A Relation Between Different Physical Parameters of a Planet and Its Consequences

Abstract

The main aim of this work is to establish a relation between various physical characteristics of a planet, which were previously considered independent. The proposed 'relation between planetary parameters' (*RPP*) elegantly shows that the ratio of axial tilt to the product of rotation period and square of the orbit radius is always constant for a planet. We also show that the relation can be obtained from a more fundamental law of physics, the *principle of conservation of angular momentum*. In other words, we can realize this relation by conserving the total angular momentum of a planet. At last, we provide some applications of this relation.

Keywords: Angular Momentum, Planetary Physical Parameters

1 Introduction

In the early seventeenth century, Mathematician Johannes Kepler developed his laws of planetary motion by a rigorous analysis of the data compiled by his mentor Tycho Brahe. The proposed relation (*RPP*) was developed similarly and was obtained from a parent equation in its initial publication [1]. This parent equation was stated without any derivation because it was created by the trial-and-error method (shown in Appendix A). However, in this paper, a few changes have been made to the parent equation, and a way to obtain it theoretically from the principle of conservation of angular momentum is also discussed. All the analyses will be based on some simple assumptions to avoid various mathematical complexities. In the end, a few of its applications will be demonstrated.

2 The Parent Equation

On analysing the data of Planetary parameters, it was observed that the ratio of specific quantities was a constant for nearly all planets. This unique ratio is called the parent equation. Which is given as follows:

$$\frac{n\theta v}{r^2 d} = \psi \tag{1}$$

In the above equation, n represents the number of 'significant moons' of a planet (will be discussed ahead), θ represents the axial tilt of a planet, v represents the magnitude of the equatorial linear velocity of a planet, r represents the orbit radius of a planet, d represents the diameter of a planet and ψ is a constant for all planets, and we call it, as the 'planetary parameters constant' (*PPC*).

The axial tilt or obliquity is the angle between a planet's rotational axis and its orbital axis. Equatorial linear velocity is the velocity a particle experiences on a planet's equator due to its spinning; its direction is tangential to its surface. The orbit radius is the average distance between a planet and the Sun. Most planets are not uniform in shape; hence the average diameter of a planet is considered.

In the above equation, most of the quantities are related to either the orbital or rotational characteristics of a planet. Hence, '*n*' seems out of place in the parent equation. However, as we verify it via substitution, the value of '*n*' exactly balances off both sides of the equation, giving a constant. Hence, it is an integral part of the parent equation.

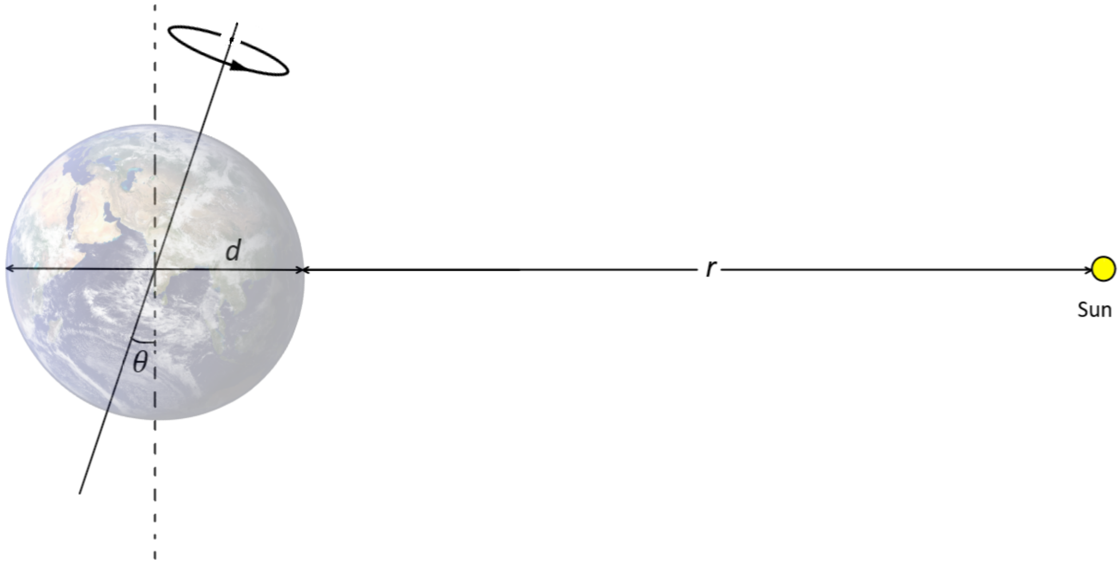

Figure 1: The side view of a planet representing the quantities of the parent equation (not to scale).

Note: Some might argue that many such equations can be formed by randomly arranging these planetary parameters; however, the chances of these randomly created equations giving a constant for all planets are less. This point makes *RPP* stand out because it gives a constant for nearly all planets. Hence, even though many equations can be formed, they won't be of significant meaning.

