# Steady Flow of Blood Plasma through a Non-deformed Artery.

## Abstract

In this paper, the variation of axial velocity of blood plasma with arterial radius has been studied in the laminar flow of blood plasma through a non-deformed artery. Since blood plasma is a Newtonian fluid, the Hagen-Poiseuille flow was used to study this variation. The Hagen-Poiseuille model was obtained by transforming the Navier- Stokes equations from rectangular coordinates system to cylindrical coordinates system and then making some approximations to the transformed equations. Furthermore, the axial velocity of blood plasma was plotted against the radius of a sampled artery, results and conclusions were made between the relationships.

**Keywords**: Newtonian fluid, blood pressure, laminar flow, axial velocity.

## 1. Introduction

The human blood circulatory system provides essential substances such as nutrients and oxygen to the cells and transports metabolic waste products away from the same cells. Human blood is composed of blood cells suspended in blood plasma. The blood plasma which constitutes 55% of blood fluid, is mostly water (92% by volume), and contains dissipated proteins, glucose, mineral ions, hormones and blood cells themselves (Blessy Thomas and K.S Sunam, 2016). The blood cells are mainly red blood cells (also called RBCs or erythrocytes) and white blood cells, including leukocytes and platelets. The red blood cells are small semisolid particles, increase the viscosity of blood and will affect the behaviour of fluid. It has been noted that plasma behaves as a Newtonian fluid (Verma and Parihar, 2010, Biswas and Chakraborty, 2010, Srivastava et. al, 2010) whereas the whole blood displays non-Newtonian character (Cokelet et. al, 1963, Goldsmith and Skalak, 1975).

There are three major types of blood vessels: the arteries through which blood is carried away from the heart at higher physiologic pressures, the capillaries, which enable the actual exchange of water and chemicals between the blood and the tissues, and the veins, which carry blood from the capillaries and back toward the heart at lower physiologic pressures. Because of their different roles, their structures and wall constituents are also different. The walls of blood vessels have a circumferentially layered structure. The most important layers are intima, media, and adventitia. The internal intima composed of the endothelium cell. The media, which is a layered one, is responsible for most of the vessel mechanical properties. The outer layer is adventitia. The artery possesses the thickest wall amongst the three major blood vessels which enables them to withstand the high pressure of arterial blood. It has a more elastic media which varies according to the size of the artery, with a thin collagenous adventitia compared to both the veins and capillaries.

Laminar flow of fluid is characterized by fluid particles following smooth paths in layers, with each layer moving smoothly past the adjacent layers with little or no mixing. Arterial blood flow is considered as a laminar flow. The study of the laminar flow of blood in arteries plays an important role in the diagnosis and clinical treatment as well as in the fundamental understanding of many cardiovascular diseases. One of the cardiovascular diseases is stenosis which is caused by an abnormal growth along the lumen of an arterial wall (Young and Tsai, 1973). Due to the presence of stenosis in an artery, the bore of the vessel is reduced and as a result, normal blood flow is disturbed appreciably. Researchers studying the blood flow through the artery usually make use of the casson model.

The Casson model for blood flow through a cylindrical tube states that

$$\sqrt{\tau} = \sqrt{\tau_y} + \sqrt{\eta\dot{\gamma}} \tag{1}$$

Equation (1) (David A. Rubenstein, 2012, p.145) is the relationship between shear stress ($\tau$) and shear rate ($\dot{\gamma}$), where $\tau_y$ is a constant yield stress and $\eta$ is an experimentally fit constant which approximates the fluid's viscosity ($\eta \cong \mu$). The shear stress is defined as

$$\tau = -\frac{r}{2}\frac{dp}{dx} \tag{2}$$

The Casson model can be rewritten as

$$\sqrt{-\frac{r}{2}\frac{dp}{dx}} = \sqrt{\tau_y} + \sqrt{\eta\dot{\gamma}} \tag{3}$$

Solving (3) for the shear rate, we have

$$-\frac{du}{dr} = \dot{\gamma} = \frac{1}{\eta}\left(\sqrt{-\frac{r}{2}\frac{dp}{dx}} - \sqrt{\tau_y}\right)^2 \tag{4}$$

Integrating (4) from $r_y$ to $R$ (tube radius), over the appropriate cross-sectional area, we have the velocity profile as

$$u(r) = \begin{cases} -\frac{1}{4\eta}\frac{dp}{dx}\left(R^2 - r^2 - \frac{8}{3}r_y{}^{0.5}(R^{1.5} - r^{1.5}) + 2r_y(R - r)\right), r_y \le r \le R \\ -\frac{1}{4\eta}\frac{dp}{dx}(\sqrt{R} - \sqrt{r_y})^3\left(\sqrt{R} + \frac{1}{3}\sqrt{r_y}\right),\ r \le r_y \end{cases} \tag{5}$$

