rate Digital FIR Kernel Filter

ABSTRACT

One ideal performance of this design is in the areas of decimation where a decimation factor of 10, 45-order and pass band ripple of 1dB and interpolation of sampled rates where a sinusoidal signal input produced a ripple free output with interpolation factor of 10, 52-order and stopband attenuation of 60dB. Owing to the multiple samples of filter length of 200, the filter performed down sampling preceded with filtering as well as up sampling preceded with filtering, hence multi-rate filter by allowing a low threshold of frequency of 0.3π to be passed, blocking a high threshold of 0.7π and vice versa. There was resampled output increased to 150% preceded by filtering. The filter coefficients for low pass and high pass Digital FIR filter, through the least square regression method, parks McClellan Algorithm and window methods were employed for easy optimization. More so, there was creation of 2-4-5 filter channel banks through the 2^{nd}-level convolution of their down sampling and up sampling filtering techniques during the multi-irate filtering to ensure the design of error-free Digital FIR Filter using MATLAB File editor(M-File) and tool boxes for writing the C-programming of the design. In the analysis, the mean and standard deviation of the low pass Digital FIR Filter output during decimation and interpolation are (0.26, 6.13) and (0.004,1.22) respectively.

Keywords: MATLAB, Multi-rate, Digital FIR Filter, Sampling, Low pass, high pass, Filter Banks.

1. INTRODUCTION

The digital filter is a discrete system, and it can do a series of mathematic processing to the input signal, and therefore obtain the desired information from the input signal [3].Digital filter is a linear time invariant system (LTI) which does not vary with time. Digital filters have high accuracy, easy to simulate and design, flexible than analog filters [4], [2]. Digital filters constitute an essential component of electronic devices like mobiles, computers, radios, wireless systems and AV receivers. As compared to analog filters, digital filters can be easily designed for a wider range of applications. Because of software programmability, superior performance-to-cost ratio, smaller area requirement and greater ease of implementation, digital filters have replaced analog filters in many applications [10].

A finite impulse response (FIR) filter is a filter whose impulse response (or response to any finite length input) is limited because it settles to zero in finite time. This contrasts with infinite impulse response (IIR) filters, which may have internal feedback and may continue to respond indefinitely [9].

Multi-rate signal processing studies digital signal processing systems which include sample rate conversion. Multi-rate signal processing techniques are necessary for systems with different input and output sample rates, but may also be used to implement systems with equal input and output rates.

Multi-rate Digital filters (MDF) is a digital filter that changes the input sampling rate of the input signal into another desired one. These filters are of an essential importance in communications, image processing, digital audio, and multimedia [7], [1]. It is necessary to decrease or to increase the sampling rate of a signal. These processes are respectively known as downsampling or upsampling, and they may affect the information contained in the signal if that signal is not properly filtered. Filtering a signal and then applying downsampling is known as decimation, whereas applying upsampling and then filtering a signal is known as interpolation.

Multi-rate systems are those that use multiple sampling frequencies in the processing of digital signals. It has been proved that using multi-rate techniques in the design of a filter generates a reduction in the number of adders and multipliers required for its implementation. There are several techniques in digital signal processing available to optimize multirate filters. For example, for M-th band FIR filters design, an algorithm was developed in to optimize a polyphase structure based on two stages for different integer sampling rate conversion. It was demonstrated in that scheme that conversions by odd factors are more efficient than conversions by even factors. In this approach there is a two-frequency system that takes advantage of the Frequency Response Masking Technique (FRM) to accomplish sharp transition bands with reduced computational load. A common application of multirate techniques is in filter bank systems [7].

2. LITERATURE REVIEW

In the proposed thesis [1], the implementation of one stage digital Low pass FIR Filter with 1087 taps was carried out using FGPA for a proposed Multi-rate decimation filter. This was interfaced to PCI card for the design case of the noise thermometer. The FGPA hip was then connected with an external oscillator of 40 MHz, on a PCI card. A decimation factor of M=64 for a sinusoidal data input was simulated using ModelSim software Program.

The design and implementation of decimation (or interpolation) filters which depends on filter length to decimation (or interpolation) ratio for a lowpass FIR filter using frequency sampling approach by optimizing only a small number of samples. The frequency sampling technique allowed a recursive implementation of the FIR filters [5]. The system shows the relationship between the frequency samples and the M polyphase components decomposition. The optimization method was used using fmincon function included in the MATLAB optimization toolbox.

A dual stage multi-rate digital filter [6] was designed and implemented for low cost VLSI wireless applications using MAC(Multiplier & Accumulator) based on FGPA performance metrics cumulating to 22% of area saving and 9.5% reduction in dynamic power dissipation. A decimation factor of 32 was designed using 2 finite FIR filter stages. The architecture design which was made of first stage higher order decimation filter (HDF) and second stage compensating filter (CF) where the CF was implemented using MAC and the HDF implemented using cascaded integrator comb (CIC) filters.

A closed-form linear phase FIR filter design concurrently possessing high efficiency and excellent transfer characteristic by employing Fourier spectrum of a convolution window was designd. There was increase in the FIR filter [8] order to 198 (i.e., filter length, L=199 and N=100) and specify passband cutoff frequency $w_p = 0.5\pi\, rad/s$, transition bandwidth 0.02π (accordingly, $M = 26$), $w_c(n)$ is derived by convolving an N-length Hamming window with an N-length rectangular window.

3. MATERIALS AND METHODS

There are two phases of this design methodology: Digital FIR Filter design, and Multi-rate filtering. First**,** in the design methodology of both Low pass and high pass Digital FIR FILTER, the filter coefficients were generated using first principle and using MATLAB filter design tool box and signal processing tool box. This is done by allowing a finite impulse response of length 3 units involving time span (n), input sequence, X (n), and output sequence y (n). The input has been assumed to be a casual sequence with the first non-zero sample occurring at n=0, and for calculating y (0) and y (1), set x (-1) =x (-2) =0.

The signal responses of impulse and frequency responses of low pass and high pass Digital FIR FILTER were generated using park-McClellan Algorithm and other window techniques for Digital FIR FILTER designs such as rectangular, Kaiser, hamming, and Han where filter design tool box, signal processing tool box and filter visualization and tool box were employed to achieve the same design for easy implementation of multi-rate filtering.

The stages involved in the design of the Real-time Digital FIR Filter is shown Figure 1 below.

