

# Jet Internal Pressure and Luminosity of Powerful Radio Galaxies

**Abstract:** We use both analytical methods and statistical methods to show that certain physical processes (such as jet internal pressure and luminosity) which occur in radio galaxies may be propelled by both the source central engine and dark/vacuum energy. We do this by carrying out linear regression analysis of observed source linear sizes ( $D$ ) of the more extended radio galaxies against their corresponding observed redshifts ( $z$ ) in our sample. In addition to that, we carry out similar analysis on the observed linear sizes of compact steep spectrum (CSS) galaxies against their corresponding observed redshifts. Results of these regressions indicate that if we take  $D$  to be distance between any two positions in the environment in which the source is domiciled, then cosmic evolution relates inversely with the distance between the two positions in question – it is given by  $(1+z) \sim D^{-\mathcal{J}}$  (where the index is different for both sources) for both the more extended EGR galaxies and CSS radio galaxies. Since “a higher redshift implies an earlier epoch”, and redshift has a direct relationship with expansion velocity between any two points in space, the results of the analyses simply suggest that at earlier epoch, the expansion rate of the universe is higher. Our results also indicate that the effect of cosmic evolution in the extended EGR galaxies is more than the effect in the CSS radio galaxies (i.e.  $D_{z(EGRg)} > D_{z(CSSg)}$ ). Since the source components (jets and lobes) of the more extended EGR galaxies are located in the intergalactic media (IGM), while the components of the CSS radio galaxies are domiciled within their individual host galaxies (i.e. the interstellar media [ISM]), the result ( $D_{z(EGRg)} > D_{z(CSSg)}$ ) can be interpreted to mean that cosmic evolution shows greater effect in the IGM (i.e. more rarefied medium) than in the ISM (i.e. less rarefied medium). Hence, from the results of the analyses, we may state that if dark energy is defined as the intrinsic tendency of vacuum (or free space) to increase in volume, then the inconsistency in  $D_{z(EGRg)}$  and  $D_{z(CSSg)}$  is simply a manifestation of dark energy. Therefore, we may state that dark energy is a forcing function behind cosmic evolution. Finally, using semi-empirical relations, we find that generally evolution of radio galaxies (which manifests in jet internal pressure and luminosity) derives from two main factors; namely, the power of central engine (which presumably harbours super massive blackhole) and dark energy according to the relation,  $\frac{P^{0.06}}{p_j^{15.6}} = \mathcal{R} \left( \frac{1}{m_h c^3 \Omega \epsilon} \right)^{0.06} \frac{(1+z)^{0.12}}{\mathcal{P}_{CE[z(EGRg)]}}$ ; where  $\mathcal{P}_{CE[z(EGRg)]}$  is power of the central engine of extended radio galaxy,  $p_j$  jet internal pressure,  $P$  source luminosity,  $\epsilon$  conversion efficiency of matter into radiation,  $\Omega$  jet opening angle,  $c$  speed of light,  $m_h$  mass of hydrogen proton, and  $\mathcal{R}$  a constant.

**Keywords:** dark energy; luminosity, central engine, cosmic evolution; linear size; radio sources; galaxies; redshifts.

## 1. Introduction

The Universe is full of galaxies. In terms of their structures, different types of galaxies exist – there are elliptical galaxies, spiral galaxies, and irregular galaxies [1]. Now, in terms of their luminosities, galaxies can be classified into groups: they are, normal galaxies and active galaxies. Active galaxies are those galaxies that radiate in excess of  $10^{36}W$  [1-4]. Unlike the normal galaxy whose radiation comes from the constituent stars, an active galaxy radiates copious amounts of radiation from its three major components: namely, central engine (believed to harbor a super massive blackhole), two-sided jets emanating from the central engine, and two-sided lobes fed by the jets [1-4].