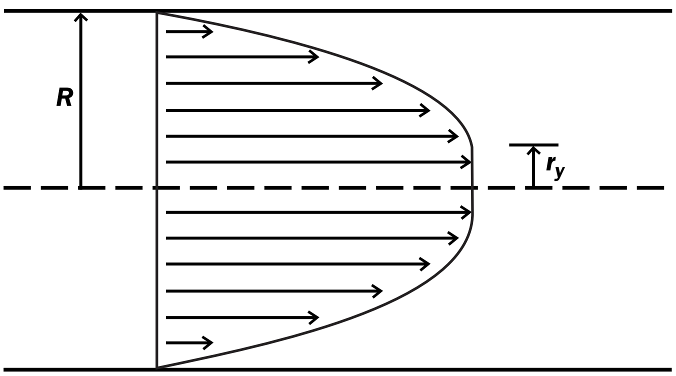



**Figure 1.** The velocity profile of blood flowing through a cylindrical vessel using the Casson model. The profile is blunter than a purely Newtonian fluid's velocity profile because the fluid that is between the centreline and $r_y$ flows as a solid. At $r_y$, the shear stress ($\tau$) exceeds the yield stress ($\tau_y$) and the viscous forces take effect.

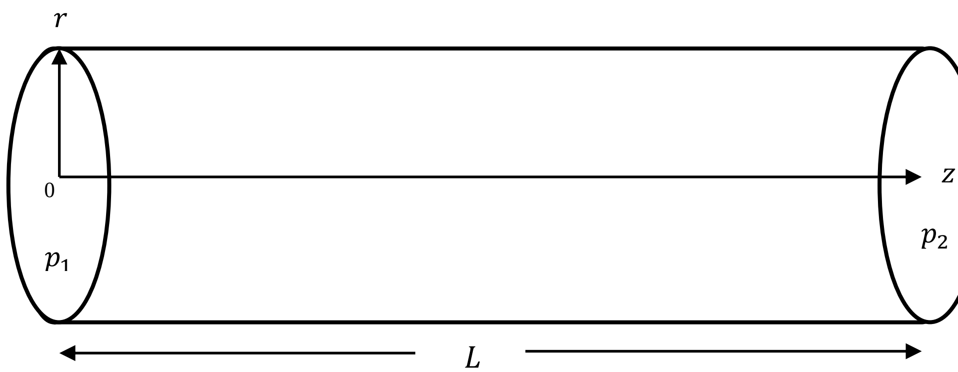
The Casson model is a very important tool in the representation of blood flow through the arterial system. We make an approximation to the Casson model by allowing the yield stress, $\tau_y = 0$. Equation (5) becomes

$$u(r) = -\frac{1}{4\eta}\frac{dp}{dx}(R^2 - r^2), \quad 0 \le r \le R \tag{6}$$

Since blood plasma behaves as a Newtonian fluid, firstly, we show that the axial velocity of blood plasma satisfy equation (6) and hence treat the flow of blood plasma through a non-deformed artery as a Hagen-Poiseuille flow.

## 2. Mathematical Formulation

Consider the one dimensional steady flow of blood plasma through a cross-section of artery of radius $r_0$, centre $o$ and length $L$.




The equations of motion for the laminar flow of blood plasma through a cross section of the artery, is given by the .Navier-Stokes equations;

$$\nabla.\vec{u} = 0 \tag{7}$$

$$\frac{D\vec{u}}{Dt} = -\frac{1}{\rho}\nabla \mathrm{p} + \ v\nabla^2\vec{u} \tag{8}$$

Equations (7) and (8) are the continuity equation and momentum equation respectively.

The boundary conditions for the solutions of equations (7) and (8) are

$$\vec{u}(r) = u \tag{9a}$$

$$\vec{u}(r_0) = 0 \tag{9b}$$

Since we are considering the flow of blood plasma through the artery (which has a cylindrical structure), we transform equations (7) and (8) from rectangular coordinates $(x, y, z)$ to cylindrical coordinates $(r, \theta, z)$.