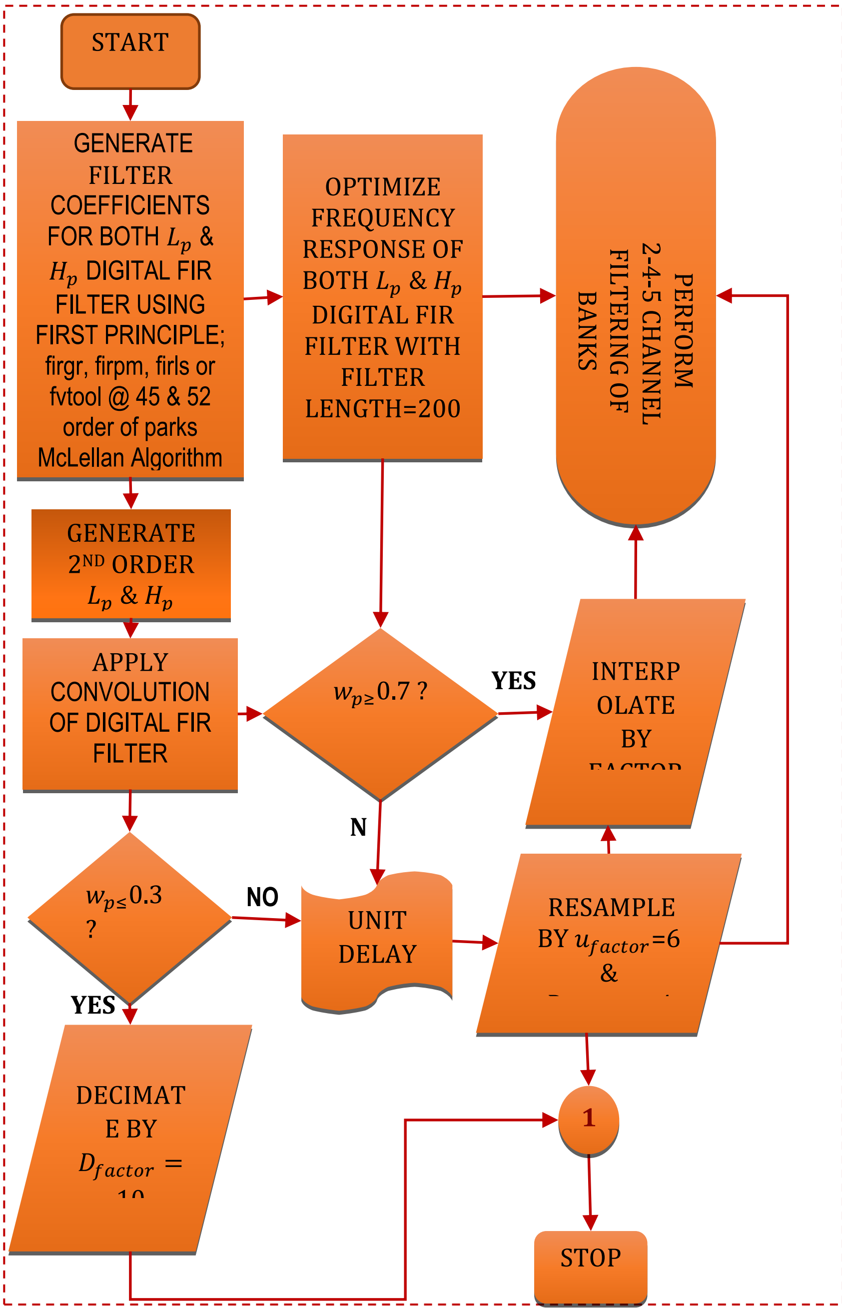

Figure 1: Flowchart Diagram of Real-Time Multi-rate Digital FIR Filter

3.1 Real-Time Digital FIR Filters Design Analysis

A. Design Objectives

- ✓ To design a Low pass discrete FIR Filter that will allow a low frequency component and block a high frequency component signals Using MATLAB Software.

- ✓ To design a high pass discrete FIR Filter that will allow a high frequency component and block a low frequency component signals Using MATLAB Software.

B. Design Techniques for Low pass and High pass Digital FIR Filter.

For simplicity, we assume the filter to be an FIR filter of length 3 with an impulse response:

h (0)=h(2)=β_0, h(1)=β_1

From the equation, gives

$y(n)=\sum_{K=N_1}^{N_2} h(k)x(n-k), N_1 < N_2$. The digital filtering is performed using the difference equation, y(n)=

$h(0)x(n)+h(1)x(n-1)+h(2)x(n-2)$ (1)

y(n)=

$\beta_0(n)+\beta_1 x(n-1)+\beta_0 x(n-2)$ (2)

Where y (n) & x(n) represent the output and input sequences respectively. Also, from our design, we chose the filter coefficients β_0 & β_1 such that the output of the filter becomes the cosine sequence with frequency 0.7π rad/time span. Now, using equation of the frequency response.

$H[e^{j\omega}]=\sum_{K=N_1}^{N_2} h(k)e^{-j\omega k}$ (3)

Which is seen to be a polynomial in $e^{-j\omega}$. The frequency response of the above filter is given by:

$H[e^{j\omega}]=h(0)e^{-j\omega 0}+h(1)e^{-j\omega 1}+h(2)e^{-j\omega 2}$ (4)

$H[e^{j\omega}]=\beta_0+\beta_1 e^{-j\omega}+\beta_0 e^{-j\omega 2}$

Re – arranging, gives

$H[e^{j\omega}]=\beta_0+\beta_0 e^{-j\omega 2}+\beta_1 e^{-j\omega}=\beta_0(1+e^{-j\omega 2})+\beta_1 e^{-j\omega}$ (5)

But, $\cos\omega=\frac{1}{2}(e^{j\omega}+e^{-j\omega})$, $2\cos\omega=e^{j\omega}+e^{-j\omega}$ (6)

$H[e^{j\omega}]=\beta_0(e^{j\omega}+e^{-j\omega})e^{-j\omega}+\beta_1 e^{-j\omega}=\beta_0(2\cos\omega)e^{-j\omega}+\beta_1 e^{-j\omega}=(2\beta_0\cos\omega+\beta_1)e^{-j\omega}$ (7)

The magnitude & phase function of the filter are $|H[e^{j\omega}]|=|2\beta_0\cos\omega+\beta_1|$

$\theta(\omega)=-\omega$

Step I: In order to stop the low frequency component from appearing at the output of the filter, the magnitude function at ω_s =0.7π rad/time span should be set to Zero. Similarly, to pass the low frequency component within any attenuation, we need to ensure to ensure that the magnitude function at ω_p =0.3π rad/ time span set to 1.

Thus, the two conditions that must be satisfied are:

$2\beta_0\cos(0.7)+\beta_1=0$ (8)

$2\beta_0\cos(0.3)+\beta_1=1$ (9)

Or converting to degrees, gives

$2\beta_0\cos 40.1+\beta_1=0$ (10)

$2\beta_0\cos 17.2+\beta_1=1$ (11)

Solving simultaneously using Crammer's rule or determinant method, gives

$\Delta_0=\begin{vmatrix}2cos40.1 & 1\\ 2cos17.2 & 1\end{vmatrix}$

=2cos40.1−2cos17.2= −0.3807

$\Delta_{\beta_0}=\begin{vmatrix}0 & 1\\ 1 & 1\end{vmatrix}=-1$

$\Delta_{\beta_1}=\begin{vmatrix}2cos40.1 & 0\\ 2cos17.2 & 1\end{vmatrix}=2cos40.1$

$=1.5298$

$\beta_0=\frac{\Delta\beta_0}{\Delta_0}=\frac{-1}{-0.3807}=2.6267$;

$\beta_1=\frac{\Delta\beta_1}{\Delta_0}=\frac{1.5298}{-0.3807}=-4.0184$

Substituting, gives

y(n)= $\beta_0 x(n)+\beta_1 x(n-1)+\beta_0 x(n-2)$, the output becomes y(n)= $2.6267x(n)-4.0184x(n-1)+2.6267x(n-2)$ (12)

OR

y(n)=2.6267[x(n)+x(n−2)]−4.0184x(n−

1) and the input becomes x(n)=$\{\cos(0.3n)+\cos(0.7n)\}u(n)$

Let, n=0: y(n)=2.6267x(n)−4.0184x(n−1)+2.6267 x(n−2) (13)

y(0)=2.6267x(0)−4.0184x(−1)+2.6267x(−2); Initial conditions are x(-1) and x(-2).Note that x(0) is the first input value and not an initial condition.

n=1:y(1)=2.6267x(1)−4.0184x(0)+2.6267x(-1);
n=2:y(2)=2.6267x(2)−4.0184x(1)+2.6267x(0)

Step II: From the first principle, for high pass digital filter; In order to stop the high frequency component from appearing at the output of the filter, the magnitude function at ω_p =0.7 rad/time span should be equal to 1. Similarly, to pass the high frequency component within any attenuation, we need to ensure to ensure that the magnitude function at ω_s=0.3rad/s is equal to 0.

$$2\beta_0\cos(0.7)+\beta_1=1 \quad (14)$$

$$2\beta_0\cos(0.3)+\beta_1=0 \quad (15)$$

Or converting to degrees, gives

$$2\beta_0\cos 40.1+\beta_1=1 \quad (16)$$

$$2\beta_0\cos 17.2+\beta_1=0 \quad (17)$$

Solving simultaneously using Crammer's rule or determinant method, gives

$\Delta_0=\begin{vmatrix}2cos40.1 & 1\\ 2cos17.2 & 1\end{vmatrix}$=2cos40.1−2cos17.2= −0.3807

$$\Delta_{\beta_0}=\begin{vmatrix}1 & 1\\ 0 & 1\end{vmatrix}=1$$

$$\Delta_{\beta_1}=\begin{vmatrix}2cos40.1 & 1\\ 2cos17.2 & 0\end{vmatrix}=-2cos17.2=-1.9106$$

$$\beta_0=\frac{\Delta_{\beta_0}}{\Delta_0}=\frac{1}{-0.3807}=-2.6267$$

$$\beta_1=\frac{\Delta_{\beta_1}}{\Delta_0}=\frac{-1.9106}{-0.3807}=5.0186$$

Substituting, gives

y(n)= $\beta_0 x(n)+\beta_1 x(n-1)+\beta_0 x(n-2)$, the output becomes:

y(n)=$-2.6267x(n)+5.0186x(n-1)-2.6267x(n-2)$ (18)

OR y(n)= −2.6267[x(n)+x(n−2)]+5.0186x(n−1) and the input becomes:

x(n)=$\{\cos(0.3n)+\cos(0.7n)\}u(n)$

Let,n=0:y(n)= −2.6267x(n)+5.0186x(n−1)−2.6267x(n−2);

y(0)= −2.6267x(0)+5.0186x(−1)−2.6267x(−2); Initial conditions are x(-1) and x(-2).Note that x(0) is the first input value and not an initial condition.

n=1: y(1)= −2.6267x(1)+5.0186x(0)−2.6267x(−1);

n=2: y(2)= −2.6267x(2)+5.0186x(1)−2.6267x(0); Initial conditions are x(-1) and x(-2).