Active galaxies consist of radio-loud sources and radio-quiet sources. The former are commonly referred to as extragalactic radio sources (EGRS). EGRS emit large amount of radio emission. They show high ratio of radio to optical emission. This ratio is generally defined by the quotient of the two flux densities given by  $S_{5\text{ GHz}}/S_{6 \times 10^5 \text{ GHz}} > 10$  [1-7]. They comprise radio galaxies, radio quasars and BL Lacertae objects [4-8]. Observationally, radio radiation from these EGRS generally assumes the morphology of two opposite sided relativistic jets connecting the base of the accretion disk to two radio-emitting lobes straddling the central engine. The jet is believed to serve as a conduit through which jets materials reach the lobe. In some sources, the lobes contain hotspots believed to be the termination points of the jets [1-8]. The jets of these sources are projected into the intergalactic

medium (IGM). It is good to note that this medium is almost a perfect vacuum where vacuum energy (or dark energy) takes effect.

Compact steep spectrum sources (CSSs), on the other hand, belong to this class of active galaxies known as extragalactic radio sources (EGRS) which radiate more in the radio wavelengths. The major difference between the CSSs and the more extended EGRSs is their miniaturized nature but yet powerful in radiation [9–14]. They are made up of radio sources with spectral indices,  $\alpha < 0.5$ ,  $S_\nu \propto \nu^{-\alpha}$ ; where  $S_\nu$  is flux density, and  $\nu$ , frequency. They are not just cores that show steep spectra, rather they are full-fledged radio galaxies and quasars complete with jets and lobes, but of sub-galactic dimensions. They have been shown to contain special characteristics that make them be considered as a separate class of objects in addition to lobe- and core-dominated Active Galactic Nuclei (AGNs). They are usually found at high redshifts, and are among high luminosity sources [9–14]. To state that CSS sources are of sub-galactic dimensions implies that their linear sizes are well below 30 Kpc – i.e. their components (jets and lobes) are buried in the interstellar media (ISM). On the contrary, the more extended EGRSs have linear sizes,  $D$ , given by  $D > 30$  Kpc assuming Hubble constant,  $H_0 = 75 \text{ kms}^{-1}\text{Mpc}^{-1}$ . In all cases, their linear sizes extend into the IGM. Their radio luminosity is in excess of  $10^{26}\text{W}$  at 5 GHz ;and overall luminosities ( $P_{bol} \geq 10^{37}\text{W}$ ) in common with the CSS [4–14].

Since presence of jets in radio sources simply suggests presence of gaseous ambient media, a number of hydrodynamic simulations of jet propagations have been performed to examine their physical properties [15–18]. These studies show that jet materials have smaller masses than those of the ambient medium; hence, suggesting that they contain relativistic electrons. Besides, Ezeugo and Ubachukwu [13] obtained a model for evolution of CSS sources (which is a subclass of EGRSs) and used it to estimate their ambient densities. Their results indicate that CSS sources are situated in denser media than their extended counterparts.

Dark energy is simply the intrinsic tendency of vacuum (or free space) to increase in volume. It brings more spaces into existence. This energy is anti-gravity, and is believed to be the driving force behind the evolution (expansion) of the universe [19].

In this paper, we show that certain physical processes (such as jet internal pressure and luminosity) which occur in radio galaxies may be propelled by both the

source central engine and dark energy. The extragalactic radio sources used in the analyses are obtained from [15]. They are made up of 85 extended radio galaxies with observed linear sizes,  $D \geq 100\text{Kpc}$ . This range of linear sizes is chosen to ensure that source components (jets and lobes) are actually located in the IGM. This means that they are in the most rarefied environment; hence, dark/vacuum energy is expected to manifest itself more here than in the environment of the sub-galactic sources. The second sample contains 31 CSS radio galaxies obtained from [12]. Their linear sizes are sub-galactic; i.e.  $D < 30\text{Kpc}$ . They are located in the ISM which is denser than the IGM.

## 2. IGM & Evolution

In this section we use the more extended EGR galaxies in the analyses. The projected linear sizes of these sources (as mentioned previously) are of extragalactic dimensions ( $D > 30$  Kpc) – their components (jets and lobes) are located in the IGM. This is because the size of a typical galaxy is  $\approx 30$  Kpc. However, we used only sources whose linear sizes,  $D \geq 100$  Kpc because, their components are well seated in the IGM. This shows that whatever result obtained in this section has been affected by the most rarefied medium – the IGM. We carry out linear regression analysis of observed source linear sizes,  $D$ , of the more extended EGR galaxies against their corresponding observed redshifts,  $z$ , (Figure 1) in our sample.