Let $e_{\mathrm{r}}$, $e_{\theta}$ and $e_{\mathrm{z}}$ be unit vectors and $h_r$, $h_\theta$ and $h_{\mathrm{z}}$ be scale factors in the cylindrical coordinates. In cylindrical coordinates,

$$h_r = h_z = 1,\ h_\theta = r$$

$$\vec{u} = u_r e_r + u_\theta e_\theta + u_z e_z$$

$u_r, u_\theta, u_z$ are the radial velocity, azimuthal velocity and axial velocity respectively.

$$\nabla \equiv \frac{e_r}{h_r}\frac{\partial}{\partial r} + \frac{e_\theta}{h_\theta}\frac{\partial}{\partial \theta} + \frac{e_z}{h_z}\frac{\partial}{\partial z}$$

$$\nabla.\vec{u} = \left[\frac{1}{h_r}\frac{\partial}{\partial r} e_r + \frac{1}{h_\theta}\frac{\partial}{\partial \theta}\ e_\theta + \frac{1}{h_z}\frac{\partial}{\partial z} e_z\right].[u_r e_r + u_\theta e_\theta + u_z e_z]$$

$$= \frac{1}{h_r}\frac{\partial u_r}{\partial r} + \frac{1}{h_\theta}\frac{\partial u_\theta}{\partial \theta} + \frac{1}{h_z}\frac{\partial u_z}{\partial z}$$

$$= \frac{1}{h_r h_\theta h_z}\left[\frac{\partial(h_\theta h_z u_r)}{\partial r} + \frac{\partial(h_r h_z u_\theta)}{\partial \theta} + \frac{\partial(h_r h_\theta u_z)}{\partial z}\right]$$

$$\nabla.\vec{u} = \frac{1}{r}\left[\frac{\partial(r u_r)}{\partial r} + \frac{\partial(u_\theta)}{\partial \theta} + \frac{\partial(r u_z)}{\partial z}\right] = 0$$

$$\nabla.\vec{u} = \frac{1}{r}\frac{\partial(r u_r)}{\partial r} + \frac{1}{r}\frac{\partial u_\theta}{\partial \theta} + \frac{\partial u_z}{\partial z} = 0 \qquad (10)$$

Equation (10) is the continuity equation in cylindrical coordinates for the steady flow of blood plasma through the artery.

Again, returning to the momentum equation

$$\frac{D\vec{u}}{Dt} = -\frac{1}{\rho}\nabla \mathrm{p} + \nu \nabla^2 \vec{u} \qquad (11)$$

But

$$\frac{D\vec{u}}{Dt} = \frac{\partial \vec{u}}{\partial t} + \nabla\left(\frac{1}{2}u^2\right) - \vec{u} \times (\nabla \times \vec{u})$$

Also

$$\nabla^2 \vec{u} = \nabla(\nabla.\vec{u}) - \nabla \times (\nabla \times \vec{u})$$

$$= -\nabla \times\ \omega$$

Where $\omega = \nabla \times \vec{u}$ is the vorticity vector, (11) can be written as

$$\frac{\partial \vec{u}}{\partial t} - \vec{u} \times \omega = -\nabla\left(\frac{\mathrm{p}}{\rho} + \frac{1}{2}u^2\right) - \nu(\nabla \times \omega)$$

$$\frac{\partial \vec{u}}{\partial t} + \nabla\left(\frac{1}{2}u^2\right) - \vec{u} \times \omega = -\frac{1}{\rho}\nabla \mathrm{p} - \nu(\nabla \times \omega) \tag{12}$$

Consider

$$\nabla \times \vec{u} = \omega_r e_r + \omega_\theta e_\theta + \omega_z e_z$$

$$= \begin{vmatrix} e_r & e_\theta & e_z \\ \frac{1}{h_r}\frac{\partial}{\partial r} & \frac{1}{h_\theta}\frac{\partial}{\partial \theta} & \frac{1}{h_z}\frac{\partial}{\partial z} \\ u_r & u_\theta & u_z \end{vmatrix}$$

$$= \begin{vmatrix} e_r & e_\theta & e_z \\ \frac{\partial}{\partial r} & \frac{1}{r}\frac{\partial}{\partial \theta} & \frac{\partial}{\partial z} \\ u_r & u_\theta & u_z \end{vmatrix}$$