The initial conditions are set to zero to satisfy linear and time-invariant properties.

C. Transfer Functions

The transfer function of an FIR filter as shown in below is the polynomial inz^{-1}, and is usually written in the form

$$H(z)=\frac{Y(z)}{X(z)}=\sum_{k=0}^{N-1}h[k]z^{-k} \quad (19)$$

Here *h*[0], *h*[1], …, *h*[*N*–1] are the coefficients of the system impulse response, and *N*–1 is the filter order.

The total number of coefficients, *N*, is usually called the filter length.

In time domain, an FIR system is characterized by the no recursive difference equation,

$$y[n]=\sum_{k=0}^{N-1}h[k]x[n-k] \quad (20)$$

Where *x*[*n*] and *y*[*n*] denote samples of the input and the output sequences, respectively.

$Y(z)=2.6267X(z)-4.0184z^{-1}X(z)+2.6267z^{-2}X(z)$ (21)

Then, $\frac{Y(z)}{X(z)}=2.6267-4.0184z^{-1}+2.6267z^{-2}; for\ z\neq 0$ for Low pass

Also, Y(z)= −2.6267X(z)+5.0186z[-1]X(z)+2.6267z[-2]X(z) (22)

Then, $\frac{Y(z)}{X(z)} = -2.6267 + 5.0186z^{-1} - 2.6267z^{-2}; for\ z \neq 0$ for High pass

D. The Frequency response of the Filter:

When the sequence {*h*[*n*]} is the impulse response of an LTI system, the *z*-transform,

$H(Z) = \sum_{n=-\infty}^{\infty} h[n]z^{-n}$ represents the transfer function of the LTI system. Zeros and poles of the LTI system are the zeros and poles of the transfer function *H* (*z*).

An LTI system is stable when the region of convergence of the transfer function *H* (*z*) includes the unit circle. The frequency response *H*(*ej*ω) of a stable system can be obtained by evaluating *H*(*z*) on the unit circle,

$$H(e^{j\omega}) = H(z)|_{z=e^{j\omega}=\sum_{n=-\infty}^{\infty} h[n]z^{-j\omega n}}$$

The transfer function of an LTI system described by a constant-coefficient difference equation is a rational *z*-transform, $H(z) = \frac{\sum_{k=0}^{M} b_k z^{-k}}{1+\sum_{k=1}^{N} a_k z^{-k}}$ (23)

The coefficients of *H* (*z*) are those of the difference equation. The order of the system is defined by max (*M*, *N*). Equation above is a general form of the rational transfer function. Being developed from the recursive difference equation above it represents the transfer function of an IIR system. In the case of an FIR coefficients a_k = 0, for *k* = 1, 2… *N*, the expression above reduces to

$$\sum_{k=o}^{M} b_k z^{-k} \quad (24)$$

Here *M* denotes the system order. All *M* poles of an FIR system are located at the origin, and therefore the FIR system is absolutely stable. The positions of the transfer function zeros are not restricted by the stability conditions, i.e., the zeros can be placed inside or outside the unit circle, or can be placed around the unit circle. A system which includes only zeros located inside the unit circle and those located around the unit circle is called the minimum-phase system. At the contrary, the system which includes zeros located outside the unit circle and those located around the unit circle is called the maximum-phase system. For representing the poles and zeros of the rational *z*-transform in the *z*-plane, we use the MATLAB. Since the region of convergence for Low Pass includes the unit circle, $H_{freq(\omega)} = H(z)|_{z=e^{j\omega}} = 2.6267 - 4.0184e^{-j\omega} + 2.6267e^{-j2\omega}$ and for high pass the unit circle, $H_{freq(\omega)} = H(z)|_{z=e^{j\omega}} = -2.6267 + 5.0186e^{-j\omega} - 2.6267e^{-j2\omega}$.

E. Cut-off and Sampling Frequency:

For Digital FIR Filter, $\Delta\omega = \frac{\omega_p - \omega_s}{\omega s} = \frac{0.3\pi - 0.7\pi}{\omega s}$, There are 200 samples i.e N=200. $N = \frac{3.3}{\Delta\omega}$, $\Delta\omega = {}^{3.3}/_{200}$=0.0165; the transition width, $\Delta\omega = 0.0165$

$\omega s = \frac{-0.4\pi}{0.0165} = -24\pi rad/samples$

Cut-off frequency, $\omega_c = \frac{\omega_s + \omega_p}{2} = \frac{0.7\pi + 0.3\pi}{2} = 0.5\pi\ rad/samples$. Therefore, Sampling frequency of Low pass Digital FIR filter, $\omega s = -24\pi rad/samples$ and the cut-off frequency, $\omega_c = 0.5\pi\ rad/samples$.

For High pass Digital FIR Filter, $,\Delta\omega = \frac{\omega_p - \omega_s}{\omega s} = \frac{0.7\pi - 0.3\pi}{\omega s}$,

$\omega s = \frac{0.4\pi}{0.0165} = \frac{24\pi rad}{samples}$ and the cut-off frequency, $\omega_c = 0.5\pi\ rad/samples$.

4. RESULTS AND DISCUSSION

The main goal of this design is to pass a desired frequency component and block another frequency component. In the design, the first 200 output samples were passed in both low pass and high pass filtering; the input sequence, x(n)=cos(0.3*n)+cos(0.7*n) was used to obtain the output sequence(n)=β_0x(n)+ β_1x(n-1)+ β_0x(n-2) where the filtering coefficient $\beta_0 = 2.6267$ and $\beta_1 = -4.0184$ for Low pass Digital FIR filter and $\beta_0 = -2.6267$ and $\beta_1 = 5.0186$ for High pass Digital FIR filter using different design techniques, employing Parks-McClellan Algorithm in the design.

4.1 Input Signal Spectrum of Digital FIR Filter

The polyphase input signal spectrum of Digital FIR FILTER which constituted from the first 200 samples, given the input sequence, x=cos (0.3*n)+cos(0.7*n). The MATLAB plot of the filter in figure 2 below