2.1 Exploring the value of *n*

Concerning equation 1, the term 'significant' in the context of n means any satellite orbiting a planet with enough mass (compared to the planet's mass) and proximity to the planet such that its presence or absence affects the planet. To understand this quantitatively, imagine a body of mass w, orbiting a planet of mass m with an average orbit radius a. Then the possibility of such a body, becoming a 'significant moon' is given by a quantity χ, which is defined as,

$$\chi = \frac{w}{ma}$$

We propose that the above relation can be used to determine whether an orbiting satellite is a significant moon or not. To find an approximate range of χ, its values for various moons of our solar system and their planets are given in Table 1. For this calculation, only the major moons of a planet are considered because their data is readily available, and they will be a significant moon to their planets.

Table 1: Calculation for Significant Moons

Name of Moon	Name of Planet	Mass of the moon (w in kg)	Mass of the planet ($m \times 10^{24}$ kg)	Orbital radius of moons (a in km)	χ (m^{-1})
Moon [2]	Earth	7.34×10^{22}	5.97	384400	3.2×10^{-11}
Phobos [3]	Mars	10.6×10^{15}	0.642	9376	1.7×10^{-15}
Deimos [3]		10.8×10^{15}		23463.2	7.1×10^{-16}
Europa [4]	Jupiter	4.8×10^{22}	1898	670900	3.7×10^{-14}
Ganymede [4]		1.48×10^{23}		1070400	7.2×10^{-14}
Io [4]		8.93×10^{22}		421700	1.1×10^{-13}
Titan [5]	Saturn	1.34×10^{23}	568	1221870	1.9×10^{-13}
Enceladus [5]		1.08×10^{20}		237948	7.9×10^{-16}
Umbriel [6]	Uranus	1.27×10^{21}	86.8	266000	5.5×10^{-14}
Titania [6]		3.4×10^{21}		435910	8.9×10^{-14}
Triton [7]	Neptune	2.13×10^{22}	102.4	354759	5.8×10^{-13}

According to the data in the above table, a body orbiting a planet will be a significant moon (at least in our solar system) to that planet if the value of χ is approximately between 8×10^{-16} and 3×10^{-11}. But the range may vary from one planet to another, and the above-given range may not be accurate. Hence, to find the range of χ, for a particular planet, a detailed study of its moons has to be done. So, now let's find out the number of significant moons for all the planets of our solar system by doing simple analyses.

2.1.1 Mercury and Venus

Mercury and Venus have zero moons; hence, the value of n will be zero for them and the value of ψ for these planets will be zero. Due to the absence of moons for Mercury and Venus, *RPP* and the parent equation do not apply to them.

2.1.2 Earth and Mars

For Mars and Earth, which have relatively lesser moons, the value of n is equal to the total

number of natural moons they currently possess. So, for Earth, the number of significant moons is 1, and for Mars, it is 2.

2.1.3 Jupiter and Saturn

For Jupiter and Saturn, which have many dispersed moons, the value of n is unequal to the total number of moons they currently possess. This is because some of their moons are small and distant, causing insignificant effects on the planet and hence can be ignored. Now let's analyse the moons of Saturn [8], to find out the number of significant moons it has.