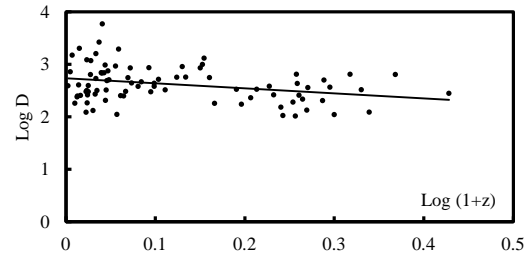


Figure 1: The scatter plot of source observed linear sizes against observed redshifts for extended radio galaxies

Results of the regression show that  $D$  relates with  $z$  according to the equation:

$$\text{Log } D = -0.956 \text{Log}(1+z) + 2.73 \quad (1)$$

The correlation is poor with coefficient, given as 0.3. If we assume the correlation is somewhat appreciable for observable data such as these, we may rewrite equation (1) to obtain

$$D \sim (1+z)^{-0.956} \quad (2)$$

Or making  $(1+z)$  subject, we obtain

$$(1+z) \sim D^{-1.1} \quad (3)$$

This shows that

$$z = z(D) \quad (4)$$

Therefore, if we take  $D$  to be distance between any two positions in the IGM, then equation (3) shows that cosmic evolution has an inverse power-law function with the distance between the two positions. This implies that at earlier epoch, the expansion rate of the universe is higher.

### 3. ISM & Evolution

Here, we use CSS galaxies in the analyses. The projected linear sizes of these sources are of sub-galactic dimensions ( $D < 30Kpc$ ) – their components (jets and lobes) are located in the ISM. Therefore, whatever result obtained in this section has been affected by the dense gases in the interstellar medium. Ezeugo and Ubachukwu [13] have shown that these radio sources are evolving in dense interstellar media unlike their extended counterparts.

On the  $D - z$  plane (Figure 2), we obtain the relation:

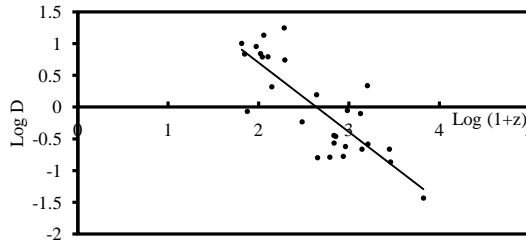


Figure 2: The scatter plot of linear size against redshift for CSS galaxies

$$\text{Log} D = -1.09 \text{Log}(1+z) + 2.88 \quad (5)$$

with correlation coefficient,  $r = 0.8$ . We may transform equation (5) to obtain

$$D \sim (1+z)^{-1.09} \quad (6)$$

Or, as before, we make  $(1+z)$  subject to get

$$(1+z) \sim D^{-0.9} \quad (7)$$

This shows that

$$z = z(D) \quad (8)$$

Therefore, just as pointed out earlier, if we take  $D$  to be distance between any two points in the ISM, then equation (7) shows that cosmic evolution has an inverse power-law function with the distance between the two positions. This also implies that at earlier epoch, the expansion rate of the universe is higher.

### 4. Dark/Vacuum Energy and Evolution

As stated earlier, dark/vacuum energy is the intrinsic tendency of vacuum (or free space) to increase in volume. It brings more spaces into existence. This energy is anti-gravity, and is believed to be the driving force behind the evolution (expansion) of the universe [19]. From the absolute values of the indices of equations (3) and (7), we find that

$$D_{z(EGRg)} > D_{z(CSSg)} \quad (9)$$

where  $D_{z(EGRg)}$  represents cosmic evolution effect in the extended radio galaxies, and  $D_{z(CSSg)}$ , the effect in CSS radio galaxies. Since most of the linear sizes of the extended radio galaxies extend much into the IGM, while the linear sizes of the CSS galaxies are situated in their individual host galaxies, therefore, equation (9) can be interpreted to mean that cosmic evolution shows greater effect in the IGM (more rarefied medium) than in the ISM (less rarefied medium). Hence, from the foregoing, we may state that if dark energy is defined as the intrinsic tendency of vacuum (or free space) to increase in volume [19], then the inconsistency in  $D_{z(EGRg)}$  and  $D_{z(CSSg)}$  is simply the manifestation of dark energy.