Hence $\omega_r$, $\omega_\theta$ and $\omega_z$ becomes

$$\omega_r = \frac{1}{r}\left(\frac{\partial}{\partial \theta}(u_z) - \frac{\partial}{\partial z}(r u_\theta)\right) = \frac{1}{r}\frac{\partial u_z}{\partial \theta} - \frac{\partial u_\theta}{\partial z}$$

$$\omega_\theta = \frac{\partial}{\partial z}(u_r) - \frac{\partial}{\partial r}(u_z) = \frac{\partial u_r}{\partial z} - \frac{\partial u_z}{\partial r}$$

$$\omega_z = \frac{1}{r}\left(\frac{\partial}{\partial r}(r u_\theta) - \frac{\partial}{\partial \theta}(u_r)\right) = \frac{u_\theta}{r} + \frac{\partial u_\theta}{\partial r} - \frac{1}{r}\frac{\partial u_r}{\partial \theta}$$

Similarly for curl $\omega$,

$$\nabla \times \omega = \xi_r e_r + \xi_\theta e_\theta + \xi_z e_z$$

$$\nabla \times \omega = \frac{1}{r}\begin{vmatrix} e_r & e_\theta & e_z \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial z} \\ \frac{1}{r}\frac{\partial u_z}{\partial \theta} - \frac{\partial u_\theta}{\partial z} & \frac{r\partial u_r}{\partial z} - \frac{r\partial u_z}{\partial r} & \frac{u_\theta}{r} + \frac{\partial u_\theta}{\partial r} - \frac{1}{r}\frac{\partial u_r}{\partial \theta} \end{vmatrix}$$

$$\xi_r = \frac{1}{r}\left[\frac{\partial}{\partial \theta}\left(\frac{u_\theta}{r} + \frac{\partial u_\theta}{\partial r} - \frac{1}{r}\frac{\partial u_r}{\partial \theta}\right) - \frac{\partial}{\partial z}\left(\frac{r\partial u_r}{\partial z} - \frac{r\partial u_z}{\partial r}\right)\right]$$

$$\xi_\theta = \frac{\partial}{\partial z}\left(\frac{1}{r}\frac{\partial u_z}{\partial \theta} - \frac{\partial u_\theta}{\partial z}\right) - \frac{\partial}{\partial r}\left(\frac{u_\theta}{r} + \frac{\partial u_\theta}{\partial r} - \frac{1}{r}\frac{\partial u_r}{\partial \theta}\right)$$

$$\xi_z = \frac{1}{r}\left[\frac{\partial}{\partial r}\left(\frac{r\partial u_r}{\partial z} - \frac{r\partial u_z}{\partial r}\right) - \frac{\partial}{\partial \theta}\left(\frac{1}{r}\frac{\partial u_z}{\partial \theta} - \frac{\partial u_\theta}{\partial z}\right)\right]$$

In determining the term $-\nu(\nabla \times \omega)$ in (12), it is found that the analysis simplifies if the zero expression $\nu(\nabla(\nabla.\vec{u}))$ is added

Recall equation (10)

$$\nabla.\vec{u} = \frac{1}{r}\frac{\partial}{\partial r}(ru_r) + \frac{1}{r}\frac{\partial u_\theta}{\partial \theta} + \frac{\partial u_z}{\partial z} = 0$$

$$= u_r\frac{\partial r}{\partial r} + r\frac{\partial u_r}{\partial r} + \frac{\partial u_\theta}{\partial \theta} + r\frac{\partial u_z}{\partial z} = 0$$

$$= u_r + r\frac{\partial u_r}{\partial r} + \frac{\partial u_\theta}{\partial \theta} + r\frac{\partial u_z}{\partial z} = 0$$

Dividing through by $r$, we obtain

$$\nabla.\vec{u} = \frac{u_r}{r} + \frac{\partial u_r}{\partial r} + \frac{1}{r}\frac{\partial u_\theta}{\partial \theta} + \frac{\partial u_z}{\partial z}$$

Therefore $\nabla(\nabla.\vec{u})$, has components

$$\frac{\partial}{\partial r}\left(\frac{u_r}{r} + \frac{\partial u_r}{\partial r} + \frac{1}{r}\frac{\partial u_\theta}{\partial \theta} + \frac{\partial u_z}{\partial z}\right)$$

$$\frac{1}{r}\frac{\partial}{\partial \theta}\left(\frac{u_r}{r} + \frac{\partial u_r}{\partial r} + \frac{1}{r}\frac{\partial u_\theta}{\partial \theta} + \frac{\partial u_z}{\partial z}\right)$$

$$\frac{\partial}{\partial z}\left(\frac{u_r}{r} + \frac{\partial u_r}{\partial r} + \frac{1}{r}\frac{\partial u_\theta}{\partial \theta} + \frac{\partial u_z}{\partial z}\right)$$