Mass of Saturn $(m_S) = 5.683 \times 10^{26}$ kg

Table 2: Analysis of Saturn's Moons

Sr. No.	Name of Moon	Mass of the moon (w in 10^{17} kg)	Orbit radius (a in km)	χ (m^{-1})
1	Aegaeon	0.000001	167493.665	1.05×10^{-24}
2	Aegir	0.001	20735000	8.48×10^{-24}
3	Albiorix	0.21	16182000	2.28×10^{-21}
4	Anthe	0.00005	197700	4.45×10^{-23}
5	Atlas	0.066	137670	8.43×10^{-20}
6	Bebhionn	0.001	17119000	1.02×10^{-23}
7	Bergelmir	0.001	19336000	9.10×10^{-24}
8	Bestla	0.002	20192000	1.74×10^{-23}
9	Calypso	0.04	294710	2.38×10^{-20}
10	Daphnis	0.002	136500	2.57×10^{-21}
11	Dione	10,970	377420	5.11×10^{-15}
12	Enceladus	1,076	238040	7.95×10^{-16}
13	Epimetheus	5.3	151410	6.15×10^{-18}
14	Erriapus	0.008	17343000	8.11×10^{-23}
15	Farbauti	0.0009	20377000	7.77×10^{-24}
16	Fenrir	0.0004	22454000	3.13×10^{-24}
17	Fornjot	0.001	25146000	6.99×10^{-24}
18	Greip	0.001	18206000	9.66×10^{-24}
19	Hati	0.001	19846000	8.86×10^{-24}
20	Helene	0.25	377420	1.16×10^{-19}
21	Hyperion	55	1500880	6.44×10^{-18}
22	Hyrrokkin	0.003	18437000	2.86×10^{-23}
23	Iapetus	17,900	3560840	8.84×10^{-16}
24	Ijiraq	0.012	11124000	1.89×10^{-22}
25	Janus	19	151460	2.20×10^{-17}
26	Jarnsaxa	0.001	18811000	9.35×10^{-24}
27	Kari	0.002	22089000	1.59×10^{-23}
28	Kiviuq	0.033	11110000	5.22×10^{-22}
29	Loge	0.001	23058000	7.63×10^{-24}
30	Methone	0.0002	194440	1.80×10^{-22}
31	Mimas	373	185540	3.53×10^{-16}
32	Mundilfari	0.002	18628000	1.88×10^{-23}
33	Narvi	0.003	19007000	2.76×10^{-23}
34	Paaliaq	0.082	15200000	9.49×10^{-22}
35	Pallene	0.0004	212280	3.31×10^{-22}
36	Pan	0.049	133580	6.45×10^{-20}
37	Pandora	1.37	141720	1.70×10^{-17}
38	Phoebe	83	12947780	1.12×10^{-18}
39	Polydeuces	0.015	377200	6.99×10^{-21}
40	Prometheus	1.59	139380	2.00×10^{-18}
41	Rhea	22,900	527070	7.64×10^{-15}
42	Siarnaq	0.39	17531000	3.91×10^{-21}
43	Skathi	0.003	15540000	3.39×10^{-23}
44	Skoll	0.001	17665000	9.96×10^{-24}

45	Surtur	0.001	22704000	7.75×10^{-24}
46	Suttungr	0.002	19459000	1.80×10^{-23}
47	Tarqeq	0.002	18009000	1.95×10^{-23}
48	Tarvos	0.027	17983000	2.64×10^{-22}
49	Telesto	0.07	294710	4.17×10^{-20}
50	Tethys	6,130	294670	3.66×10^{-15}
51	Thrymr	0.002	20314000	1.73×10^{-23}
52	Titan	1342000	1221870	1.93×10^{-13}
53	Ymir	0.049	23040000	3.74×10^{-22}

If we observe the above table, the moons which have very little mass and are distant from the planet are giving the value of χ in the range 1.05×10^{-24} to 9.96×10^{-24}. Hence, we can assume that all the moons lying in this range are insignificant. Now, to bring the value of ψ for Saturn closer to the value of ψ obtained for Earth and Mars, we need the value of n to be at least 32. So, if we take *the range of χ for significant moons,* from 1.93×10^{-13} to 2.86×10^{-23}, we get the value of n as 32. Hence, out of 53 analysed moons, only 32 are significant for Saturn. One more thing should be considered: here, we have examined only 53 out of 83 confirmed moons of Saturn (due to the lack of their data). So, the above range is not very accurate. But, the number of significant moons should always be around 32, independent of the range of χ. Now, let's have a look at Jupiter's moons.

Jupiter currently has 79 confirmed moons [9]. Out of them, we need at least 74 moons to be significant, because if we take n to be 74, then we get the value of ψ to be 6.00×10^{-28}, which is close to all other planets' ψ. We can also analyse the moons of Jupiter, as we have done for Saturn in the above table. But the analysis is not very important because the number of significant moons has to be very close to 74, independent of the range of χ.

2.1.4 Uranus and Neptune

For Uranus and Neptune (according to the recently available data [10]), the number of significant moons required is more than the number of current known moons. For Uranus, we need at least 57 significant moons, but only 27 [10] moons have been discovered so far. Considering the trend of outer planets having many moons, Uranus might have many more moons, but we haven't found them yet.

For Neptune, the number of significant moons required is immense. It needs at least 452 significant moons. To explain this colossal number of moons, we have two speculations. Firstly, the orbit of Neptune passes through the inner edge of the Kuiper belt [11]; hence some Kuiper belt objects (KBOs) might be acting as significant moons to the planet. The second speculation is that due to the considerable distance between Earth and Neptune, we might have some errors in our readings of the physical parameters of Neptune. The error in other readings is causing the equation to give the value of n to be very high.

2.1.5 Limitations of *RPP*

According to the International Astronomical Union, a planet is any celestial body that [12]

(a) is in orbit around the Sun

(b) has sufficient mass for its self-gravity to overcome rigid body forces so that it assumes a hydrostatic equilibrium shape
(c) has cleared the neighbourhood around its orbit.

Due to this definition of a planet, *RPP* does not apply to Moons because they do not directly orbit the Sun. Also, *RPP* does not apply to dwarf planets like Pluto because of their unclear neighbourhood. For *RPP* to work for a planet, it should have at least one significant moon.

