Evaluation of Generalized Extreme (GEV), Log-Pearson Type 3 (LP3), Pearson Type 3(P3) and Gumbel (EV1) Distributions For Development of IDF Equations for Warri, Nigeria.

Abstract:

The application of Gumbel (EVI) to the development of rainfall intensity – duration – frequency (IDF) curves has often been criticized on theoretical and empirical grounds as it may underestimate the largest extreme rainfall amounts. The consequences of underestimation are economic losses, property damages, and loss of life. Therefore, it is important that water resources engineering infrastructure be accurately design to avoid these consequences. This paper evaluates the performances of four probability distributions; GEV, EV1, LP3 and P3 using the annual maxima precipitation series of 26 years for Warri Metropolis obtained from Nigerian Meteorological Agency (NiMet). The strength and weakness of the four probability distributions were examined with the goodness of fit (GOF) module of Easyfit software which implemented Kolmogorov - Smirnov (KS) and Anderson - Darling (AD) tests at 5% significance level. The Easyfit software fitted the precipitation series data to the four probability distributions and ranked the four probability distributions across the fifteen rainfall durations. Results show that for both KS and AD tests, GEV distribution was found to be best-fit distribution and it was applied to the development of IDF curves in Warri Metropolis, Nigeria. Furthermore, the IDF values obtained were applied in the development of three-parameter IDF models for return periods of 10 -, 15 -, 20 -, 25 -, 50 -, and 100-years. The mean absolute error, Nash – Sutcliffe Efficiency (NSE) and Root Mean Square Error (RMSE) indices computed for the IDF models increase with increasing return periods. The IDF curves and models depicted the general attributes of IDF curves and models. This study could be of significant academic value and improvement to professional practice in the design of storm water drainage systems. Therefore, the developed IDF curves and models are recommended to the Warri Urban Authority for inclusion in her stormwater handbooks and manuals.

Key words: IDF curves, Frequency analysis, Goodness – fit – tests, Warri, Probability distributions.

INTRODUCTION:

The IDF Equation is a mathematical relationship connecting rainfall intensity (I), duration (D) and return period (T) or its inverse, the exceedance probability (P). It is a standard water resources engineering tool in most countries for planning, design and operation of hydraulic structures and storm water drainage systems [1]. The development of IDF equations has reached a level of maturity as IDF equations and curves are presented and discussed in many water resources engineering related texts, e.g. [2] to [5] etc. The Asian Pacific FRIEND [6] reported the application of IDF equations/curves as standard tool for design of hydrologic, hydraulic and water resources systems amongst the South East Asia and Pacific countries. Fordjour et al.,[7] compared Gumbel (EV1) and Log-Pearson Type 3 distributions in the development of IDF curves for Koforidua city in Ghana and found Gumbel EV1 more suitable.

Ewea et al. [8] derived intensity – duration-frequency curves for the kingdom of Saudi Arabia using Gumbel EV1 as default distribution. In Nigeria, the applications of rainfall intensities at various return periods as input into rainfall-runoff modes (e.g. RFM) for design of a variety of civil Infrastructure, especially in urban environment, has become a common and standard practice. It is important that these civil infrastructure be appropriately sized to avoid economic losses, higher risks and loss of human life [9]. Consequently, the accurate estimation of IDF curves/equations is crucial to proper sizing of water infrastructure such as urban storm – water drainage systems.

There is wealth of literature dealing with the Developing of IDF Curves/Equations in Nigeria.

Some of the reviewed literature include [10] – [16], etc,. and all the reviewed literature adopted Gumbel extreme type 1 (EV1) the default distribution in the development of IDF curves and models in Nigeria. But Gumbel EVI may significantly underestimate the largest extreme rainfall amount (albeit their predictions for small return periods of 5-10 years are satisfactory) [17]. Consequently, the applicability of Gumbel (EV1) has often been criticized both on theoretical and empirical grounds. The use of return periods (≤10 years) are no longer in vogue in view of climate change and urbanization causing non-stationarity of observed rainfall series [18]. The objective of this study, therefore is to evaluate the four selected probability distribution functions; GEV,LP3,P3 and Gumbel (EV1).