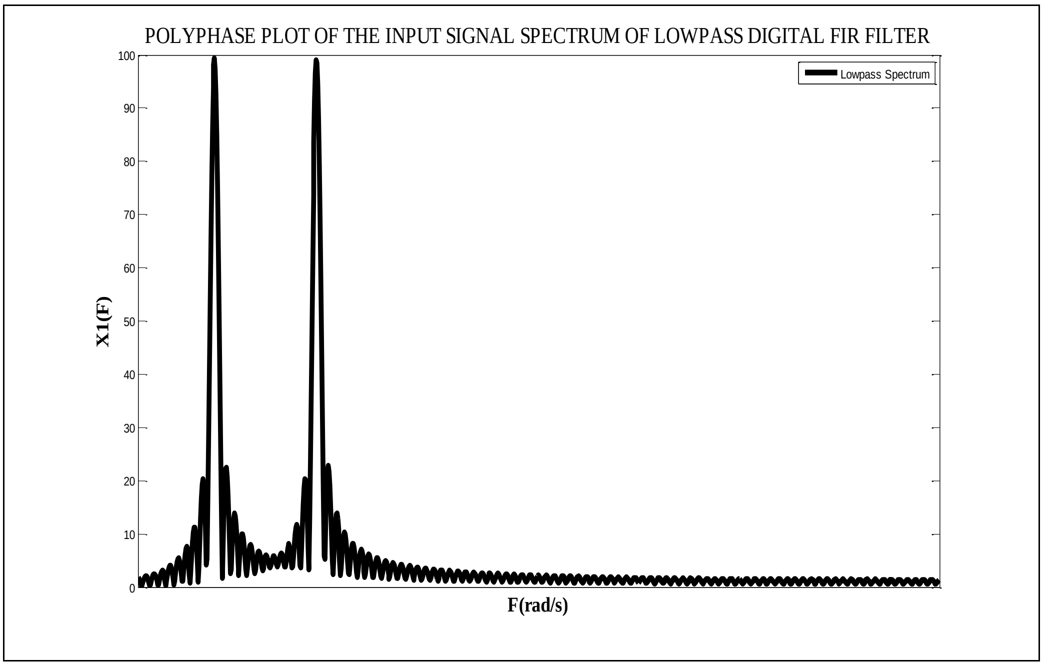

Figure 2: Polyphase Plot of Input Signal Spectrum of Digital FIR Filter

4.2 Frequency Response of The Optimized Digital FIR Filter

The frequency responses for the optimized Low pass and High pass Digital FIR filter are designed MATLAB tool box by allowing a Low frequency component of 0.3 and blocking a high frequency component of 0.7 for Low pass and allowing a High frequency component of 0.7 and blocking a low frequency component of 0.3 for High pass with the frequency response generated using frequency sampling method as shown in figure 3 below. Owing to finite length of Digital FIR Filter, there was convolution of the both filters in frequency domain as plotted in figure 4 below to show their filtering actions Also the MATLAB plot of figure 5 for the estimated power spectral density for the both filters was generated using different windows. The power spectrum describes the distribution of signal power over a frequency spectrum. The most common way of generating a power spectrum is by using a discrete Fourier transform, but other techniques such as the maximum entropy method can also be used. The power spectrum can also be defined as the Fourier transform of auto correlation function.

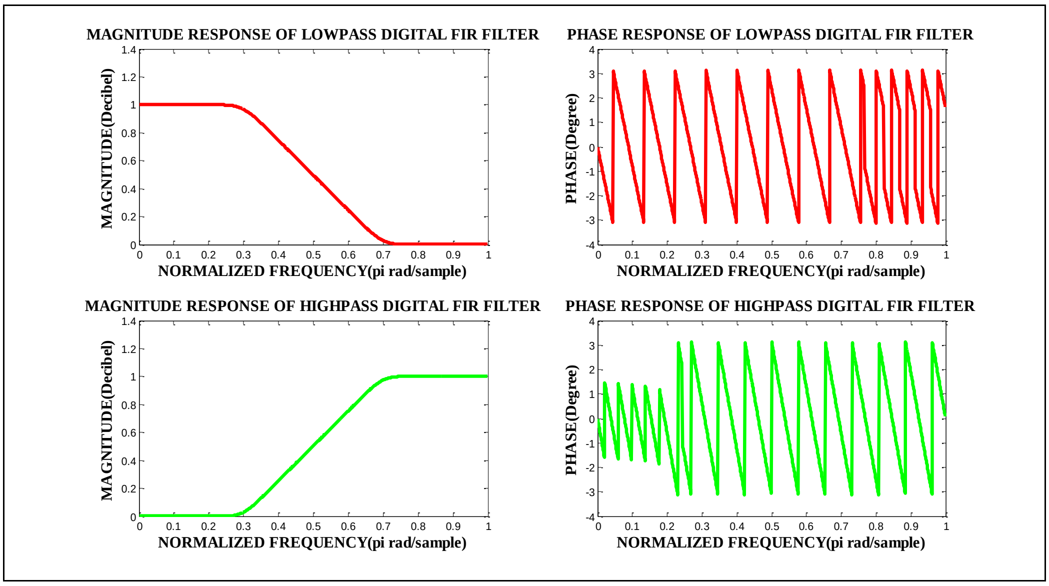

Figure 3: Frequency Response of Digital FIR Filter Using Frequency Sampling Method

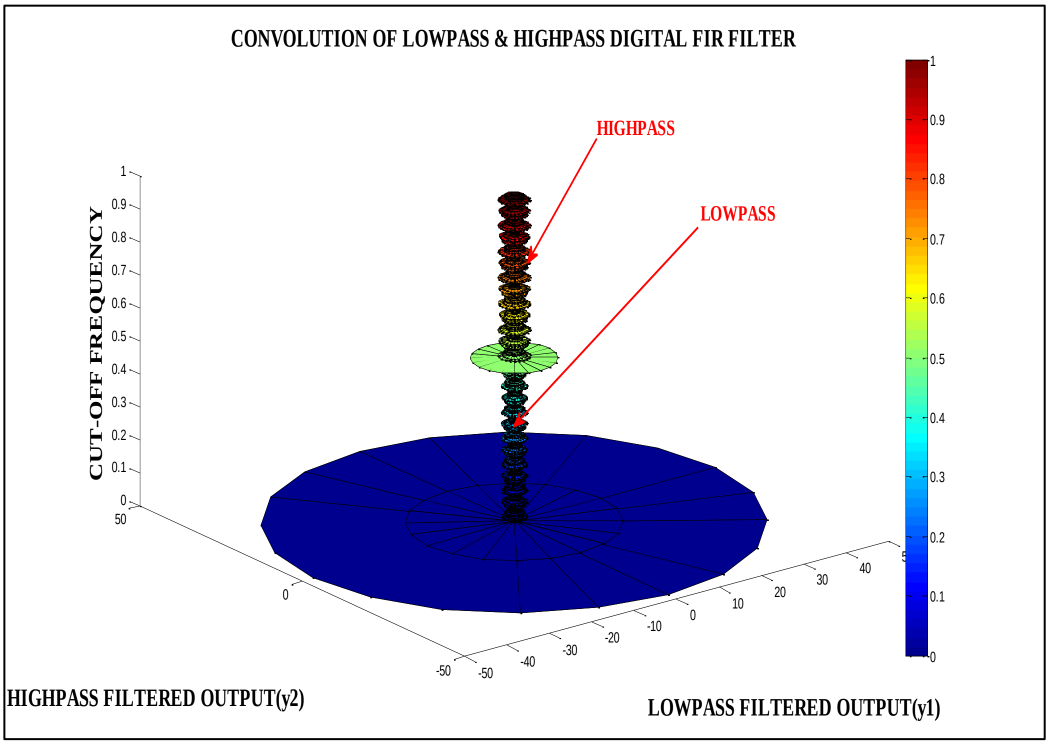

Figure 4: Convolution of Digital FIR Filter

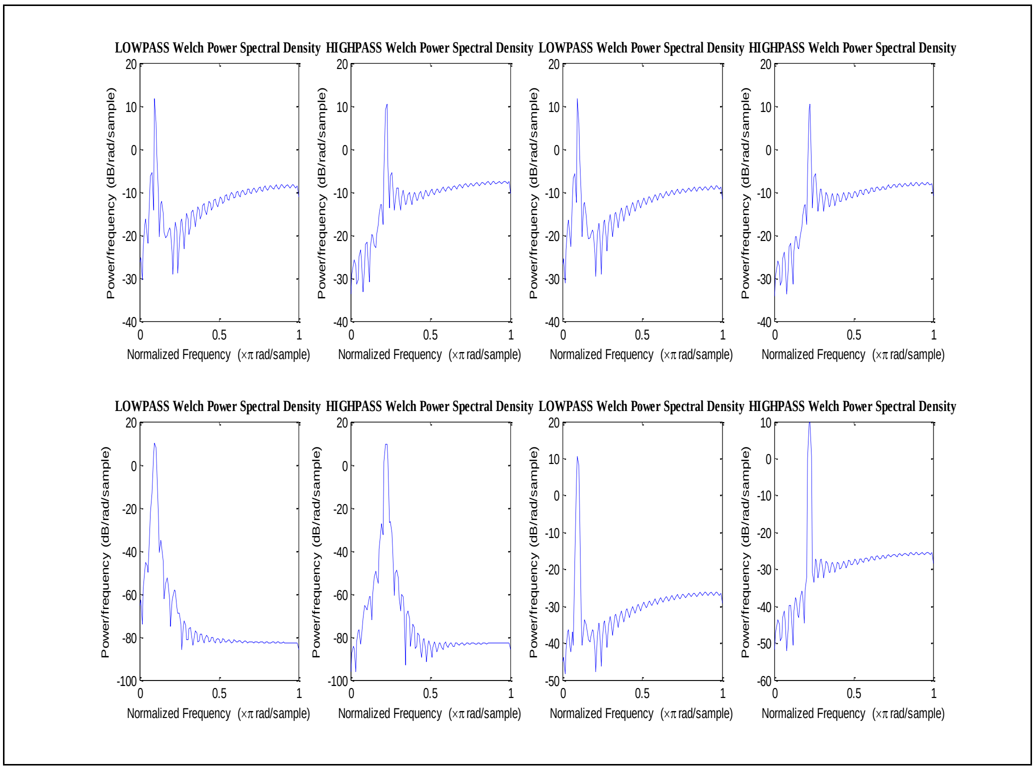

Figure 5: Power Spectrum of Digital FIR Filter Using Rectangular, Kaiser, Han, & Hamming Window Methods