In verification of the Parent equation, we consider the current physical parameters of the planets. Hence, we cannot comment on whether *RPP* was valid during the early chaotic stages of planet formation because of the lack of data on planetary parameters during that time.

All the equations discussed in the manuscript are purely classical. Hence they do not account for all the relativistic effects caused by the Sun and other heavenly bodies in the solar system.

The moon provides stability to Earth's rotational axis [13], which means the moon prevents the wobbling of Earth's axial tilt. The moon's effect on Earth's axial tilt can be mathematically realized from the parent equation. Hence, the presence of 'n' in equation 1 is justified because the moon affects a planet's physical characteristics in real life.

2.2 Verification of the parent equation by the method of substitution

Without any derivation, the only way to verify the parent equation is by substituting the planetary parameters in it and comparing the obtained value of ψ for different planets. The example below shows this method for Earth.

For Earth [14], we have
$n = 1,$
$\theta = 0.410$ rad,
$v = 465.1$ m/s,
$r = 149.6 \times 10^9$ m,
$d = 12742 \times 10^3$ m

On substituting the above values in equation 1,

$$\psi = \frac{1 \times 0.410 \times 465.1}{(149.6 \times 10^9)^2 \times 12472 \times 10^3}$$

$$\therefore \psi = 6.68 \times 10^{-28} \text{ rad}(\text{m}^{-2}\text{s}^{-1})$$

Similarly, the value of ψ has been calculated for other planets (in Appendix B) and is given in table 3.

Table 3: Values of PPC for Different Planets

Name of the planet	Value of ψ ($\text{rad}(\text{m}^{-2}\text{s}^{-1})$)
Earth	6.68×10^{-28}
Mars	6.02×10^{-28}

Jupiter	6.00×10^{-28}
Saturn	6.14×10^{-28}

As observed in the above table, the value of *PPC* ranges from 6.0×10^{-28} to 6.68×10^{-28} for all given planets. To use ψ in our calculations, its average value will be considered, which can be found using the data from table 3.

Average value of

$$\psi = \frac{(6.68 + 6.02 + 6.00 + 6.14) \times 10^{-28}}{4} \text{ rad}(\text{m}^{-2}\text{s}^{-1})$$

$$\Rightarrow \psi = 6.16 \times 10^{-28} \text{ rad}(\text{m}^{-2}\text{s}^{-1})$$

Hence, for any further calculation, the value of ψ will be considered as $6.16 \times 10^{-28}\, rad(m^{-2}s^{-1})$.

The parent equation is expressed in terms of equatorial linear velocity (v) and diameter (d), which are related quantities. Hence, the parent equation can be simplified further.

3 The Relation between Planetary Parameters (*RPP*)

To simplify the parent equation, we will substitute the expression $v = \frac{\pi d}{t}$ in equation 1, which gives us,

$$\boldsymbol{\frac{\theta}{r^2 t} = \frac{\psi}{n\pi}} \tag{2}$$

where t = Rotation Period of a Planet (about its axis)
The rest of the quantities are the same as stated before

Equation 2 is the simplified form of the parent equation, and it is also the expression for *RPP*. On the RHS, there are n, ψ, and π. Out of which ψ and π are constants across all the planets in our solar system and n is assumed to be a constant for a planet (its value changes for each planet). Hence, for a particular planet, the RHS is constant, making the equation as follows:

$$\frac{\theta}{r^2 t} = \text{constant} \tag{3}$$

From the above equation, we realize that,

"For a planet, the ratio of axial tilt to the product of the square of orbit radius and rotation period is always a constant."

4 Realization of *RPP* from the Conservation of Angular Momentum

To obtain *RPP's* parent equation from the principle of conservation of angular momentum, we make the following assumptions first.

1. The orbits of planets are perfectly circular, with the Sun stationary at its centre.
2. The planets orbit the Sun at a constant speed.

3. The planets rotate about their axis with constant angular velocity and constant equatorial linear velocity.

4. The planets are perfectly spherical.

Now consider a planet with mass '*m*', orbital radius '*r*', radius '*R*', equatorial linear velocity '*v*', orbital velocity 'v_o', the tilt of the rotational axis (axial tilt) 'θ' and orbiting around its Sun of mass '*M*'. The system is shown in figure 2.