The best-fit-distribution (the one selected) will be used to develop rainfall Intensity-Duration-Frequency (IDF) relationships for Warri, Nigeria.

Using the Easyfit software, the rainfall data is fitted to the four probability distribution functions, perform goodness – of – fit (GOF) tests using Kolmogonov-Smirnov, and Anderson-Darling tests and finally rank the distributions. To the best of the Author's knowledge and literature search, no attempt had been made in the past to evaluate candidate probability distribution functions first, thereafter the best fit distribution is applied in the development of IDF curves for Warri, Nigeria. Meanwhile the Warri metropolis continues to suffer from the devastating impacts of urban flooding, causing loss of property due to absence good drainage systems and poor planning practices [19]).

2. MATERIALS AND METHODS
2.1 Description of Study Area and Data Description

The rainfall station at Warri is located at an elevation of 2.44m above mean sea level and the coordinates fall within Latitude $05^o31'$, Longitude 05° 44'.Warri Metropolis itself is geographically located between $5^o30'N$ and $5^o35'N$ and $5^o29'E$ to $5^o48'E$. The study area is bounded to the north by Okpe and Sapele Local Government Areas; to the southern axis by Warri South West and the Atlantic Ocean; to the east, the metropolis is bounded by Ughelli South Local Government Area while it shares its western boundary with Warri North Local Government Area. Figure 1 shows the map of Warri metropolis [19]

The rainfall data were obtained from Nigerian Meteorological Agency (NiMet) office Abuja, Nigeria. The rainfall intensities were extracted from FORM MET 414 (Tabulation of Autographic Rain gauge Records. The length of data is 26years (1962-1990), with five years missing due to the Nigeria civil war. The data had been screened by in – house data management. NiMet has the responsibility of measuring, analyzing, hydro meteorological data storage and forecasting the weather in Nigeria.
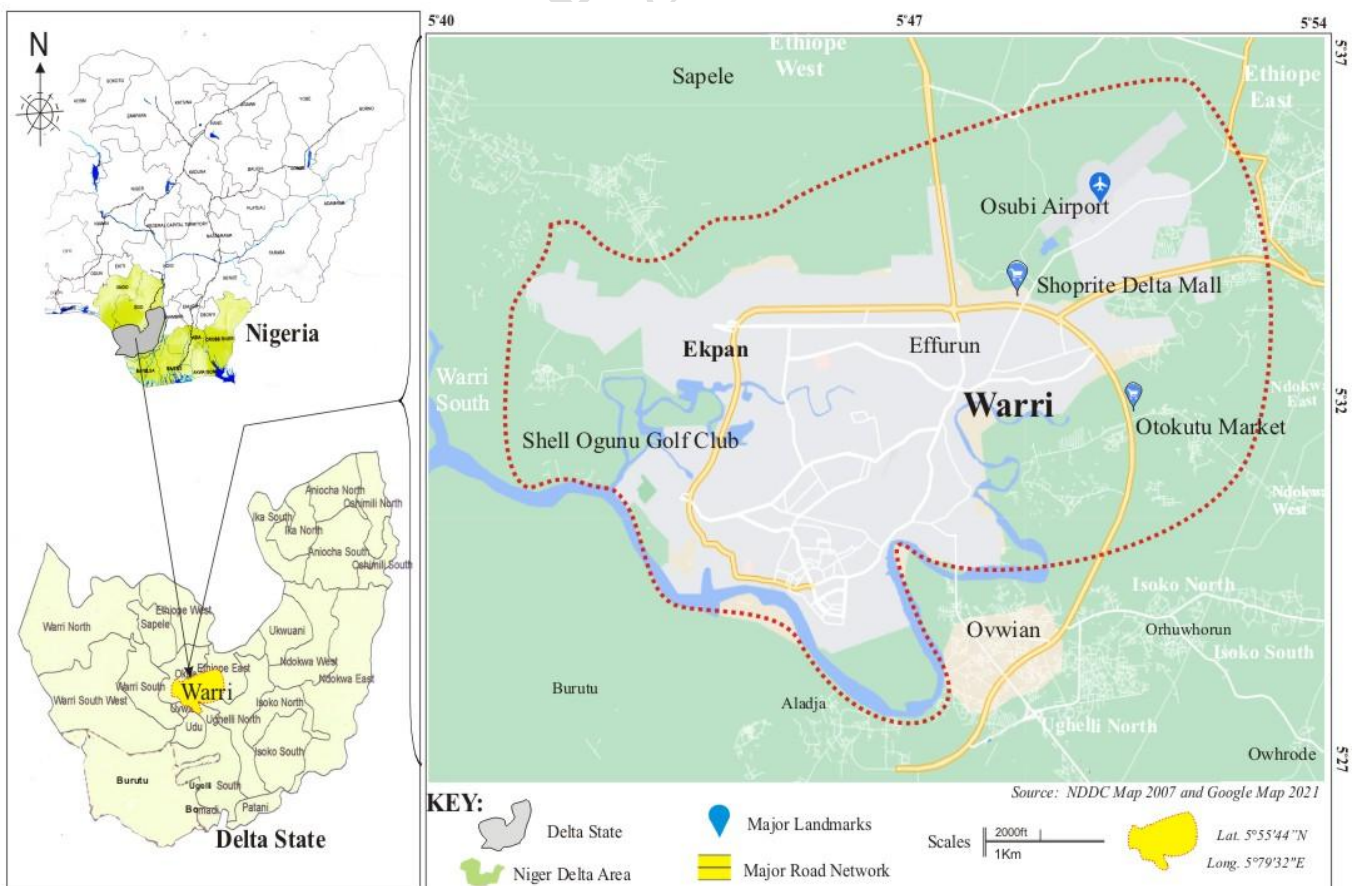