4.3 Multi-rate Real-Time Implementation of Digital FIR Filter

4.3.1 Implementation of Decimation Process Using MATLAB

AIM: program to verify the decimation of given sequence

SOFTWARE: MATLAB

THEORY:

"Decimation" is the process of reducing the sampling rate**.**

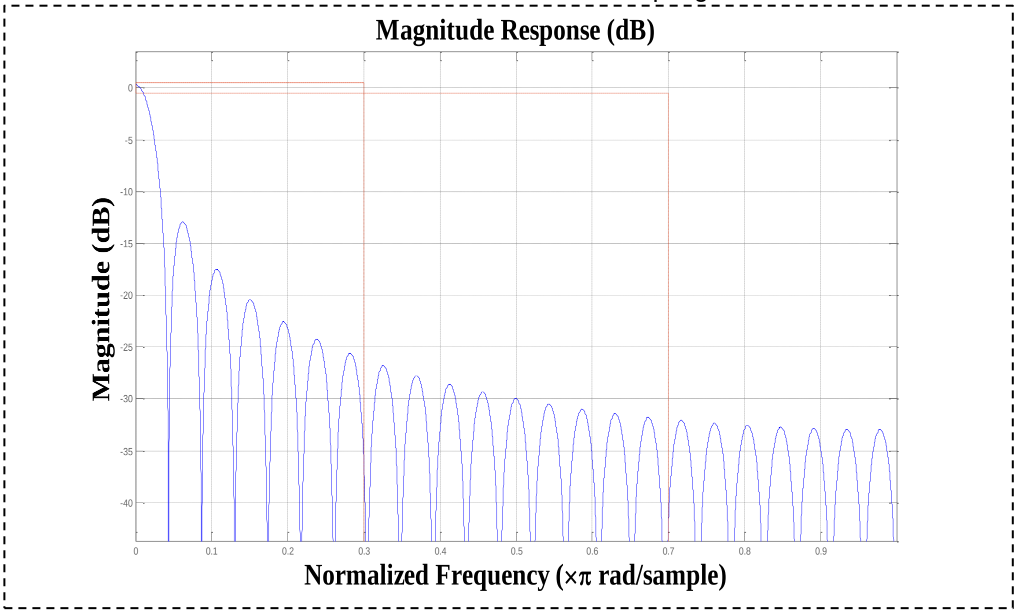

Figure 6: Direct-Form FIR Polyphase Decimator

"Down sampling" is a more specific term which refers to just the process of throwing away samples, without the low pass filtering operation. The direct form FIR Polyphase decimator of the Low Pass FIR Filter with filter order of 45 with decimation factor of 10 and pass band ripple of 1dB is shown in figure 6. The down sampling and decimation of both Low pass and High pass Digital FIR Filter are implemented in figure 7 and figure 8. This was done by assigning a suitable down sampling factor of 10 over a finite filter sampled length of 200, keeping the filter input sequence constant all through the implementation while the sampled output was reduced by 10%, preceded by filtering. The most immediate reason to decimate is simply to reduce the sampling rate at the output of one system so a system operating at a lower sampling rate can input the signal. But a much more common motivation for decimation is to reduce the cost of processing: the calculation and/or memory required to implement a DSP system generally is proportional to the sampling rate, so the use of a lower sampling rate usually results in a cheaper implementation.

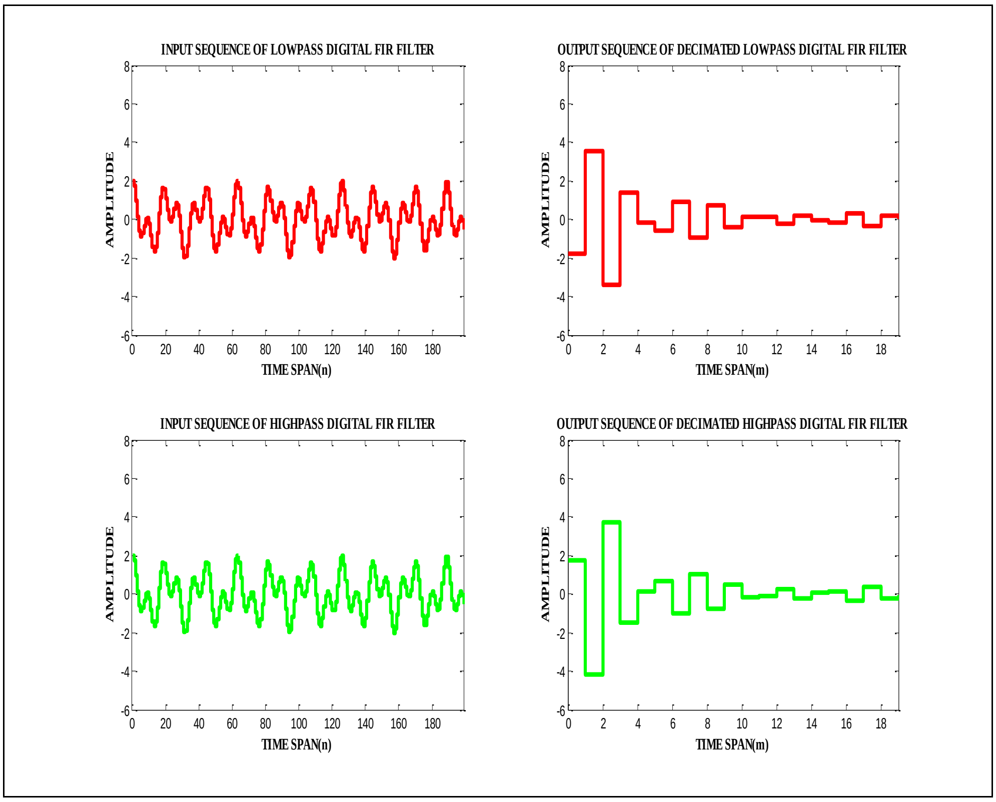

Figure 7: MATLAB Implementation of the Decimation Process

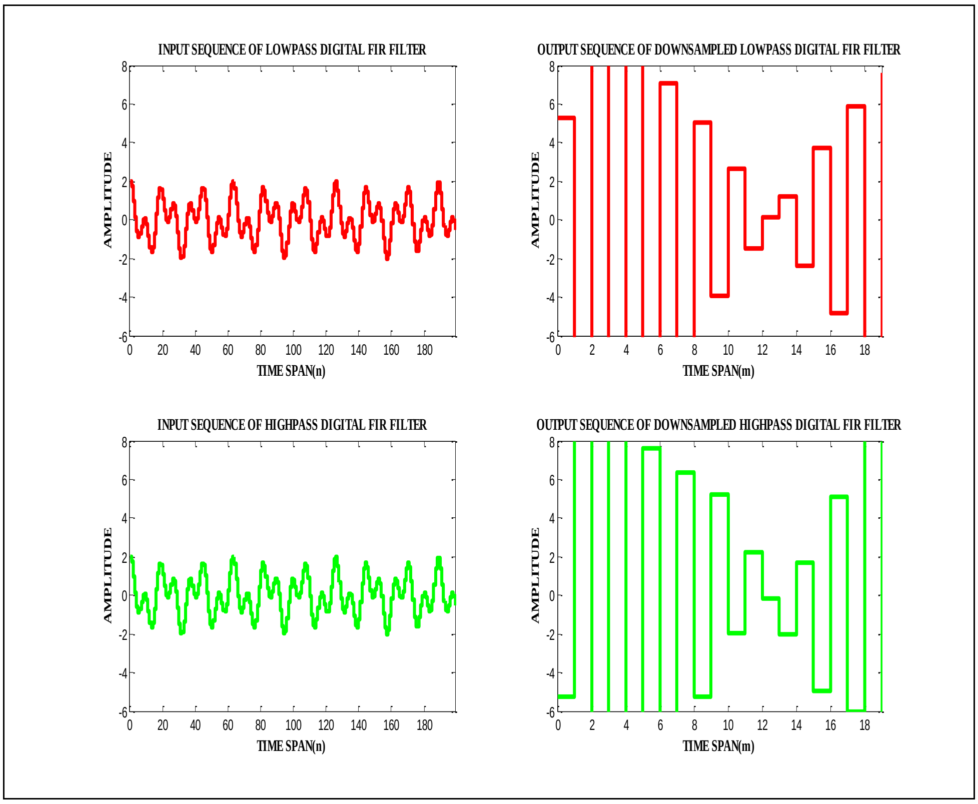

Figure 8: MATLAB Implementation of the Down sampling Process

4.6 Implementation of Interpolation Process Using MATLAB

AIM: program to verify the decimation of given sequence.