### 5. Theory on the Central Engine

The standard beam model for EGRS in general states that jet materials (presumably electrons) are ejected from the central engine [1,8,12,13]. They plough their way through the ambient medium until they terminate with strong shocks (i.e. hotspots) which are thermalized to form lobes [1,8,12,13]. Therefore, dynamical evolution of a radio source should be expected to depend (in addition to other factors) on the following factors: (i) power of the central engine, (ii) time, and (iii) the density of the ambient medium. Hence, we can write

$$D \sim \mathcal{P}_{CE}^\gamma \mathcal{T}^\mu \rho^\sigma \quad (10)$$

where  $D$  is projected source linear size,  $\mathcal{P}_{CE}$  power of the central engine,  $\rho$  density of ambient medium, and  $\mathcal{T}$  time. The indices are not yet known. Assuming a uniform medium, jet velocity,  $v_j$ , may be defined as

$$v_j = \frac{dD}{d\mathcal{T}} \quad (11)$$

so that source age may be expressed as

$$\mathcal{T} = \int_{\mathcal{T}_1}^{\mathcal{T}_2} \frac{dD}{v_j} \quad (12)$$

where  $\mathcal{T}_1$  represents the time jet materials started shooting out from the central engine; while  $\mathcal{T}_2$  represents the present epoch. From equations (10) and (12), we obtain

$$D \sim \mathcal{P}_{CE}^\gamma \rho^\sigma \left( \int_{\mathcal{T}_1}^{\mathcal{T}_2} \frac{dD}{v_j} \right)^\mu \quad (13)$$

For simplicity, we assume ram-pressure balance

between the jet and the ambient medium; therefore, we have [8,11,12,13]

$$p_j \approx \rho m_h v_j^2 \quad (14)$$

where  $p_j$  is jet internal pressure, and  $m_h$  hydrogen mass. Or for jet velocity, we obtain

$$v_j \approx \sqrt{\frac{p_j}{\rho m_h}} \quad (15)$$

Combining equations (13) and (15), we have

$$D \sim \mathcal{P}_{c\epsilon}^\gamma \rho^\sigma \left( \int_{T_1}^{T_2} \sqrt{\frac{\rho m_h}{p_j}} dD \right)^\mu \quad (16)$$

which yields

$$\sqrt{\frac{\rho m_h}{p_j}} \sim \frac{d \left( \frac{D}{\mathcal{P}_{c\epsilon}^\gamma \rho^\sigma} \right)^\frac{1}{\mu}}{dD} \quad (17)$$

Assuming

$$\frac{d \left( \frac{D}{\mathcal{P}_{c\epsilon}^\gamma \rho^\sigma} \right)^\frac{1}{\mu}}{dD} \approx (\mathcal{P}_{c\epsilon}^\gamma \rho^\sigma)^{-\frac{1}{\mu}} \quad (18)$$

then, equation (17) becomes

$$\sqrt{\frac{\rho m_h}{p_j}} \sim (\mathcal{P}_{c\epsilon}^\gamma \rho^\sigma)^{-\frac{1}{\mu}} \quad (19)$$

Solving for the power of the central engine, we obtain

$$\mathcal{P}_{c\epsilon} \sim \rho^{-\frac{\sigma}{\gamma}} \left( \frac{\rho m_h}{p_j} \right)^{-\frac{\mu}{2\gamma}} \quad (20)$$

Or in simple terms, we may write

$$\mathcal{P}_{c\epsilon} \sim \rho^{-\left(\frac{2\sigma+\mu}{2\gamma}\right)} p_j^{\frac{\mu}{2\gamma}} \quad (21)$$

where hydrogen mass,  $m_h$ , is a constant. If we let

$\beta \equiv -\left(\frac{2\sigma+\mu}{2\gamma}\right)$  and  $\psi \equiv \frac{\mu}{2\gamma}$ , the last equation becomes

$$\mathcal{P}_{c\epsilon} \sim \rho^\beta p_j^\psi \quad (22)$$

Equation (22) may be interpreted to mean that if every other factor is constant, the radio jet derive its power from the central engine; while the magnitude of the power of the central engine depends on the density of the source ambient medium.