Now, $v(\nabla(\nabla.\vec{u}) - \nabla \times \omega)$, has components

$$v\left(\frac{\partial^2 u_r}{\partial r^2} + \frac{1}{r}\frac{\partial u_r}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u_r}{\partial \theta^2} + \frac{\partial^2 u_r}{\partial z^2} - \frac{u_r}{r^2} - \frac{2}{r^2}\frac{\partial u_\theta}{\partial \theta}\right) = v\left(\nabla^2 u_r - \frac{u_r}{r^2} - \frac{2}{r^2}\frac{\partial u_\theta}{\partial \theta}\right)$$

$$v\left(\frac{\partial^2 u_\theta}{\partial r^2} + \frac{1}{r}\frac{\partial u_\theta}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u_\theta}{\partial \theta^2} + \frac{\partial^2 u_\theta}{\partial z^2} - \frac{u_\theta}{r^2} - \frac{2}{r^2}\frac{\partial u_r}{\partial \theta}\right) = v\left(\nabla^2 u_\theta - \frac{u_\theta}{r^2} - \frac{2}{r^2}\frac{\partial u_r}{\partial \theta}\right) \quad (13)$$

$$v\left(\frac{\partial^2 u_z}{\partial r^2} + \frac{1}{r}\frac{\partial u_z}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u_z}{\partial \theta^2} + \frac{\partial^2 u_z}{\partial z^2}\right) = v(\nabla^2 u_z)$$

Where $\nabla^2 \equiv \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2} + \frac{\partial^2}{\partial z^2}$

Similarly, we obtain that $\nabla\left(\frac{1}{2}u^2\right) - u \times \omega$ has components

$$\frac{\partial}{\partial r}\left(\frac{1}{2}{u_r}^2 + \frac{1}{2}{u_\theta}^2 + \frac{1}{2}{u_z}^2\right) - u_\theta\left(\frac{u_\theta}{r} + \frac{\partial u_\theta}{\partial r} - \frac{1}{r}\frac{\partial u_r}{\partial \theta}\right) + u_z\left(\frac{\partial u_r}{\partial z} - \frac{\partial u_z}{\partial r}\right)$$

$$\frac{1}{r}\frac{\partial}{\partial \theta}\left(\frac{1}{2}{u_r}^2 + \frac{1}{2}{u_\theta}^2 + \frac{1}{2}{u_z}^2\right) - u_z\left(\frac{1}{r}\frac{\partial u_z}{\partial \theta} - \frac{\partial u_\theta}{\partial z}\right) + u_r\left(\frac{u_\theta}{r} + \frac{\partial u_\theta}{\partial r} - \frac{1}{r}\frac{\partial u_r}{\partial \theta}\right)$$

$$\frac{\partial}{\partial z}\left(\frac{1}{2}{u_r}^2 + \frac{1}{2}{u_\theta}^2 + \frac{1}{2}{u_z}^2\right) - u_r\left(\frac{\partial u_r}{\partial z} - \frac{\partial u_z}{\partial r}\right) + u_\theta\left(\frac{1}{r}\frac{\partial u_z}{\partial \theta} - \frac{\partial u_\theta}{\partial z}\right)$$

Simplifying to

$$u_r\frac{\partial u_r}{\partial r} + \frac{u_\theta}{r}\frac{\partial u_r}{\partial \theta} + u_z\frac{\partial u_r}{\partial z} - \frac{{u_\theta}^2}{r}$$

$$u_r\frac{\partial u_\theta}{\partial r} + \frac{u_\theta}{r}\frac{\partial u_\theta}{\partial \theta} + u_z\frac{\partial u_\theta}{\partial z} - \frac{u_r u_\theta}{r} \qquad (14)$$

$$u_r\frac{\partial u_z}{\partial r} + \frac{u_\theta}{r}\frac{\partial u_z}{\partial \theta} + u_z\frac{\partial u_z}{\partial z}$$

