A Stochastic Model for Stock Market Price Variation

Abstract

This paper develops a differential equation model that could consider environmental effects for decision making and incorporated stochastic parameter in the model. These analyses were logically extended to vector stochastic differential equation that would help in predicting different commodity price processes, and the result obtained by using principal component analysis which is a function of the drift and by imposing a condition on the stochastic part. Furthermore, the results show the level of proportion accounted by first Principal Component Analysis (PCA) a function of the drift. In the circumstance, Kolmogorov-Smirnov (KS) test was carried out; and there exist a difference between distributions of volatility and drift.

Keywords: Stock, Market, Price, Drift, Volatility, Stochastic Differential Equation, Principal component analysis

1. Introduction

A stock in a stock market or in an investment denotes a share based on ownership of a unified company, stocks signifies ownership of an incorporated company. Investors ventures in buying of stocks on the premise of acquiring more dividends so as to maximize enough profit. Thus, market price is an act at which the current price of an asset is sold. Hence, economic theory advocates that the market prices become stationary at point of interception between supply and demand. Therefore, the most recent price at which the stock was traded is called market price.

Normally, financial analysts, investors and financial markets are geared towards making profit or loss of the stock throughout the trading days; which are the changes as a result of prices. So modeling stock market prices can be achieved through changes of the unstable market quantities or variables over time. This is having a more convenient ways of predicting stock price fluctuation, so that investors, corporative owners can issue stocks in their cooperation. The foundation of this work lies in the observation of [4] in its experiment of Brownian motion which has the same properties to that of stock prices.

Now, behavior of market price has the characteristics to that of stochastic process known as Brownian motion or Wiener process with drift. It is vital illustration of stochastic processes which satisfies a stochastic differential equation.

However, incorporating random terms into a differential equation makes it a stochastic differential equation. The random terms in the model can be modeled into discrete or continuous variables for the purpose of predicting real-life behavior of any random dynamic properties [6]. In the area of science and engineering technology, accurate evaluation, design and proper assessment of the model must be taken account of, for instance environmental effects such as white noise. Randomness is almost characterized in many real life situations such as stock market price fluctuations, noise in population dynamics etc. Stock price modeling is linked with modeling new stocks based on information which deals with price formation. Two crucial points to bear in mind when modeling stock prices are as follows: probability distribution and information respectively. All these are important and play good roles in modeling future stock prices. In all, future stock prices are predicted based on the availability and effectiveness ascribed to the stochastic model for the purpose of practical findings.

More so, stock exchange market will require some elements of principal component analysis for predicting the volatility variables or quantities. Principal component analysis is a statistical method that transforms a set of values of uncorrelated or correlated variables through linear combination. The transformation is carried out such that the first principal component of the data set has the largest possible variance. It is principal component if the data set is jointly normally distributed.

[1] Studied the financial modeling of stock market prices; and applied stochastic model. The result showed that stochastic differential equation is better used in modeling financial markets.[2] investigated the stock market price models. They used stochastic analysis in terms of stochastic differential equation and their results proved that; for the production of stock prices the proposed model is efficient.[3] studied stochastic volatility model on stock prices. Monte Carlo was applied with efficient method of moments (EMM). Their results showed that efficient method of moment provided better results than other models.

[9] Investigated the economy of developing nation and effect of capital flight. Stochastic model was applied to the study. They concluded that good measures should be instituted in order to make domestic economy more lucrative. In the work of [10] a stochastic volatility were derived of option pricing.

[12] Worked on stock price modeling and applied stochastic model to explain the time series of stock prices. In the same vain, [7] examined the stochastic analysis of stock prices and concluded that geometric Brownian motion has less accuracy in short term modeling of stock prices.[11] considered measurement of stock price changes which precise conditions were obtained to determined equilibrium price and growth rates

It is obvious that, [6] have formulated stochastic model for describing the stability of a system and exploring the properties of fundamental matrix solution for the analysis. The main advantage of this work as against the work of [6] is that it considers the levels of proportion accounted by first Principal Component Analysis (PCA) a function of the stock drift which is the annual rate of return and analysis of two key parameters of Stochastic Differential Equation(SDE) using Kolmogorov-Smirnov (KS). Our novel impact improves the previous knowledge.