## 6. Size/Luminosity Relation (Theory)

We can show that source luminosity is diminished by the particles of the medium as distance from the central engine increases; this is given by

$$P \approx (m_h c^3 \Omega \epsilon)^{-1} \frac{1}{D^2 \rho} \quad (23)$$

where  $P$  is source luminosity,  $c$  speed of light,  $\Omega$  jet opening solid angle,  $D$  source linear size, and  $\epsilon$  conversion efficiency of matter into radiation. Combining equations (22) and (23), we obtain

$$\mathcal{P}_{c\epsilon} \sim \left( \frac{1}{m_h c^3 \Omega \epsilon D^2 P} \right)^\beta p_j^\psi \quad (24)$$

Equation (24) also shows that the magnitudes of source size and luminosity depend on the power of the central engine. This may be expressed as

$$\mathcal{P}_{c\epsilon} \sim (D^2 P)^\beta \quad (25)$$

where the indices in equations (24) and (25) are to be determined.

Let  $\mathcal{O}$  be an unknown constant; therefore, equation (24) can be rewritten as

$$\mathcal{P}_{c\epsilon} = \mathcal{O} \left( \frac{1}{m_h c^3 \Omega \epsilon D^2 P} \right)^\beta p_j^\psi \quad (26)$$

Furthermore, solving for source projected linear size, we have

$$D = \left[ \frac{1}{m_h c^3 \Omega \epsilon} \left( \frac{\mathcal{P}_{c\epsilon}}{\mathcal{O} p_j^\psi} \right)^\frac{1}{\beta} \right]^\frac{1}{2} P^{-0.5} \quad (27)$$

The last equation simply suggests that the indices,  $\beta$  and  $\psi$ , may be estimated from  $D - P$  data.

## 7. Size/Luminosity Relation (Empirical)

From  $D - P$  data for the more extended radio galaxies (Figure 3), we obtain the relation,

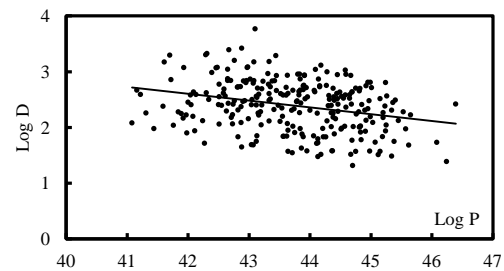
$$\text{Log } D = -0.12 \text{Log } P + 7.75 \quad (28)$$

with correlation coefficient,  $r = 0.3$  – we take this value to be substantial enough for observable data. The equation connects the source linear size,  $D$ , and luminosity,  $P$ . Making  $D$  subject, we obtain

$$D \sim P_{EGRg}^{-0.1} \quad (29)$$

where  $P \equiv P_{EGRg}$ . This shows that observed source size has an inverse power-law function with observed luminosity. Writing the constant in power of ten, equation (29) becomes

$$D = 10^{7.8} P_{EGRg}^{-0.1} \quad (30)$$



**Figure 3:** The scatter plot of source observed linear sizes against observed luminosities for extended radio galaxies

Moreover, we carry out linear regression analysis of observed source linear sizes of CSS galaxies against

their individual observed luminosities (Figure 4).

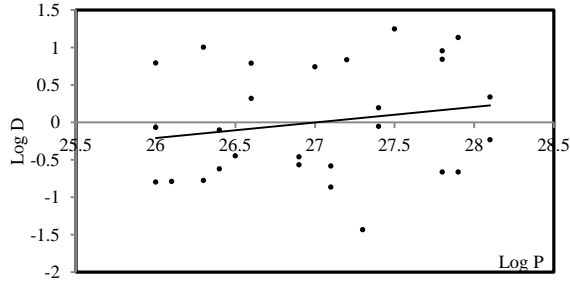


Figure 4: The scatter plot of source observed linear sizes against observed luminosities for CSS Galaxies

Result of the regression shows that  $D$  relates with  $P$  according to the expression:

$$\text{Log} D = -0.21 \text{Log} P - 5.61 \quad (31)$$

The correlation is not appreciable; the coefficient,  $r = 0.2$ . So this result is not good enough to be used in further analyses.