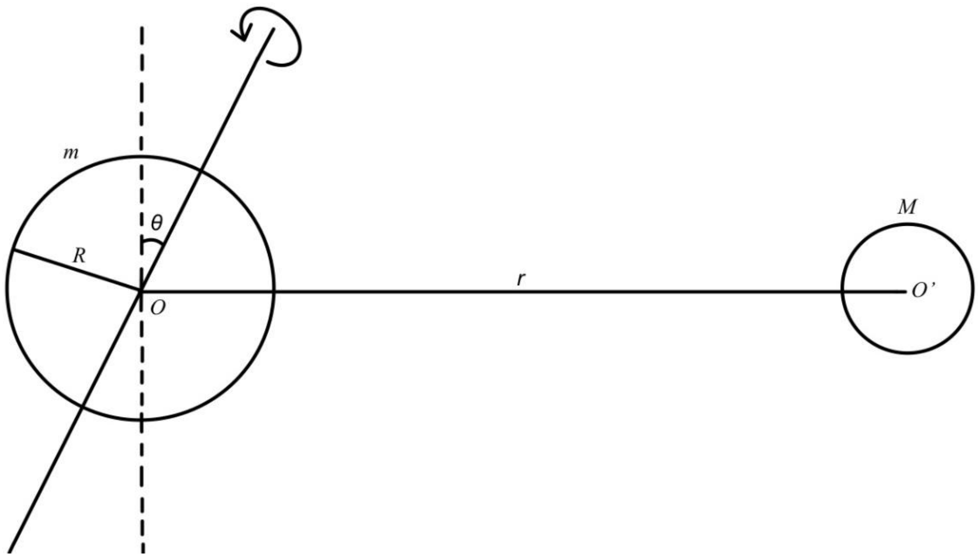

Figure 2: The Planetary System (not to scale)

From classical mechanics, we know that the angular momentum of such a system remains constant with time, as gravity is a central force [15]. Now, let's calculate the total angular momentum for the planet about point O'. A planet usually possesses two kinds of angular momenta. One is orbital angular momentum, which is due to its orbital motion around the point O'. The second is the 'spin' angular momentum due to the planet's rotation about its axis. The situation is shown in figure 3.

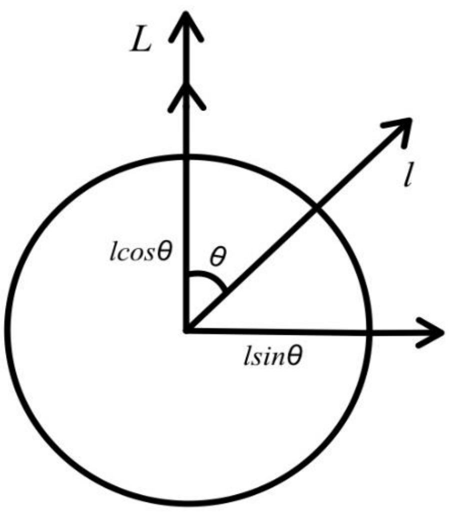

Figure 3: Angular Momenta of a Planet

In figure 3, L is the orbital angular momentum, and l is the spin angular momentum. From classical mechanics, we know that L is constant with respect to time, and hence all the planets orbit the Sun in the same plane. Here, l is also a constant because we have assumed that the planet is rotating about its axis with a constant angular velocity ω.

So, here,

$$L = mrv_o$$

And

$$l = I\omega$$

Where I is the moment of inertia of the planet (sphere) about its axis of rotation.

Hence the total angular momentum in the vertical direction can be written as,

$$L + l\,cos\theta = \alpha \tag{4}$$

And the total angular momentum in the horizontal direction is,

$$l\,sin\theta = \beta \tag{5}$$

Here, α and β are two arbitrary constants. On substituting the values of l and L, we can rewrite equations 4 and 5 as,

$$mrv_o + l\,cos\theta = \alpha \tag{6}$$

And

$$I\omega\,sin\theta = \beta \tag{7}$$

Now, substituting the value of v_o as $\frac{2\pi r}{T}$ (T is the orbital period of a planet) in equation 6,

$$\frac{2\pi m r^2}{T} = \alpha - l\,cos\,\theta$$

$$\text{or, } 2r^2 = \frac{(\alpha - l\,cos\,\theta)T}{\pi m}$$

Multiplying R on both sides,

$$(2R)r^2 = \frac{(\alpha - l\,cos\,\theta)RT}{\pi m}$$

$$\text{or, } dr^2 = \frac{(\alpha - l\cos\theta)RT}{\pi m} \tag{8}$$

Now from equation 5,

$$\frac{2}{5}mRv\sin\theta = \beta$$

$$\text{or, } v\sin\theta = \frac{5\beta}{2mR} \tag{9}$$

Dividing equation 9 with equation 8, we get,

$$\frac{v\sin\theta}{dr^2} = \left(\frac{5\beta}{2mR}\right)\frac{\pi m}{(\alpha - l\cos\theta)RT}$$

$$\text{or, } \frac{v\sin\theta}{dr^2} = \frac{5\beta\pi}{2(\alpha - l\cos\theta)TR^2} \tag{10}$$

For a particular planet, everything is a constant on the RHS of the equation, and the LHS resembles the parent equation. Hence, it can be said that

$$\frac{5\beta\pi}{2(\alpha - l\cos\theta)TR^2} \approx \frac{\psi}{n} \tag{11}$$

And obviously,

$$\frac{\boldsymbol{v}\sin\boldsymbol{\theta}}{\boldsymbol{dr^2}} \approx \frac{\boldsymbol{\psi}}{\boldsymbol{n}}$$

$$\text{or, } \frac{\boldsymbol{nv}\sin\boldsymbol{\theta}}{\boldsymbol{dr^2}} \approx \boldsymbol{\psi}$$

So, this is how the parent equation can be realized from the conservation of angular momentum. Now, let's discuss another way to realize the parent equation.