The paper is aimed at studying the stochastic analysis of stock market prices, determining the levels of proportion of total variance by the first PCA as it affects stock market prices and subjecting the two key parameters of stochastic differential equation to Kolmogorov-Smirnov (KS) test to know if they come from a common distribution.

This paper is arranged as follows: Section 2 presents the mathematical preliminaries, formulation of the problem is seen in Subsection 2.1, Subsection 2.3 is PCA of the stock variables, Subsection 2.4 is KS test, Data analysis and results is presented in Section 3 while Section 4 is discussions of results and the paper is concluded in Section 5.

2 Mathematical Preliminaries

let $S(t)$ represents the price of asset at time t ,and μ , rate of return and dt as a rate change throughout the trading days such that the dynamics of stock price is governed by a stochastic differential equation.

$$dS(t)=\mu S(t)dt+\sigma S(t)dW_t \quad (1)$$

Where, μ is drift and σ the volatility of the stock, W_t is a Brownian motion or Wiener's process on a probability space $(\Omega,\xi,\wp)$, ξ is a $\sigma-$algebra generated with $W_t, t\geq 0$.

Theorem 1.1: (Ito's formula) Let $(\Omega,\beta,\mu,\mathrm{F}(\beta))$ represents a filtered probability space $X=\{X,t\geq 0\}$ be an adaptive stochastic process on $(\Omega,\beta,\alpha,\mathrm{F}(\beta))$ processing a quadratic variation (X) with SDE defined as:

$$dX(t)=g(t,X(t))dt+f(t,X(t))dW(t) \quad (2)$$

$t\in\Re$ and for $u=u(t,X(t)\in C^{1\times 2}(\Pi\times\square))$

$$du(t,X(t))=\left\{\frac{\partial u}{\partial t}+g\frac{\partial u}{\partial x}+\frac{1}{2}f^2\frac{\partial^2 u}{\partial x^2}\right\}d\tau+f\frac{\partial u}{\partial x}dW(t) \quad (3)$$

Using theorem 1.1 and equation (3) gives a complete solution of SDE given below:

$$S(t)=S_0\ \exp\left\{\sigma dW(t)+\left(\alpha-\frac{1}{2}\sigma^2\right)t\right\}, \forall t\in[0,1]$$

2.1 Formulation of the Problem

Given a finite time horizon $T>0$,we look at a complete probability space((Ω,f,P) whose stock price dynamics is in(1). We consider stock drift of the expected returns for investors in financial market. From the model equation (1) in Sections 2, we now developed and consider a vector valued SDE, where the securities invested on some bond processes are correlated and to obtain the proportion of total variance accounted by the principal component analysis.

We consider the SDE given by

$$dS_1(t)=\mu_1 S_1(t)dt+\mu_2 S_2(t)dt+S_1(t)\left(\sigma_{11}dw_1(t)+\sigma_{12}dw_2(t)\right) \quad (4)$$

$$dS_2(t) = \mu_1 S_1(t)dt - \mu_2 S_2(t)dt + S_2(t)\big(\sigma_{21}dw_1(t) + \sigma_{22}dw_2(t)\big)$$
$$\vdots \qquad \vdots \qquad \vdots \qquad \vdots \qquad \vdots$$
$$dS_n(t) = \mu_n S_n(t)dt - \mu_n S_n(t)dt + S_n(t)\big(\sigma_n dw_1(t) + \sigma_{n2}dw_2(t)\big)$$

Here, it is assumed that $\sigma_{21} = \sigma_{22} \neq 0$

Since both processes $S_1,..,S_n$ are correlated, to develop a vector equation for (4) write the equations in matrix form by

Letting $S = (S_1,\dots,S_n)^T$

$$A(t) = \begin{pmatrix} \mu_{11} & \mu_{12} \\ \mu_{21} & \mu_{22} \end{pmatrix}, \ \sum B_i(t), \ \ B_1(t) = \begin{pmatrix} \sigma_{11} & 0 \\ 0 & \sigma_{21} \end{pmatrix}, \ B_2(t) = \begin{pmatrix} \sigma_{21} & 0 \\ 0 & \sigma_{22} \end{pmatrix}, \quad (5)$$

A generalized equation for the vector valued SDE can now be put in the form

$$dx(t) = A(t)x(t)dt + \sum_{i=1}^{n} B_i(t, x(t))\, dw_i(t), \qquad (6)$$
$$x(0) = x_0$$

Where $A(t) \in \mathbb{R}^{n\times n}, B_i(t) \in \mathbb{R}^{n\times n}, w_i(t) \in \mathbb{R}^n$ is an n-dimensional Brownian motion, $x(t) \in \mathbb{R}^n$ is a process of price volatility.