We equate the indices of the terms in the brackets of equation (27) to those of equation (30) to obtain  $\beta \approx 0.06$  and  $\psi = 15.6$  for the extended radio galaxies. Putting the values of the indices into equation (26), we obtain for extended radio galaxies,

$$\mathcal{P}_{\mathcal{CE}[z(EGRQ)]} = \mathcal{R} \left( \frac{1}{m_h c^3 \Omega \epsilon D^2 P} \right)^{0.06} p_j^{15.6} \quad (32)$$

$\mathcal{R}$  is a new constant that takes care of the old ( $\mathcal{O}$ ) and new constants that may have appeared during the last mathematical operation. The last relation may be interpreted to mean that in absence of external factors, the source central engine fuels the observed physical properties of these radiogalaxies.

Combining equations (2) & (32), we get

$$\mathcal{P}_{\mathcal{CE}[z(EGRg)]} = \mathcal{R} \left( \frac{(1+z)^2}{m_h c^3 \Omega \epsilon P} \right)^{0.06} p_j^{15.6} \quad (33)$$

Hence, for simplicity we obtain,

$$\frac{P^{0.06}}{p_j^{15.6}} = \mathcal{R} \left( \frac{1}{m_h c^3 \Omega \epsilon} \right)^{0.06} \frac{(1+z)^{0.12}}{\mathcal{P}_{\mathcal{CE}[z(EGRg)]}} \quad (34)$$

This shows that evolution of radio galaxies (which manifests in jet internal pressure and luminosity) derives from two main factors; namely, the central engine which harbours super massive blackhole and dark energy.

## 8. Discussion and Conclusion

We have carried out linear regression analysis of observed source linear sizes ( $D$ ) of the more extended radio galaxies against their corresponding observed redshifts,  $z$ , (Figure 1) in our sample. Results of the

regression analysis show that  $D$  relates with  $z$  according to equation (2),  $(1+z) \sim D^{-0.956}$ , with correlation coefficient, 0.3. The correlation is poor. However, we assume it is appreciable for observable data such as those found in the field of astronomy. Therefore, if we take  $D$  to be distance between any two positions in the IGM, then the relation shows that cosmic evolution has an inverse power-law function with the distance between the two positions. Since “a higher redshift implies an earlier epoch”, and redshift has a direct dependence on expansion velocity between any two positions (according to Hubble’s law), then the result of the analysis simply suggests that at earlier epoch, the expansion rate of the universe is higher.

Moreover, on the  $D - z$  plane (Figure 2), we obtain the relation,  $(1+z) \sim D^{-1.09}$  (i.e. equation [6]), which connects the observed linear sizes of CSS galaxies and their respective redshifts. The correlation is good with,  $r = 0.8$ . Therefore, just as pointed out earlier, if we take  $D$  to be distance between any two positions in the ISM, then equation (7) shows that cosmic evolution has an inverse power-law function with the distance between the two positions. This also shows that since “observation of a higher redshift implies observation at an earlier epoch” (and as pointed out earlier – according to Hubble’s law, redshift has a direct dependence on expansion velocity between any two points in space), then the result of the analysis simply suggests that at earlier epoch, the expansion rate of the universe is higher.

From equations (3) and (7) (using absolute values of the indices), we find that the effect of cosmic evolution in the extended galaxies is more than the effect in the CSS galaxies (i.e.  $D_{z(EGRg)} > D_{z(CSSg)}$ ). Since the linear sizes of the extended radio galaxies are projected into the IGM, while the linear sizes of the CSS galaxies are confined within their individual host galaxies, equation (9) can be interpreted to mean that cosmic evolution shows greater effect in the IGM (more rarefied medium) than in the ISM (less rarefied medium). Hence, from the results of the analyses, we may state that if dark/vacuum energy is defined as the intrinsic propensity of vacuum (or free space) to increase in volume, then the discrepancy in  $D_{z(EGRg)}$  and  $D_{z(CSSg)}$  is simply a manifestation of dark energy. Therefore, we may conclusively state that dark energy constitutes a driving parameter behind cosmic evolution. The particle number density of the ISM of CSS galaxies has been estimated by Ezeugo and Ubachukwu [13]. Their results show that the estimated ambient (ISM) densities of CSS galaxies generally, by far, outweigh those of their extended counterparts.