4.1 Reverse approach to realize the Parent equation

From equation 11, it is clear that $\frac{\psi}{n}$ is related to $\frac{l}{L}$, and if l and L are directly divided for a planet without invoking the conservation principle, it is obtained that,

$$\frac{l}{L} = \frac{\frac{2}{5}mvR}{mrv_o}$$

On substituting the value of v_o in the above equation,

$$\frac{l}{L} = \frac{2vR}{5r}\frac{T}{2\pi r}$$

$$\text{or,} \frac{l}{L} = \frac{vRT}{5\pi r^2}$$

On rearranging the above equation,

$$\frac{5\pi l}{LRT} = \frac{v}{r^2}$$

On multiplying $\frac{1}{2R}$ to both RHS and LHS,

$$\frac{5\pi l}{2LTR^2} = \frac{v}{dr^2} \tag{12}$$

The RHS of the above equation somewhat resembles the parent equation. On substitution, it is found that for all planets the LHS of equation 12 is nearly equal to $\frac{\psi}{n\theta}$. One example is shown below,

For Earth [14],

$l = 7.07 \times 10^{33}\ kgm^2s^{-1}$

$L = 2.7 \times 10^{40}\ kgm^2s^{-1}$

π

$R = 6371 \times 10^3$ m

$T = 3.154 \times 10^7$s

$\psi = 6.16 \times 10^{-28}\ \mathrm{rad}(\mathrm{m}^{-2}\mathrm{s}^{-1})$
$n = 1$
$\theta = 0.410\ \mathrm{rad}$
On substitution,

$$\frac{5\pi l}{2LTR^2} = \frac{5 \times 3.14 \times 7.07 \times 10^{33}}{2 \times 2.7 \times 10^{40} \times 3.154 \times 10^{7} \times (6371 \times 10^{3})^2} = 1.60 \times 10^{-27}\ m^{-2}s^{-1}$$

$$\frac{v}{dr^2} = \frac{6.16 \times 10^{-28}}{1 \times 0.410} = 1.50 \times 10^{-27}\ m^{-2}s^{-1}$$

$$\therefore \frac{5\pi l}{2LTR^2} = \frac{v}{dr^2}$$

Hence, the relation is obtained as

$$\frac{v}{dr^2} = \frac{\psi}{n\theta} = \frac{5\pi l}{2LTR^2} \qquad (13)$$

The above equation is the parent equation and a rearranged form of equation 11.

5 Applications

The first application is a set of planetary hypotheses. In that subsection, we will make some hypotheses based on *RPP*. If any of these hypotheses are proved via observations, that can be considered as proof of *RPP*.

5.1 Planetary hypotheses

Previously, the physical characteristics of a planet like the axial tilt, orbit radius, and rotational period were considered independent of each other. Hence, it was believed that the change in one quantity would not affect the others directly. This concept changes with the introduction of *RPP* because it shows a direct relation amongst these physical characteristics.

So, from equation 3, we have,

$$\frac{\theta}{r^2 t} = k$$

Where k is a constant. Hence, we can write,

$$\boldsymbol{\theta \propto r^2\, t} \qquad (14)$$

By considering each variable to be a constant one at a time, three relations are obtained:

1. $\theta \propto r^2$

   This result is obtained by assuming that t is a constant, and it suggests that if a planet’s distance from the Sun is changed, keeping its rotational period constant, then its axial tilt will also change, and vice versa. This result shows that by simply moving a planet close or away from the Sun, a planet’s axial tilt can be changed, directly affecting that planet's seasons.

2. $\theta \propto t$

This result is obtained by assuming that r is a constant. It suggests that if we change a planet's axial tilt such that its orbit radius remains the same, then its rotation period also changes, and vice versa. According to this relation, the axial tilt of a planet directly affects the duration of the day on it.

3. $t \propto \frac{1}{r^2}$

This result is obtained by assuming that θ is a constant. It suggests that by changing a planet's orbit radius such that its axial tilt remains the same, its rotation period also varies, and vice versa. According to this relation, as a planet moves closer to the Sun (perihelion), then the duration of its day increases. Similarly, as the planet moves away from the Sun (aphelion), the day's duration decreases. However, the change in the day's duration may not be very significant because the difference in 'r' is not very drastic (due to our solar system's lower eccentricity of orbits). Kepler's third law of planetary motion shows a relation between orbit radius and orbital period [16]. Similarly, the *RPP* offers an association between orbit radius and rotation period.