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Figure 1 : Map of Warri Metropolis, Nigeria.

2.2 Derivation of IDF Curves

The derivation of IDF relationships involved fitting a theoretical extreme value distribution to the observations and then use the theoretical distribution to estimate the rainfall events with given exceedance probabilities. The IDF were derived using the method of frequency analysis as follows.

(i) Gather time series records of different duration. (eg. 5.10,15,20,30,60,90,120 min, etc.,)

(ii) Extract annual precipitation extremes from the record of each duration

(iii) Fit the annual precipitation extremes of each duration to the selected probability distribution; GEV, Gumbel EV1, LP3 and P3, using open source easyfit software, version 5.6.

(iv) The best –fit distribution of each duration was determined using Kolmogorov – Smirnov (KS) and Anderson – Darling tests of goodness – of – fits at 5% significance level.

(v) Rank the four distributions to determine the best – fit – distribution in each duration.

(vi) Following steps (i) to (iv), select the best – fit – distribution across the fifteen (15) durations.

(vii) Finally, the selected best – fit distribution in (vi) will be applied to the development of Rainfall Intensity-Duration –Frequency (IDF) relationship.

2.3 Procedure for Fitting GEV distribution to Annual Precipitation Series.

The main objective of frequency analysis is to fit geophysical data to a probability distribution to establish a relationship between the event magnitude and its exceedance probability, and then use the quantile relation as basis for extrapolation to higher return periods. The steps followed in fitting GEV distribution are detailed in subsection 2.3.1.

2.3.1 Generalized Extreme Value (GEV) Distribution

The generalized extremes value (GEV) is a three-parameter distribution; shape (k), location (μ), and scale (α). The three – parameters of the (GEV) distribution; shape (k), location (beta), and scale (a) may be estimated from the sample moments; mean (E(Q)), variance (Var [Q]), and skew coefficient (Cs) using Equations 1 – 3 as follows;

$$\alpha = \left(\frac{K^2 \, Var[Q]}{\Gamma \, (1+2K) - \Gamma^2 (1+K)} \right)^{1/2} \qquad (1)$$

$$\beta = E[Q] - \frac{\alpha}{K} \left[1 - \Gamma(1+K) \right] \qquad (2)$$

In which г = gamma function
The shape parameter (k) is calculated from the skew coefficient (Cs) using the equation given by [20] for -2 < Cs < 1.1396 (EV3).

$$K = 0.277648 - 0.32201Cs + 0.060278 C_s^2 +$$
$$0.016759 C_s^3 - 0.005873 C_s^4 - 0.00244 C_s^5$$
$$- 0.00005 C_s^6 \qquad (3)$$