Moreover, we show that source luminosity is reduced by the particles of the medium as distance from the central engine increases; this is given by  $P \approx (m_h c^3 \Omega \epsilon)^{-1} \frac{1}{D^2 p}$  (see equation [23]); where the symbols have their usual meanings. Combining equations (22) and (23), we obtain equation (24); i.e.  $\mathcal{P}_{CE} \sim \left( \frac{1}{m_h c^3 \Omega \epsilon D^2 p} \right)^\beta p_j^\psi$ . This equation shows that the magnitudes of source size and luminosity depend on the power of the central engine. The indices,  $\beta$  and  $\psi$ , are to be estimated. We let  $\mathcal{O}$  be an unknown constant; therefore, (24) may be rewritten as  $\mathcal{P}_{CE} = \mathcal{O} \left( \frac{1}{m_h c^3 \Omega \epsilon D^2 p} \right)^\beta p_j^\psi$ . Furthermore, we solve for source projected linear size, and get equation (27); i.e.  $D = \left[ \frac{1}{m_h c^3 \Omega \epsilon} \left( \frac{\mathcal{P}_{CE}}{\mathcal{O} p_j^\psi} \right)^{\frac{1}{\beta}} \right]^{\frac{1}{2}} P^{-0.5}$ . This equation simply shows that  $\beta$  and  $\psi$  may be estimated from linear regression of the  $D/P$  data.

From  $D - P$  data for the more extended radio galaxies (Figure 3), we have the relation, (with correlation coefficient,  $r = 0.3$ ), which connects linear size and luminosity (equation [30]), i.e.  $D = 10^{7.8} P_{EGRg}^{-0.1}$ , where  $P \equiv P_{EGRg}$ . This implies that observed source size shows an inverse power-law function with observed luminosity. Here, we assume the correlation is good enough for observed physical data such as those in astronomy.

Moreover, we carry out linear regression analysis of observed source linear sizes of CSS galaxies against their individual observed luminosities (see Figure 4). Result of the regression shows that correlation is not appreciable; the coefficient,  $r = 0.2$ . So this result is not good enough to be used in further analyses.

We equate the indices of the terms in the brackets of equation (27) to those of equation (30) to obtain  $\beta \approx 0.06$  and  $\psi = 15.6$  for the extended radio galaxies. Putting the values of the indices into equation (26), we obtain for extended radio galaxies, equation (32) i.e.  $\mathcal{P}_{CE[z(EGRQ)]} = \mathcal{G} \left( \frac{1}{m_h c^3 \Omega \epsilon D^2 p} \right)^{0.06} p_j^{15.6}$ ; where  $\mathcal{R}$  is a new constant that takes care of the old ( $\mathcal{O}$ ) and new constants during the last mathematical operation. The last relation may be interpreted to mean that in absence of external factors, the source central engine drives the observed physical properties of these radio galaxies.

Combining equations (2) and (32), we obtain equation (33); i.e.  $\mathcal{P}_{CE[z(EGRg)]} = \mathcal{R} \left( \frac{(1+z)^2}{m_h c^3 \Omega \epsilon P} \right)^{0.06} p_j^{15.6}$ . Hence,

for simplicity we obtain equation (34); i.e.  $\frac{p^{0.06}}{p_j^{15.6}} = \mathcal{R} \left( \frac{1}{m_h c^3 \Omega \epsilon} \right)^{0.06} \frac{(1+z)^{0.12}}{\mathcal{P}_{CE[z(EGRg)]}}$ . This shows that evolution of radio galaxies (which manifests in jet internal pressure and luminosity) is propelled or powered by two main factors; namely, the central engine which harbours super massive blackhole and dark/vacuum energy.