All the above relations show the interdependence between planetary parameters. Using the above relations, we can understand how the change in one of the parameters affects the others during the final stages of planetary formation [17].

6 Conclusion

We started this paper with a relation between different planetary parameters and the connection between those parameters was intriguing, especially the inclusion of n in *RPP.* The proper definition of n was critical to understand, as it affects other parameters of a planet. Then we discussed a theoretical approach to obtain *RPP*. In the end, the applications of *RPP* showed the beautiful ways in which we can use it in our ongoing research. *RPP* relates quantities that seemed independent hence it has opened new horizons in planetary research.

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8 Author contributions

Rajat Saxena came up with the parent equation and the analyses of moons. Sagar Kumar Biswal developed the theoretical approach to the parent equation and the applications of *RPP*. Both authors contributed equally to the design of the manuscript.

Competing Interests: The authors declare that they have no competing interests.

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Appendix A

To understand our thought process while developing the parent equation as shown in Table A1:

Table A1: Obtaining the parent equation

Planet Name	$\frac{1}{r^2}$ (m^{-2})	$\frac{\theta}{r^2}$ $(\mathrm{rad}(\mathrm{m}^{-2}))$	$\frac{\theta}{dr^2}$ $(\mathrm{rad}(\mathrm{m}^{-3}))$	$\frac{\theta v}{dr^2}$ $(\mathrm{rad}(\mathrm{m}^{-2}\mathrm{s}^{-1}))$	$\frac{nv\theta}{dr^2}$ $(\mathrm{rad}(\mathrm{m}^{-2}\mathrm{s}^{-1}))$
Earth [14]	4.46×10^{-23}	1.83×10^{-23}	1.43×10^{-30}	6.68×10^{-28}	6.68×10^{-28}
Mars [14]	1.92×10^{-23}	8.45×10^{-24}	1.24×10^{-30}	3.01×10^{-28}	6.02×10^{-28}
Jupiter [14]	1.64×10^{-24}	9.01×10^{-26}	6.44×10^{-34}	8.12×10^{-30}	6.00×10^{-28}
Saturn [14]	4.86×10^{-25}	2.26×10^{-25}	1.94×10^{-33}	1.92×10^{-29}	6.14×10^{-28}

The result of $\frac{1}{r^2}$ for the selected planets was coming close. In an attempt to bring the value of $\frac{1}{r^2}$ even closer, different quantities were multiplied to it until a constant was obtained. The result of this trial and error was the Parent equation. In Table A1, the data obtained is very small hence, plotting it on a graph is difficult. To make a graph, we'll find the logarithm of each entry, as shown in Table A2.

Table A2: Obtaining the parent equation logarithmically

Planet Name	$A = \left\lvert\log_{10}\left(\frac{1}{r^2}\right)\right\rvert$	$B = \left\lvert\log_{10}\left(\frac{\theta}{r^2}\right)\right\rvert$	$C = \left\lvert\log_{10}\left(\frac{\theta}{dr^2}\right)\right\rvert$	$D = \left\lvert\log_{10}\left(\frac{\theta v}{dr^2}\right)\right\rvert$	$E = \left\lvert\log_{10}\left(\frac{n\theta v}{dr^2}\right)\right\rvert$
Earth [14]	22.35	22.73	29.84	27.17	27.17
Mars [14]	22.71	23.07	29.90	27.52	27.22
Jupiter [14]	23.78	25.04	33.19	29.09	27.22
Saturn [14]	24.31	24.64	32.71	28.71	27.21

Based on the data of Table A2, we will plot Figure A1.