SOFTWARE: MATLAB

THEORY "Upsampling" is the process of inserting zero-valued samples between original samples to increase the sampling rate. (This is called "zero-stuffing".) "Interpolation", is the process of upsampling followed by filtering. The filtering removes the undesired spectral images. The primary reason to interpolate is simply to increase the sampling rate at the output of one system so that another system operating at a higher sampling rate can input the signal. The direct form FIR Polyphase interpolator of the High Pass FIR Filter with filter order of 52 with decimation factor of 10 and stopband attenuation of 60dB is shown in Figure 9.The up sampling and interpolation of both Low pass and High pass Digital FIR Filter are implemented in figure 10. This was done by assigning a suitable up sampling factor of 10 over a finite filter sampled length of 200, keeping the filter input sequence constant all through the implementation while the sampled output was increased to 1000% preceded by filtering.

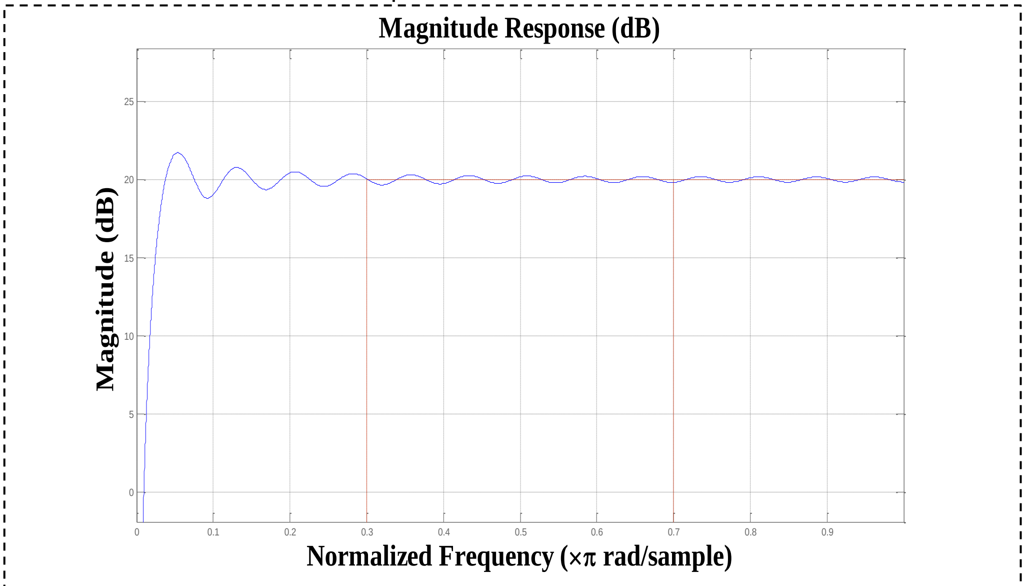

Figure 9: The Direct-Form FIR Polyphase interpolator

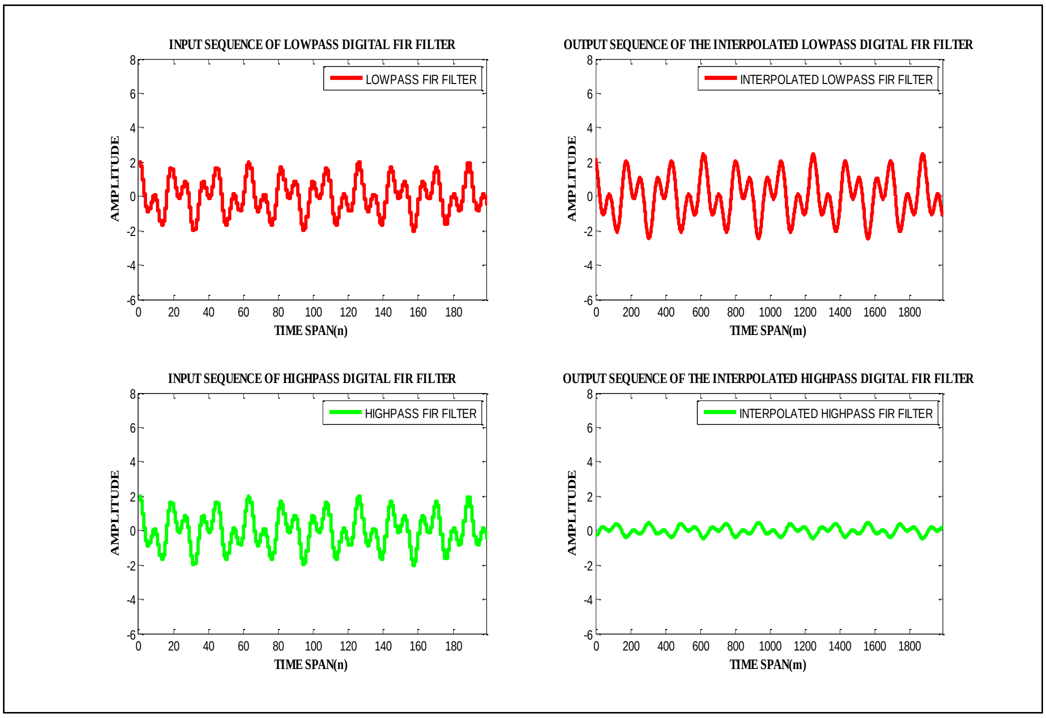

Figure 10: MATLAB Implementation of the Interpolation Process

4.6 Implementation of the I/D Sampling Rate Converter of the Lowpass & Highpass Digital FIR Filter Using MATLAB.

AIM: program to implement sampling rate conversion.

SOFTWARE: MATLAB

THEORY: "Multi-rate" means "multiple sampling rates". A multi-rate DSP system uses multiple sampling rates within the system. Whenever a signal at one rate has to be used by a system that expects a different rate, the rate has to be increased or decreased, and some processing is required to do so. Therefore "Multi-rate DSP" refers to the art or science of changing sampling rates.

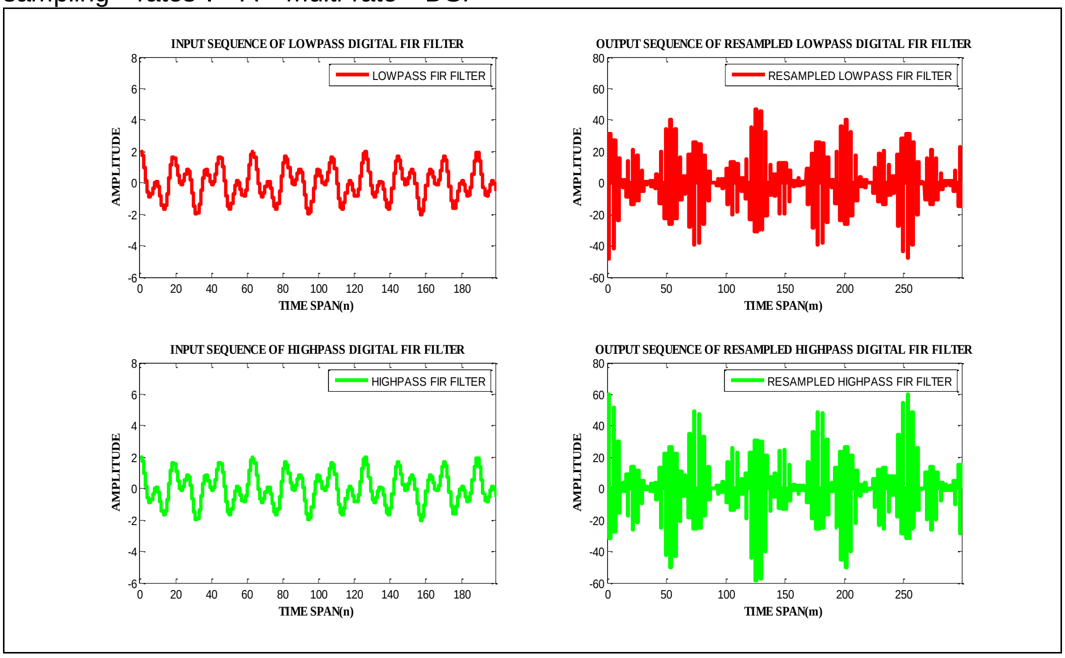

Figure 11: MATLAB Implementation of Resampling of Lowpass & Highpass Digital FIR Filter

"Resampling" means combining interpolation and decimation to change the sampling rate by a rational factor. Resampling is done to interface two systems with different sampling rates. C programming of the resampling was done using I/D factor of 2:3 over a finite filter sampled length of 200, keeping the filter input sequence constant all through the implementation while the resampled output was increased to 150% preceded by filtering. The analytical plot of the MATLAB program is shown in figure 11. The spline interpolation of the filter length is shown in figure 12.