The following steps may be followed to compute the GEV quantiles:

i) Using the MS Excel built-in-functions compute the first three sample moments; the mean E[Q], variance, Var [Q], and coefficient of skewness (Cs).

ii) Using the coefficient of skewness (Cs), select the appropriate range of inequality, thus Equation 3 was selected based on the estimated shape parameter, k according to **[20]**.

iii) Estimate the other two method of moments (MoM) estimators; α and β from Equations 1 and 2 respectively.

iv) Once the GEV (α, β and k) parameters have been estimated, the precipitation quantiles are estimated using Equation 4 for different return periods.

v) Compute the T-year quantile estimate as:

$$Q_T = \beta + \frac{\alpha}{k} \left[1 - \left\{ -\ln\left(\frac{T-1}{T} \right) \right\}^K \right] \qquad (4)$$

2.5 Calibration of IDF Equation Parameters
2.5 Calibration of IDF Equation Parameters
The IDF data is fitted to Equation 5 as:

$$i(D) = \frac{\alpha}{(D+\theta)\delta} \qquad (5)$$

Equation 5 is a three parameter function, and the optimum values are estimated by least squares method. Plots of rainfall intensity (I) versus duration (D) for each return period is then produced from the fitted IDF data to Equations 5. Taking logarithms on both sides of Equation 5, gives:

$$\log(i,D) = \log \alpha - \delta \log(D+\theta)$$

The optimum values of α, δ and θ are those for which the error sum of the square deviations is minimum. That is

$$S = \sum \left[\log(i,D) - \left\{ \log \alpha - \delta \log(D+\theta) \right\} \right]^2$$

Partial differentiation of S with respect to α and δ yields:

$$\sum \log 1 = n \log \alpha - \eta \sum \log(D+\theta) \qquad (6)$$

$$\sum \left[\log(i,D) \times \log(D+\theta) = \log y \right.$$
$$\left. \sum \log(D+\theta) - n \sum \log(D+\theta) \right] \qquad (7)$$

where n is the number of observations. Equations 6 and 7 was solved simultaneously to find α and δ for

any assumed value of θ and the best value of θ itself will be found by trial and error [21].

2.6 Goodness of fit tests

The performance of the selected distribution fits are ranked using two goodness of fit tests namely; Kolmogorov –Smirnov test and Anderson Darling estimate. The two test were carried out using Easyfitsoftware,at:http://www.mathwave.com/easyfit-distribution-fitting.html.

2.6.1 Kolmogorov-Smirnov Test

The Kolmogorov-Smirnov (KS) Statistics (D) is based on the largest vertical different between the theoretical and empirical cumulative distribution function (CDF):

$$D = \underset{1 \le i \le b}{Max}\left(F(Xi) - \frac{i-1}{n}, \frac{i}{n} - F(Xi) \right) \quad (8)$$

The hypothesis is rejected, if the KS *Statistics* is greater than the critical value at a chosen significance level $\alpha = 0.05$

2.6.2 Anderson –Darling Estimate (AD)

The Anderson –Darling Estimate compares the fit of an observed cumulative distribution function (CDF) to an expected cumulative distribution function. The method gives greater weight to the tail of the distribution than the KS statistics test. The (AD) statistics (A^2) is expressed as;

$$A^2 = -n - \frac{1}{n}\sum_{i=1}^{n}(2i-1) \times \left(InF(Xi) + In(1 - F(Xn - i + 1))\right)$$
$$(9)$$

The test hypothesis is rejected if the AD statistics is greater than a critical value of 2.5018 at a given significance level $\alpha = 0.05$. The quantile equation (Equations 4), was used to calculate maximum precipitation for return periods 10,15,20,25, 50 and 100 years in Warri metropolis The analysis was executed in Microsoft Excel 2010. The fitting of the probability distributions, goodness of fit tests and ranking of the probability distributions across the fifteen durations were performed using Easy fit Software.