Substituting (12) and (13) into equation (11), the momentum equation becomes

$$\frac{\partial u_r}{\partial t} + \frac{u_r u_\theta}{r}\frac{\partial u_r}{\partial r} + \frac{u_\theta}{r}\frac{\partial u_r}{\partial \theta} + u_z\frac{\partial u_r}{\partial z} - \frac{{u_\theta}^2}{r} = -\frac{1}{\rho}\frac{\partial \mathrm{p}}{\partial r} + v\left(\nabla^2 u_r - \frac{u_r}{r^2} - \frac{2}{r^2}\frac{\partial u_\theta}{\partial \theta}\right) \qquad (15)$$

$$\frac{\partial u_\theta}{\partial t} + u_r\frac{\partial u_\theta}{\partial r} + \frac{u_\theta}{r}\frac{\partial u_\theta}{\partial \theta} + u_z\frac{\partial u_\theta}{\partial z} + \frac{u_r u_\theta}{r} = -\frac{1}{\rho r}\frac{\partial \mathrm{p}}{\partial \theta} + v\left(\nabla^2 u_\theta - \frac{u_\theta}{r^2} - \frac{2}{r^2}\frac{\partial u_r}{\partial \theta}\right) \qquad (16)$$

$$\frac{\partial u_z}{\partial t} + u_r\frac{\partial u_z}{\partial r} + \frac{u_\theta}{r}\frac{\partial u_z}{\partial \theta} + u_z\frac{\partial u_z}{\partial z} = -\frac{1}{\rho}\frac{\partial \mathrm{p}}{\partial z} + v\nabla^2 u_z \qquad (17)$$

Equations (15), (16) and (17) are the $r$, $\theta$ and $z$ coordinates respectively of the momentum equation in cylindrical coordinates.

## 3. Method of Solution

The motion of plasma in the artery is induced by axial pressure gradient. It is called a Hagen-Poiseuille flow.

Since the blood plasma flow parallel to the axis of the artery

$$u_\theta = u_r = 0, \;\; u_z \neq 0$$

From the continuity equation

$$\frac{1}{r}\frac{\partial (r u_r)}{\partial r} + \frac{1}{r}\frac{\partial u_\theta}{\partial \theta} + \frac{\partial u_z}{\partial z} = 0$$

$$\frac{\partial u_z}{\partial z} = 0$$

Therefore $u_z$ is a function of $r$ alone $\left(u_z \neq u_z(z)\right)$

Since the flow is steady, it holds that $\frac{\partial \vec{u}}{\partial t} = 0$

From the momentum equation,

Equation (15) becomes

$$-\frac{1}{\rho}\frac{\partial \mathrm{p}}{\partial r} = 0 \tag{18}$$

Equation (16) becomes

$$-\frac{1}{\rho r}\frac{\partial \mathrm{p}}{\partial \theta} = 0 \tag{19}$$

Equation (17) becomes

$$-\frac{1}{\rho}\frac{\partial \mathrm{p}}{\partial z} + v(\nabla^2 u_z) = 0$$

$$-\frac{1}{\rho}\frac{\partial \mathrm{p}}{\partial z} + v\left(\frac{\partial^2 u_z}{\partial r^2} + \frac{1}{r}\frac{\partial u_z}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u_z}{\partial \theta^2} + \frac{\partial^2 u_z}{\partial z^2}\right) = 0$$

$$-\frac{1}{\rho}\frac{\partial \mathrm{p}}{\partial z} + v\left(\frac{\partial^2 u_z}{\partial r^2} + \frac{1}{r}\frac{\partial u_z}{\partial r}\right) = 0$$

$$-\frac{\partial \mathrm{p}}{\partial z} + \mu\left[\frac{1}{\mathrm{r}}\frac{\partial}{\partial r}\left(r\frac{\partial u_z}{\partial r}\right)\right] = 0 \tag{20}$$

From (18) and (19)

$$\mathrm{p} \neq \mathrm{p(r)},\ \ \mathrm{p} \neq \mathrm{p}(\theta)$$

$$\mathrm{p} = \mathrm{p(z)},\ \ \frac{\partial \mathrm{p}}{\partial z} = \frac{d\mathrm{p}}{dz} = \mathrm{constant}$$

From (19)

$$-\frac{d\mathrm{p}}{dz} + \mu\left[\frac{1}{\mathrm{r}}\frac{\partial}{\partial r}\left(r\frac{\partial u_z}{\partial r}\right)\right] = 0$$

$$\mu\left[\frac{1}{\mathrm{r}}\frac{\partial}{\partial r}\left(r\frac{\partial u_z}{\partial r}\right)\right] = \frac{d\mathrm{p}}{dz}$$

$$\frac{\partial}{\partial r}\left(r\frac{\partial u_z}{\partial r}\right) = \frac{\mathrm{r}}{\mu}\frac{d\mathrm{p}}{dz}$$

$$r\frac{\partial u_z}{\partial r} = \frac{\mathrm{r}^2}{2\mu}\frac{d\mathrm{p}}{dz} + c_1 \tag{21}$$

$$\frac{\partial u_z}{\partial r} = \frac{\mathrm{r}}{2\mu}\frac{d\mathrm{p}}{dz} + \frac{c_1}{\mathrm{r}}$$

$$u_z(r) = \frac{\mathrm{r}^2}{4\mu}\frac{d\mathrm{p}}{dz} + c_1 \ln r + c_2 \tag{22}$$