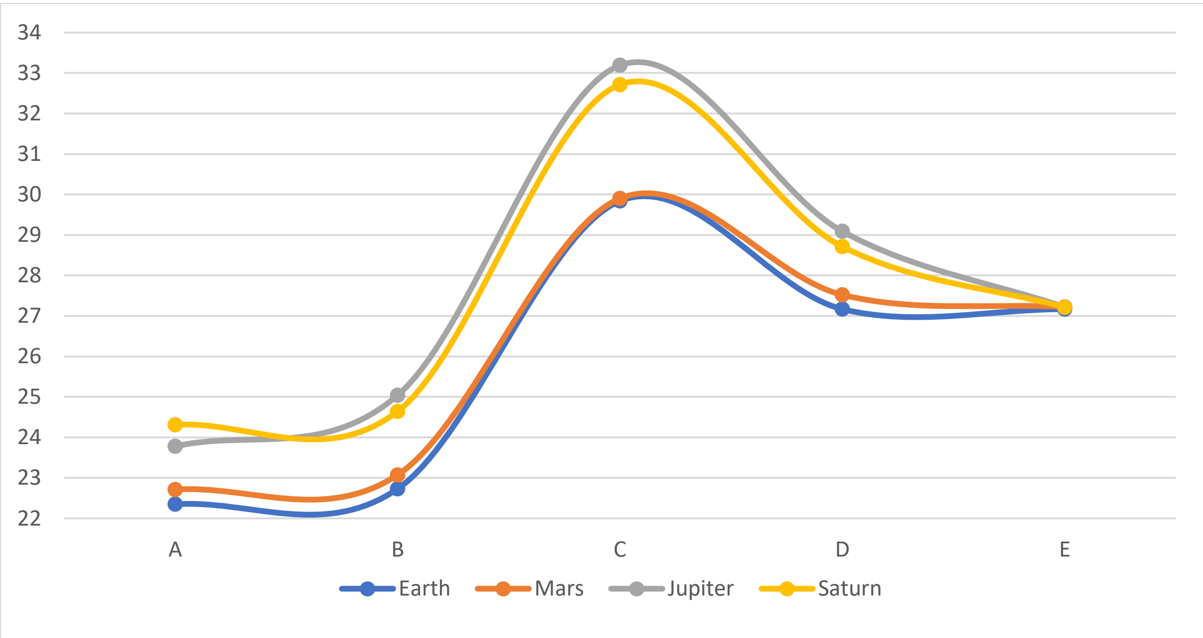

Figure A 1: The convergence of different physical parameters of a planet

It can be seen that all planets follow a similar curve.

Appendix B

The value of ψ calculated for different plane

For Earth we know [14],
$n = 1$
$\theta = 0.410$ rad
$v = 465.1$ m/s
$r = 149.6 \times 10^9$ m
$d = 12742 \times 10^3$ m
On substituting the values in equation 1,

$$\psi = \frac{1 \times 0.410 \times 465.1}{(149.6 \times 10^9)^2 \times 12472 \times 10^3} \quad \text{(B1)}$$

$$\therefore \psi = 6.68 \times 10^{-28}\, rad(m^{-2}s^{-1})$$

For Mars we know [14],
$n = 2$
$\theta = 0.439$ rad
$v = 241.17$ m/s
$r = 227.9 \times 10^9$ m
$d = 6779 \times 10^3$ m
On substituting the values in equation 1,

$$\psi = \frac{2 \times 0.439 \times 241.17}{(227.9 \times 10^9)^2 \times 6779 \times 10^3} \quad \text{(B2)}$$

$$\therefore \psi = 6.02 \times 10^{-28}\, rad(m^{-2}s^{-1})$$

For Jupiter we know [14],
$n = 74$
$\theta = 0.05462$ rad
$v = 12600$ m/s
$r = 778.57 \times 10^9$ m
$d = 139820 \times 10^3$ m
On substituting the values in equation 1,

$$\psi = \frac{74 \times 0.05462 \times 12600}{(778.57 \times 10^9)^2 \times 139820 \times 10^3}$$

$$\therefore \psi = 6.00 \times 10^{-28}\, rad(m^{-2}s^{-1})$$

For Saturn we know [14],
$n = 32$
$\theta = 0.466$ rad
$v = 9870$ m/s
$r = 1433.53 \times 10^9$ m
$d = 116460 \times 10^3$ m
On substituting the values in equation 1,

$$\psi = \frac{32 \times 0.466 \times 9870}{(1433.53 \times 10^9)^2 \times 116460 \times 10^3}$$

$$\therefore \psi = 6.14 \times 10^{-28}\, rad(m^{-2}s^{-1})$$

For Uranus we know [14],
$n = 57$ (say)
$\theta = 1.70$ rad
$v = 2590$ m/s
$r = 2870.97 \times 10^9$ m
$d = 50724 \times 10^3$ m
On substituting the values in equation 1,

$$\psi = \frac{57 \times 1.7 \times 2590}{(2870.97 \times 10^9)^2 \times 50724 \times 10^3}$$

$$\therefore \psi = 6.00 \times 10^{-28}\, rad(m^{-2}s^{-1})$$

For Neptune we know [14],
$n = 452$ (say)
$\theta = 0.4942$ rad
$v = 2680\, m/s$
$r = 4.5 \times 10^{12}$ m
$d = 49244 \times 10^3$ m
On substituting the values in equation 1,

$$\psi = \frac{452 \times 0.4942 \times 2680}{(4.5 \times 10^{12})^2 \times 49244 \times 10^3}$$

$$\therefore \psi = 6.00 \times 10^{-28}\, rad(m^{-2}s^{-1})$$

Observation:
In equations B1 and B2, the axial tilt for both planets is similar; however, the value of n for Mars is double Earth's n. To compensate for this doubling, the equatorial linear velocity of Mars becomes nearly half of Earth's v. This mechanism ensures that we always get the value of ψ similar for all planets.