2.6.3 Evaluation of IDF Models Efficiency

The indices used are Mean Absolute error (MAE), Root Mean Square Error (RMSE), and Nash-Sutcliffe Efficiency (NSE). The computational forms of the above indices are given below;

$$MAE = N^{-1}\sum_{i=1}^{N}\left| I_p - Io \right| \ldots\ldots\ldots \quad (10)$$

$$RMSE = \left[N^{-1}\sum_{i=1}^{N} \left(I_p - Io \right)^2 \right]^{\frac{1}{2}} \ldots (11)$$

$$NSE = 1 - \frac{\sum_{i=1}^{N} \left(I_o - Ip \right)^2}{\sum_{i=1}^{N} \left(I_o - \overline{1}_o \right)^2} \ldots\ldots (12)$$

Where N is the sample size, I_o is the observed rainfall intensity. I_p is the predicted rainfall intensity. Io is the average observed rainfall intensity. The Nash-Sutcliffe Efficiency (NSE) lies between 1.0 (perfect fit) and - ∞. MAE and RMSE work well for continuous long-term simulations and commonly used in model performance evaluation. RMSE and MAE are among the best overall measures of model performance because they summarized the mean difference between observed (Io) and predicted (I_p) values [22].

.(3) Results and Discussion

Tables 1 – 4, Figures 2 show the results of this study. Table 1 shows the descriptive statistic and GEV parameters computed from the annual precipitation series across the rainfall durations considered in this study. The coefficients of variation (cv) show moderate variability, generally less than 0.5. This implies that within each rainfall duration, the rainfall amounts are comparative in magnitudes expect for 10min duration (cv = 0.78), where the magnitudes are wide apart. The data exhibit asymmetry with positive and negative skewness coefficient. Therefore, it is plausible to model the rainfall series with non- normal distributions. Using the kurtosis coefficient in conjunction with the excess coefficient (E), a platykurtic- type distribution was obtained, further confirming the non-nomality of the data. The shape parameter (k) is generally greater than zero (0) leading to a EV-III distribution.

Table 2 shows the outcome of the goodness of fit tests conducted using KS and AD tests at 5% significance level. The chi-squared test was not considered because it is weaker test compared to KS and AD tests and also not distribution free. The results of fitting of KS and AD tests to GEV, Gumbel (EVI), LP3 and P3 distributions and ranking their performances across the fifteen (15) rainfall durations considered in this study, are shown in Table 2.

Table 2 revealed that in terms of KS GOF test, GEV is best fit distribution in eleven (11) durations out of fifteen (15), Gumbel (EVI), scored (0) zero, LP3 scored one (1) and P3 scored 3. Similarly, in terms of AD GOF test, GEV scored ten (10) out of fifteen (15), Gumbel (EV1) scored one (1), LP3 scored zero (0) and P3 scored four (4) out of fifteen (15). Consequently, GEV distribution is the best-fit probability distribution in this study. Therefore, it is selected for frequency analysis and development of IDF curves for Warri Metropolis, Nigeria. Rainfall

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intensities for different durations and return periods using the GEV quantile relation (Equation 4) are presented in Table 3. Figures 2 shows the comparison of rainfall amounts calculated by the four probability distributions. It comes out that for the durations considered, GEV gives the maximum values. Furthermore, using Figures 2, in conjunction with Table 2, shows that the best-fit distribution actually produces the maximum rainfall values. The computed intensities in Table 3, were applied in the derivation of the IDF models shown in Table 4. Figure 3 is the graphical alternative to the IDF models. From it reveals that rainfall intensity is a decreasing function of rainfall duration for a given return period [23]. Figure 3 shows that rainfall intensity and duration are inversely related, meaning that as the duration increases, the intensity reduces. The performances of the IDF models were evaluated using the statistical indices; MAE, NSE and RMSE. In terms of MAE index, the accuracy of the IDF models decreases with increasing return periods. For a record length of 26 years, the predictive power is about 50 years (about 2N), where N is the length of record. Beyond 50 years, the uncertainties amplify which reduce the predictive power. Consequently, the computed error for return period of 100years almost tripled the values for lower return periods ([24],pg.146]). Similarly, for NSE index, the computed efficiencies diminish with increasing return periods. A similar trend was also observed for the RMSE index. The error values are higher because of squaring the differences between observed and predicted