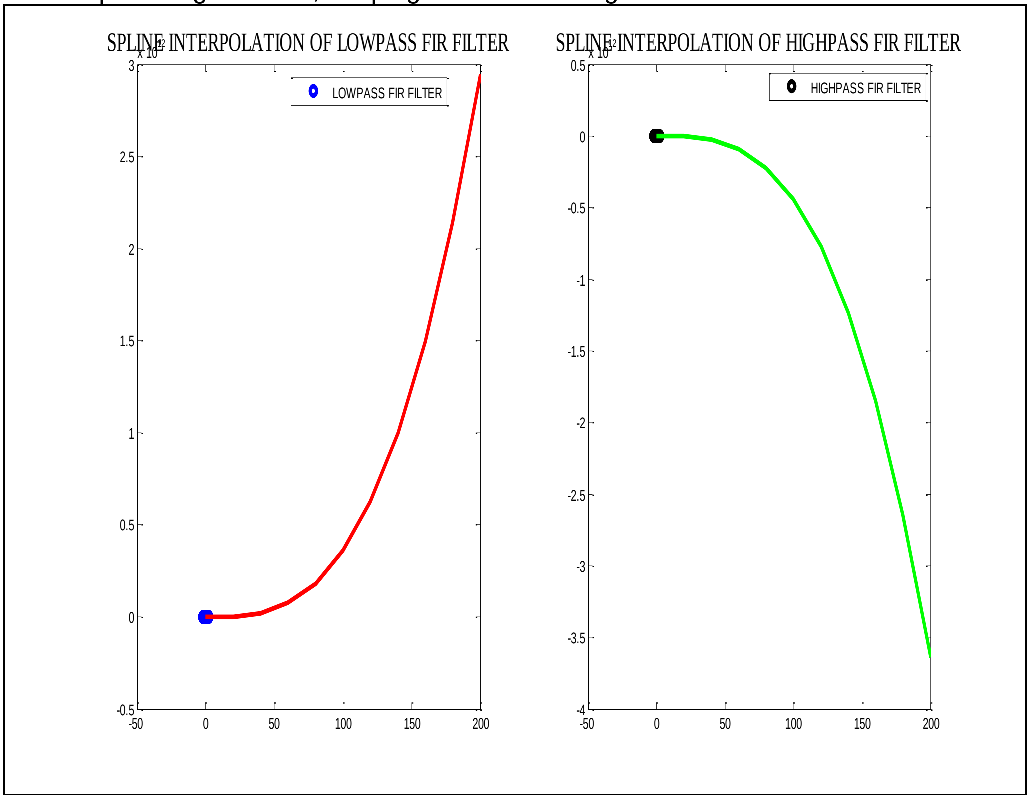

Figure 12: Spline Interpolation of Lowpass & Highpass Digital FIR Filter

4.6 Intelligent Implementation of Digital FIR Multi-rate Filter Banks

The underlying principles involved when the sampling frequency is changed from one value to another, and devices that can be used for the conversion, known as decimators and interpolators, are described Figure 6 and Figure 9 respectively. The application of decimators and interpolators in the design of filter banks is then considered especially in communications systems, spectrum analyzers, and speech synthesis with free ripple distortion for digital signal controls as regards sub-band coding of speech and images. Owing to the trend toward lower cost, higher speed microprocessors, digital solutions are becoming attractive for a wide variety of applications. Filter banks allow signals to be decomposed into sub-bands, often facilitating more efficient and effective processing. They are particularly visible in the areas of image compression, speech coding, and image analysis.

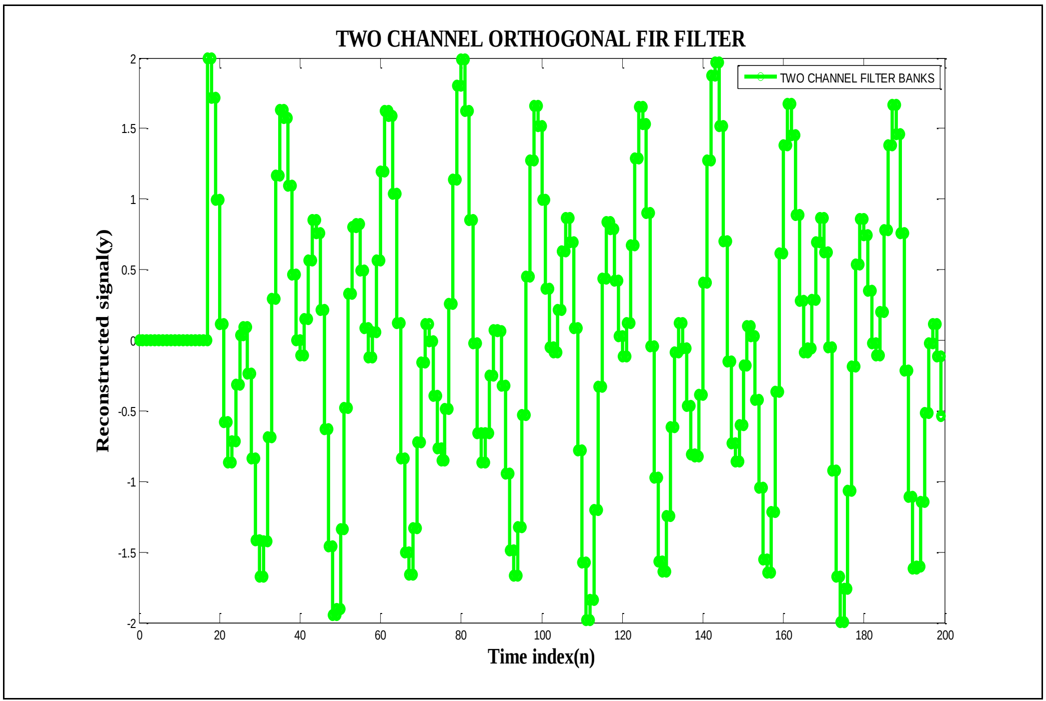

Figure 13: Two-Channel Reconstructed Orthogonal Lowpass/Highpass Digital FIR Filter

Figure 13 shows the implementation plot for Two-Channel Reconstructed Orthogonal Lowpass/Highpass Digital FIR Filter where the filter length was reduced by 9%. Filter design tool box and Signal processing tool box were employed to generate MATLAB codes for the Multi-rate System designs Chebyshev for the implementation of Reconstructed Signal of Lowpass/ Highpass Orthogonal Two-channel, and Perfect reconstructed Four-channel and Magnitude-preserving Five-channel Digital FIR Filter Bank through the 2nd-level convolution of their downsampling and upsampling filtering techniques as shown in figure 13, figure 14 and figure 15(a and b) respectively. The mean and standard deviation during the convolution and mutli-rate filtering id depicted in table 1 below.