## References

- [1]. Robson, I. (1996) Active Galactic Nuclei, John Wiley and Sons Ltd, England.
- [2]. Urry, C.M. (2004) AGN Unification: An Update. Astronomical Society of the Pacific conference series 1. No vol.
- [3]. Ezeugo, J.C. (2021) On Cosmic Epoch and Linear Size/Luminosity Evolution of Compact Steep Spectrum Sources. American Journal of Astronomy and Astrophysics. 9(1): 8–12.
- [4]. Ezeugo, J.C. (2021) Jet in the More Extended Radio Sources and Unification with Compact Steep Spectrum Sources. The Pacific Journal of Science and Technology. 22: 14 – 19.
- [5]. Ubah, O.L., Ezeugo, J.C. (2021) Relativistic Jet Propagation: Its Evolution and Linear Size Cosmic Dilation. International Astronomy and Astrophysics Research Journal. 3(3): 1–6.
- [6]. Ezeugo, J.C. (2021) On the Intergalactic Media Densities, Dynamical Ages of Some Powerful Radio Sources and Implications. Journal of Physical Sciences and Application. 11 (1): 29–34.
- [7]. Jackson, J.C. (1999) Radio Source Evolution and Unified Schemes. Publications of Astronomical Society of the Pacific. 16: 124–129.
- [8]. Readhead, A.C. (1995) Evolution of Powerful Extragalactic Radio Sources. In proc. Colloquium on Quasars and Active Galactic Nuclei, ed. Kohn, M., and Kellermann, K. (USA: National Academy of Sciences, Berkman Center, Irvine), 92, 11447–11450.
- [9]. Ezeugo, J.C. (2015) Compact Steep-Spectrum Radio Sources and Ambient Medium Density. International Journal of Astrophysics and Space Science. 3(1): 1–6.
- [10]. Ezeugo J.C. (2015) On the Dependence of Spectral Turnover on Linear Size of Compact Steep-Spectrum Radio Sources. International Journal of Astrophysics and Space Science. 3(2): 20–24.
- [11]. Fanti, C., Fanti, R., Dallacasa, D. Schilizzi, R.T. Spencer, R.E. and Stanghellini, C. (1995) Are compact steep spectrum sources young? Astronomy and Astrophysics. 302: 317–326.
- [12]. O'Dea, C.P. (1998) The Compact Steep Spectrum and Gigahertz peaked spectrum radio sources. Publications of the Astronomical Society of the Pacific. 110: 493–532.

- [13]. Ezeugo, J.C. and Ubachukwu, A.A. (2010) The Spectral Turnover–Linear Size Relation and the Dynamical Evolution of Compact Steep Spectrum Sources. *Monthly Notices of the Royal Astronomical Society*. 408: 2256–2260.
- [14]. Ezeugo J.C. (2021) Compact Steep Spectrum Source Size and Cosmological Implication. *Journal of Research in Applied Mathematics*. 7(2): 1–4.
- [15]. Nilsson, K. (1998) Kinematical Models of Double Radio Sources and Unified Scheme. *Monthly Notices of the Royal Astronomical Society*. 132: 31–37.
- [16]. Kawakatu, N. and Kino, M. (2007) The Velocity of Large-scale Jets in a Declining Density Medium. In *Serie de Conferencias. Triggering Relativistic Jets*, ed. W.H. Lee and E. Ramirez-Ruiz. 27: 192–197.
- [17]. Mahatma, V.H., Hardcastle, M.J. and Williams, W.L. (2019). LoTSS DR1: Double-double Radio Galaxies in the HETDEX Field. *Astronomy and Astrophysics*. 622: A13.
- [18]. Mingo, B., Croston, J.H. and Hardcastle, M. J. (2019) Revisiting the Fanaroff-Riley Dichotomy and Radio Galaxy Morphology with the LOFAR Two-Meter Sky Survey (LoTSS). *Monthly Notices of the Royal Astronomical Society*. 488:2701–2721.
- [19]. Friedman, J.A., Turner, M.S. Huterer, D. (2008) Dark Energy and the Accelerating Universe. *Annual Review of Astronomy*. 46: 385–432.