intensities. Some of the recent studies include [14], [25] and [26]. David et al. [14] modeled rainfall intensities using optimization techniques in Abeokuta City, Nigeria and found Gumbel (EV1) the best – fit distribution. Akpen et al.[25] developed rainfall intensity - duration – frequency models for Lokoja, Nigeria, and found LP3, the best - fit distribution . Also [26] conducted Predictive Performance Analysis of IDF model types using EV1, LP3 and Normal (N) distributions and found LP3 to be the best – fit distribution from fourteen stations across Nigeria. The findings of the above studies cannot be corroborated with the findings in this study, because they did include GEV distribution in their investigations. This study agrees with [27] and [28], who applied GEV distribution for the construction of IDF curves using the annual maxima rainfall of Netherland, England and Wales respectively. They used GEV distribution due to the superiority of the distribution in describing the upper tail characteristics. A challenging problem to municipal engineers in developing countries is the ability to design storm water drainage systems. It is hope that the IDF curves/models developed in this study will be made available to local engineers, urban planners and managers to cope with the vulnerability of urban areas to flooding, budgeting for flood planning and timely response.

3.1 Descriptive Statistics and GEV Parameters

Table 1:   Descriptive Statistics and GEV Parameters

Duration(min)	Mean	Stdev	CV	Skewness	Kurtosis	A	B	K
10	11.21	8.744	0.78	2.062	4.547	4.962	7.257	-0.184
20	21.97	9.079	0.413	0.255	-0.050	8.632	18.43	0.1997
30	31.47	9.691	0.308	0.869	0.723	8.029	27.21	0.0498
45	40.00	16.34	0.408	0.864	1.114	13.55	32.83	0.0507
60	55.61	14.20	0.255	-0.053	-0.678	14.32	50.67	0.0295
90	42.08	24.91	0.592	0.530	-0.501	22.31	31.70	0.126
120	54.42	20.58	0.378	0.077	-0.926	20.27	46.87	0.253
180	57.99	24.70	0.426	0.602	-0.138	21.76	47.56	0.108
240	56.40	26.38	0.468	1.410	2.695	19.25	44.35	-0.005.
300	60.68	18.25	0.301	-0.958	1.252	19.88	57.38	0.624
360	59.14	21.87	0.37	0.578	-0.404	19.37	49.94	0.1142
420	54.62	23.74	0.435	0.970	0.687	19.22	44.08	0.0230
480	63.67	27.53	0.443	0.832	1.082	23.00	51.63	0.0573
540	65.38	24.00	0.367	1.520	4.088	17.01	54.37	-0.066
600	67.65	27.18	0.402	0.595	1.560	23.98	56.18	0.111

Table 2:   Ranking of Probability Distributions Across The Fifteen Durations.