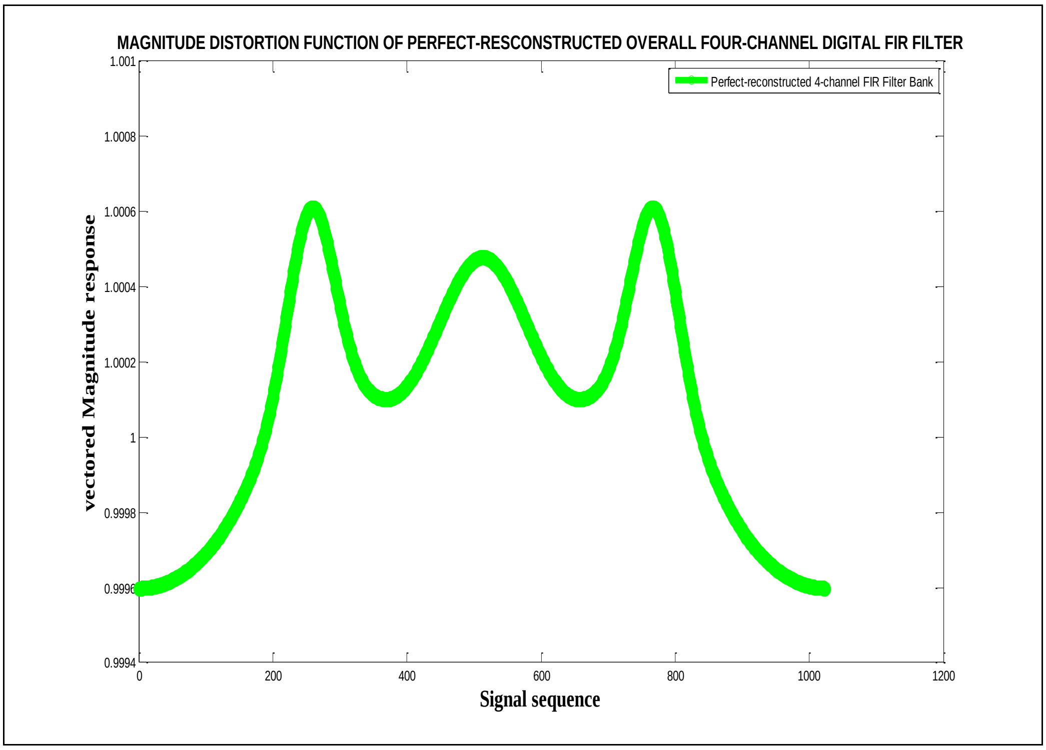

Figure 14: Four-Channel Perfect-Reconstructed Lowpass/Highpass Digital FIR Filter

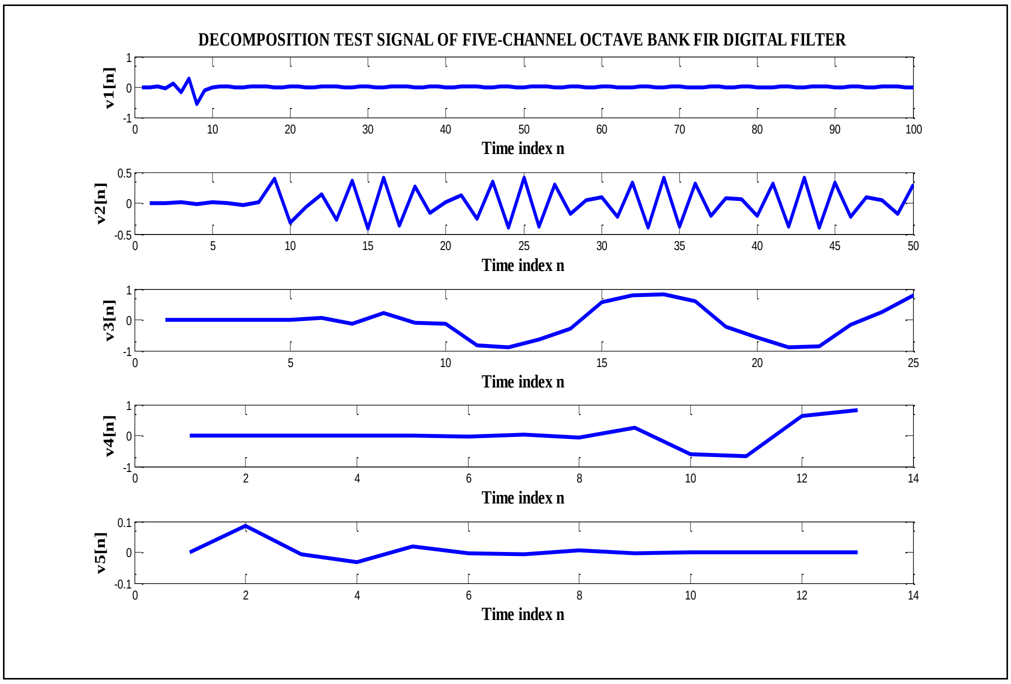

Figure 15a: Decomposition Test Signal of Five-Channel Octave Bank FIR Digital Filter

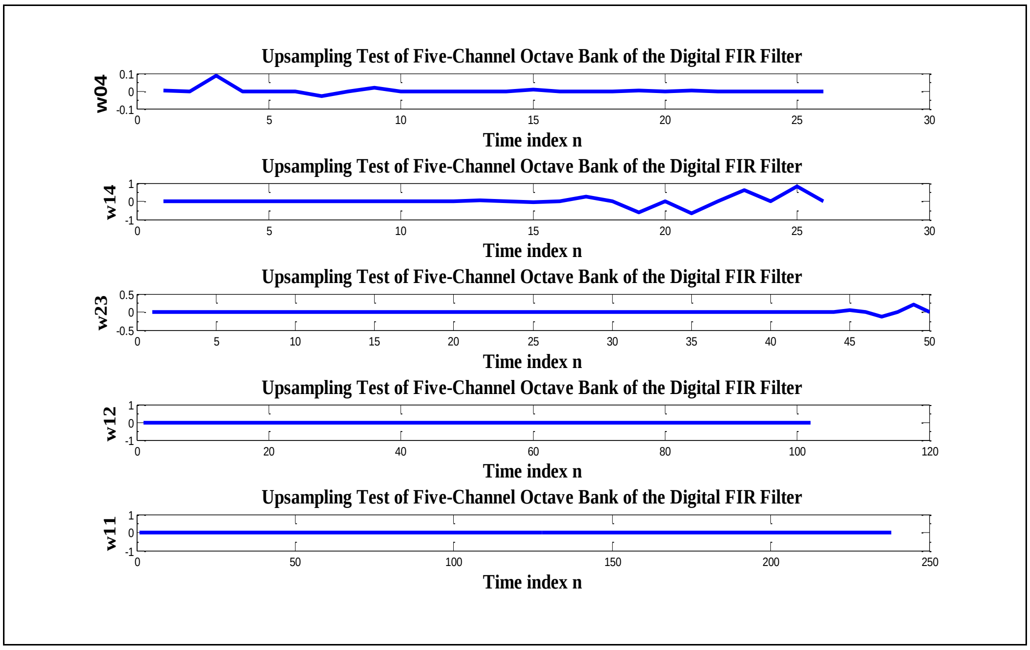

Figure 15b: Upsampling Test of Five-Channel Octave Bank of the Digital FIR Filter

Table 1: The Implemented Results Showing Their Mean and Standard Deviation

Digital Filtering		**Low Pass**	**High Pass**
Input sequence	Std	0.99	
	Mean	0.0095	
Output B/4 Convolutn	Std	0.83	0.8891
	Mean	0.009	0.0036
Output A/4 Convolutn	Std	3.30	
	Mean	0.0031	
Decimated Input	Std	1.030	
	Mean	0.144	
Decimated Output	Std	6.13	6.8285
	mean	0.26	-0.1855
Interpolated Input	std	0.99	
	Mean	0.0028	
Interpolated Output	Std	1.22	0.2761
	Mean	0.004	-7.2815

5. CONCLUSION

In this paper, multi-rate applications of Digital FIR filter in decimation and interpolation of sample rates with filter length of 200 samples was utilized by employing optimization criteria of parks-McClellan Algorithm and different window techniques in the design. The paper centers its discussion on low pass and high pass digital FIR filter by setting the cut-off frequency components at desired amplitudes and filter coefficients generated using MATLAB filter tool boxes and least square method as well as use of M-File for the programming of the software based filter. The creation of 2-4-5 magnitude-preserving channel through the 2nd-level convolution of their down sampling and up sampling filtering techniques helps to ensure perfect reconstructed signals by minimizing ripple distortion and error in the digital FIR Filter. Since this non-recursive filter requires no feedback, it can practically be applied in image compression, speech coding and image analysis.

5.1 Challenges/Problems encountered

The development of this article was posed with problems in the area of the type of optimization criteria to employ for the error/distortion control during the multi-rate filtering. More so, the use of the MATLAB tool boxes such as the filter tool and DSP tool boxes to filter speech and images was not easily achievable. The cost implication of adopting this research is high.

5.2 Further Research Work

This research work used the different optimization methods: frequency sampling, window techniques and Parks- McClellan Algorithm in the design. The research should extend to control applications of multi-rate systems in speech and image control analysis using fuzzy inferences. Hence, this work is subject to more research and development with a view to come up with a better system.

COMPETING INTERESTS

Authors have declared that no competing interests exist.

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