Boundary conditions

$$u_z \text{ is finite, } r = 0 \text{ (along the axis)} \tag{23a}$$

$$u_z = 0,\ r = r_0 \text{ (no slip condition)} \tag{23b}$$

Applying the boundary conditions

$$c_1 = 0 \tag{24}$$

$$c_2 = -\frac{{r_0}^2}{4\mu}\frac{dp}{dz} \tag{25}$$

Substituting (24) and (25) into (22), we have

$$u_z = \frac{r^2}{4\mu}\frac{dp}{dz} - \frac{{r_0}^2}{4\mu}\frac{dp}{dz}$$

$$= \frac{1}{4\mu}\frac{dp}{dz}(r^2 - {r_0}^2),\ 0 \le r \le r_0 \tag{26}$$

Obviously equation (26) agrees with equation (6).

But $\frac{dp}{dz} = \frac{p_2 - p_1}{L}$, $p = p_1$ at z = 0 and $p = p_2$ at z = L

Where L is the length of the cross-section of the artery and $p_1$, $p_2$ are the pressures at both ends of the cross-section of the artery.

Equation (26) can be rewritten as

$$u_z(r) = \frac{p_2 - p_1}{4\mu L}(r^2 - {r_0}^2),\ 0 \le r \le r_0 \tag{27}$$

Equation (27) is the axial velocity of blood plasma through a cross-section of artery of length L.

## 4. Results and Discussions

The axial velocity of blood plasma through a cross section of an artery of length L is defined as:

$$u_z(r) = \frac{p_2 - p_1}{4\mu L}(r^2 - {r_0}^2),\ 0 \le r \le r_0$$

We consider a cross-section of a radial artery of

L = 10cm = 100mm

$r_0$ = 1mm

$\mu$ = 1.30mpa.s = 0.0013pa.s

$p_1$ = 50mmHg

$p_2$ = 40mmHg

$\Delta p$ = -10mmHg = -1333.32pa

**Table 1** The relation between axial velocity of blood plasma and radius of the artery

| r (mm) | $u_z$ (mm/s) |
|---|---|
| 0 | 2564.079 |
| 0.2 | 2461.516 |
| 0.4 | 2153.827 |
| 0.6 | 1641.011 |
| 0.8 | 923.069 |
| 1 | 0 |

Data obtained from Nnamdi, Azikiwe, University Teaching Hospital, Nnewi, Anambra State, Nigeria.