	Kolmogorov - Smirnov(KS)				Anderson - Darling(AD)			
Dur.(mins)	GEV	GUM	LP3	P3	GEV	GUM	LP3	P3
10	0.0875^{1}	0.1358^{4}	0.0990^{2}	0.0996^{3}	0.2551^{1}	0.8948^{4}	0.4601^{3}	0.2653^{2}
20	0.0879^{1}	0.1376^{4}	0.0974^{2}	0.1035^{3}	0.2439^{1}	0.5918^{4}	0.2654^{3}	0.2599^{2}
30	0.0742^{1}	0.0843^{4}	0.0817^{3}	0.0757^{2}	0.1848^{1}	0.1855^{2}	0.1884^{4}	0.1872^{3}
45	0.1036^{3}	0.0943^{2}	0.1048^{4}	0.0942^{1}	0.2182^{2}	0.2288^{3}	0.2415^{4}	0.2054^{1}
60	0.0876^{1}	0.1681^{5}	0.1117^{2}	0.1224^{3}	0.2265^{1}	1.0666^{5}	0.3095^{2}	0.3602^{3}
90	0.0960^{1}	0.1232^{4}	0.1027^{2}	0.1145^{3}	0.2773^{1}	0.4212^{4}	0.4068^{3}	0.3673^{2}
120	0.1291^{2}	0.1576^{4}	0.1275^{1}	0.1462^{3}	0.3500^{1}	0.7085^{4}	0.3536^{2}	0.4096^{3}
180	0.0960^{1}	0.1146^{4}	0.0969^{2}	0.1087^{3}	0.2124^{1}	0.2635^{4}	0.2126^{2}	0.2310^{3}
240	0.1024^{1}	0.1031^{2}	0.1087^{3}	0.1294^{5}	0.2928^{1}	0.3557^{4}	0.2947^{2}	0.3637^{5}
300	0.0918^{1}	0.2216^{4}	0.1483^{2}	0.1718^{3}	0.3064^{1}	1.8834^{3}	14.254^{4}	0.6012^{2}
360	0.1689^{3}	0.1693^{4}	0.1610^{2}	0.1574^{1}	0.7128^{3}	0.6860^{1}	0.6897^{2}	0.7892^{4}
420	0.0930^{1}	0.1089^{3}	0.1154^{4}	0.1085^{2}	0.2541^{1}	0.2685^{3}	0.3047^{4}	0.2685^{2}
480	0.1338^{3}	0.1287^{2}	0.1380^{4}	0.1256^{1}	0.3824^{2}	0.4149^{3}	0.4279^{4}	0.3579^{1}
540	0.0997^{1}	0.1134^{3}	0.1149^{4}	0.1073^{2}	0.3656^{2}	0.3854^{3}	0.4252^{4}	0.3471^{1}
600	0.1147^{1}	0.1231^{3}	0.1339^{4}	0.1193^{2}	0.3856^{2}	0.5770^{3}	4.366^{4}	0.3334^{1}
Score	11/15	0/15	1/15	3/15	10/15	1/15	0/15	4/15

3.2   **Results of Rainfall Intensity – Duration Frequency**

Table 3: Estimates of Intensity – Duration – Frequency values

Dur.(hrs)	10-Year	15-Year	20- Year	25- Year	50-Year	100-Year
10	88.7632	108.2066	140.3973	152.1532	179.8677	236.2118
20	76.0003	91.8664	117.4926	138.9007	163.1145	198.2079
30	72.6138	86.6291	98.7661	121.9114	131.379	170.4549
45	67.5303	80.9486	86.9238	94.4520	109.0674	127.2366
60	62.2209	70.1564	79.0043	80.3216	90.6455	100.7251
90	50.9429	55.5611	59.9844	60.2871	66.9785	73.0594
120	43.3222	48.6236	49.5942	52.6541	59.0591	61.4738
180	33.6695	35.6878	36.9380	38.4568	43.2630	45.45814
240	28.7647	31.8034	34.9333	37.9544	40.9572	43.3138
300	25.0839	27.4475	29.8506	32.3834	34.6915	37.4890
360	22.5628	24.7615	26.9549	29.1399	31.3211	33.6288
420	20.8425	22.7793	24.9556	26.9557	28.9775	30.9507
480	19.3987	21.3309	23.1810	25.7312	27.5065	29.4552
540	18.4062	20.0148	22.3570	23.9974	25.5683	27.5393
600	17.5982	19.2725	20.9936	22.5835	23.9248	26.0736

Table 4: Derived IDF Models for Various Return Periods.

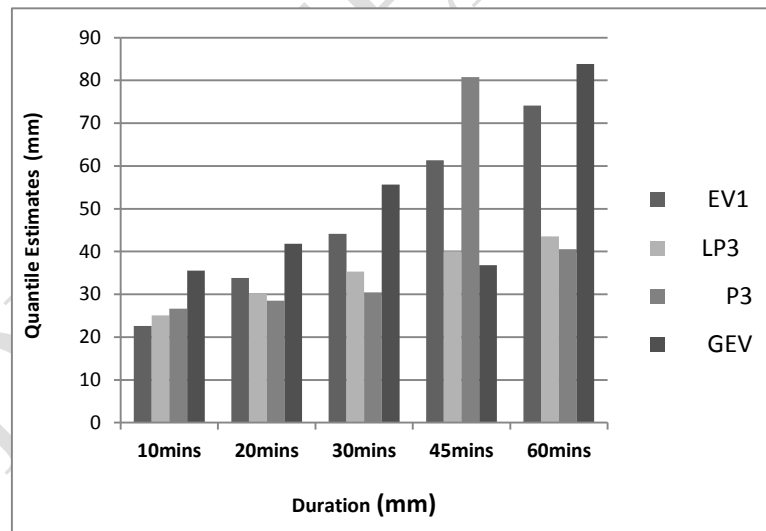| Return Period	IDF Model	Mean Absolute Error (MAE) – (mm/hr)	Nash-Sutcliffe Efficiency	Root Mean Square error
10	$I_{10} = \dfrac{2321.89}{(D+65)^{0.749}} \; mm/hr$	1.24	0.997	3.05
15	$I_{15} = \dfrac{2249.62}{(D+50)^{0.745}} \; mm/hr$	1.56	0.995	2.08
20	$I_{20} = \dfrac{1225.00}{(D+20)^{0.640}} \; mm/hr$	1.50	0.991	2.05
25	$I_{25} = \dfrac{1429.20}{(D+20)^{0.654}} \; mm/hr$	3.17	0.983	4.79
50	$I_{50} = \dfrac{1709.58}{(D+18)^{0.672}} \; mm/hr$	3.10	0.887	4.82
100	$I_{100} = \dfrac{2552.37}{(D+18)^{0.729}} \; mm/hr$	6.30	0.88	8.71

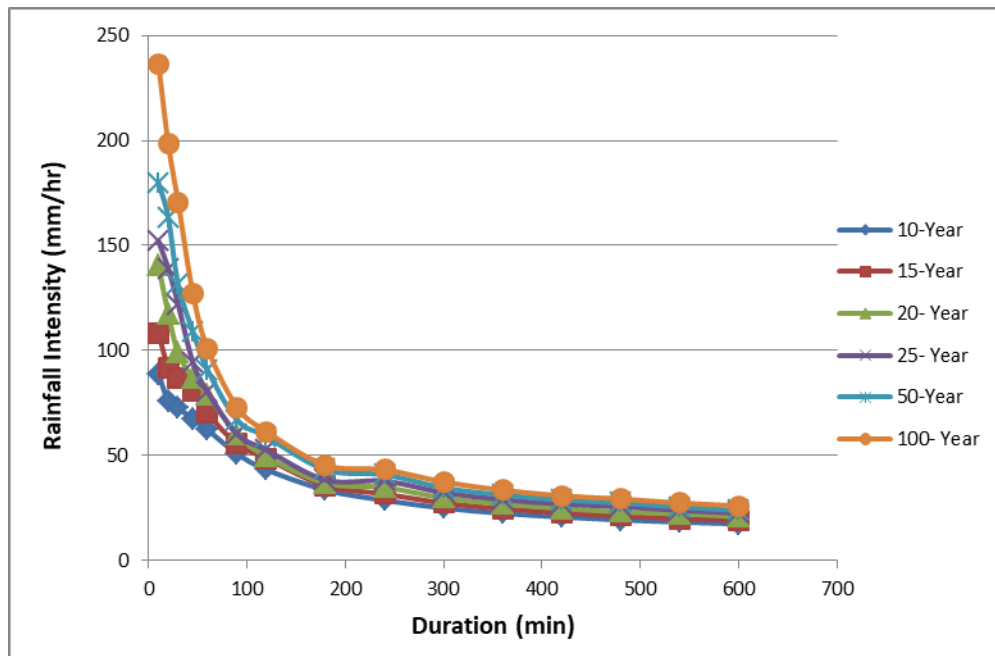Figure 2: Outcome of Ranking for 50 -Year Return Period

Figure 3. IDF Curves for Various Return Periods

.CONCLUSION

The study was conducted to evaluate four probability distributions functions; GEV, Gumbel(EVI), Log-Pearson type III and Person type III and then use the best-fit distribution to derive curves/models for Warri Metropolis.

This study shows that GEV is the best-fit distribution, seconded by Pearson Type distributions. Consequently, GEV distribution was used for the development of IDF curves and Models for Warri Metropolis, Nigeria. The IDF curves and models displayed the general characteristics of IDF curves and models. The MAE, NSE and RMSE indices computed for the IDF models generally increase with increasing return period. Intensity-duration-frequency curves and models are standard tools widely applied for the design of structures such as municipal storm-water drainage systems.

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