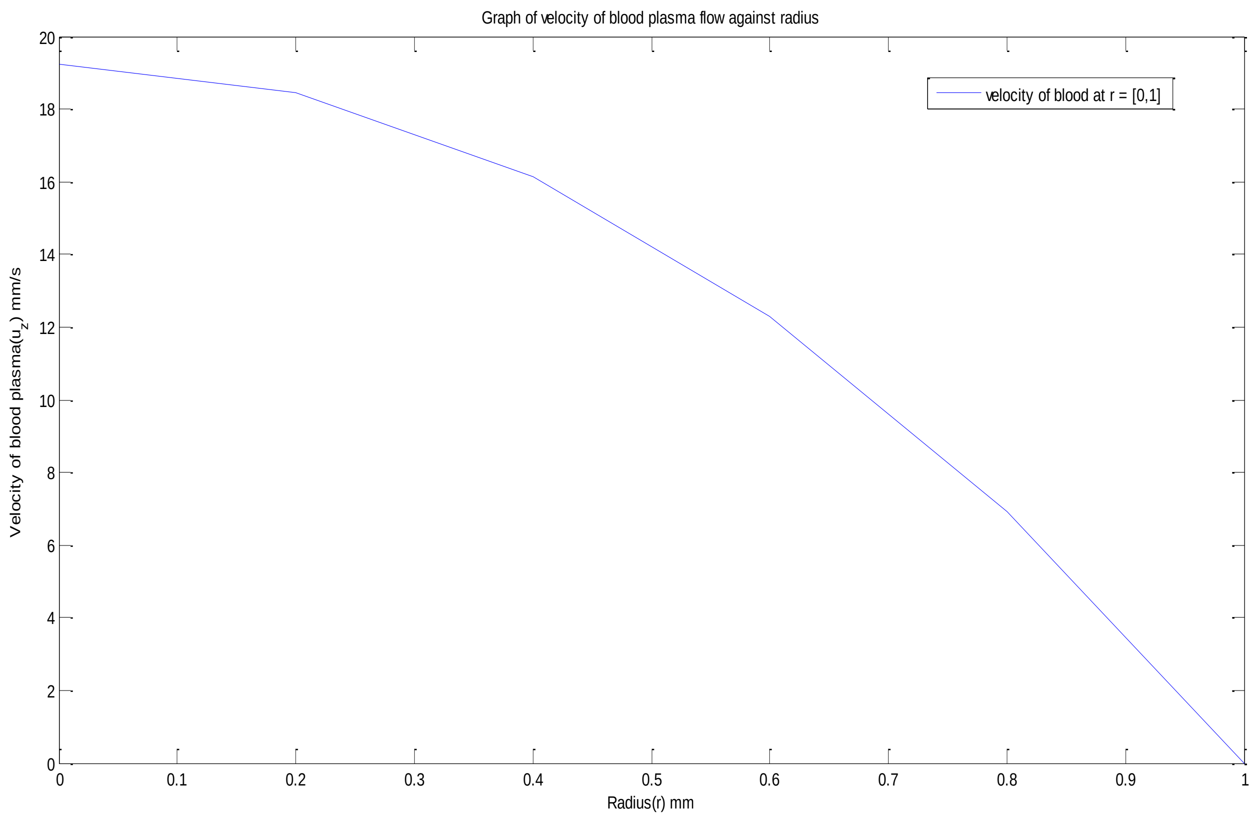



**Figure 2.** Variation of axial velocity of blood plasma $(u_z)$ with radius of artery $(r)$.

From figure 2, we see that as the radius of the artery increases, the axial velocity decreases and vice versa. The velocity is highest at the centre of the artery where $r = 0$ and reduces as $r \rightarrow r_0$, it is zero at $r = r_0$, which is at the walls of the artery. The zero velocity at the walls of the artery is as a result of viscous forces in the fluid, which are very high at the vicinity of the walls of the artery (Batchelor, 1967, p.149). This result is used to explain why the velocity of flow is at the highest value at the centre of a river and solid materials tend to move to the shore of the river because the velocity is low to move the materials. This can also be seen in turning tea in a tea cup, the velocity of the tea is highest at the centre, which is why there is a depression at the centre of the tea.

In medicine, this result can be used to explain the situation in stenosis, where the blood flow into a region of reduced radius in a blood vessel is lowered, that is the blood flow is strongly proportional to the blood vessel radius. The resistance $(R)$ to blood flow into a region in a blood vessel is given by (Poiseuille's Law); $R = \frac{8\mu L}{\pi r^4}$.

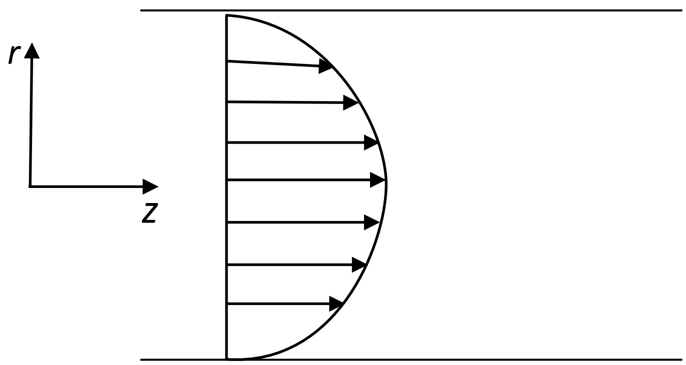



**Figure 3.** The velocity profile of blood plasma through a cross section of an non-deformed artery.

## 5. Conclusion

The model in equation (27) and the graph in figure 2 show the relationship of the arterial velocity of blood plasma flow to the radius of the artery. This result shed light into the dynamics of blood flow where blood flow into a region in a blood vessel depends on the radius of the vessel in that region.

Consumption of fatty foods should be lowered so as to avoid cholesterol building up on the walls of the artery. Smoking causes an increase in blood pressure since the chemicals in tobacco damages the blood vessel walls, causing inflammation and narrowing the arteries.

This work will enlighten readers of the need to avoid habits that would result in the shrinkage of the arterial walls. Preventive measures and perhaps prescriptive measures can be adopted to ensure that the rate of health issues resulting from shrinkage of arterial walls declines.

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