It is known in [6], that $x(t)$ for equation (6) is normally distributed because the Brownian motion is just multiplied by time-dependent factors.

Let $A(t) \in \mathbb{R}^{n\times n}$ be Covariance matrix of the homogenous stochastic differential equation (6).

Then the integral solution of (6) is given by

$$X(t) = X(t)x_0 + \int_0^t X(t)X^{-1}(s)\sum_{i=1}^{n} B_i(t, x(t)dw_i(s)$$

2.2 Principal component Analysis of the stock variables

Definition 1.2: Suppose $\underline{X}$ has a joint distribution which has a variance matrix $\sum$ with eigenvalues $\lambda_1, \geq \lambda_2 \geq \dots \geq \lambda_p \geq 0$. consider the random variables $y_1 \dots y_p$ which are linear combination of the $X_1's$ ie:

$$\left.\begin{aligned} y_1 &= \underline{l}_1'\underline{X} = l_{11}X_1 + \dots + l_{p_1}\lambda_p \\ &\vdots \\ y_p &= \underline{l}_1'\underline{X} = l_1 p X_1 + \dots + l_{p_1}\lambda_p \end{aligned}\right\} \qquad (7)$$

The y_i 's will be PC if they are uncorrelated and the variances of $y_{1,}y_2$ are as large as possible. Recall that if $y_i = l_i' X$. In order to look at the amount of information that is in y_1 .We can consider the proportion of the total population variance due to y_i

$$\frac{\lambda_i}{\lambda_1 + \lambda_2 + \cdots + \lambda_p}, i = 1, ..., p \quad (8$$

2.3 Kolmogorov-Smirnov (KS) Test

The KS statistic belongs to nonparametric test in statistics. It is a very useful test for testing normality assumption and based on the largest vertical difference between the hypothesized and empirical distribution, [5]. Besides its use in the test for assumptions of normality, it is popular in the test to verify if two independent samples are drawn from a common distribution. We test as follows:

H_0: The Drift and Volatility stocks are from the same distribution.

H_1: They are not from the same distribution.

3.Data Analysis and Results

In this Section we present the computational results for the problems formulated in Section 2. The problems were solved analytical and graphical solutions obtained using matlab programming language.

To illustrate the stock price fluctuations, we used ten years stock price data extracted from [2] which shows the initial stock prices, drift and volatility with respect to their trading days. We computed the values of the stock drift to obtain covariance matrix solution given as:

$$A(t) = \begin{pmatrix} 0.09377 & 0.04545 \\ 0.04545 & 0.364 \end{pmatrix}$$

Which was extended to compute the levels of proportion accounted by first principal component analysis a function of the drift parameter.

To obtain the values of the volatilities, we use the last term of right hand side of equation(6) which gave the following:

$$\sum B_I = B_1(\mathrm{t}) + B_2(\mathrm{t}) = \begin{pmatrix} 0.03384 & 0 \\ 0 & 0.04811 \end{pmatrix} + \begin{pmatrix} 0.04811 & 0 \\ 0 & 0.364 \end{pmatrix} = \begin{pmatrix} 0.08195 & 0 \\ 0 & 0.41211 \end{pmatrix}$$

In this paper, we have used the notion of the Brownian motion model to determine the principal component analysis of the dynamics of stock price solution to the stochastic model. The main idea is to use the geometric Brownian motion model of the stock price which is otherwise the solution of the stochastic model to develop and analyze condition that the total variance will be accounted.

Hence, solving the Principal Component Analysis (PCA) of stock drift

$$A(\mathrm{t}) = \begin{pmatrix} 0.09377 & 0.04545 \\ 0.04545 & 0.364 \end{pmatrix} \quad (9)$$

Solving for the $|\mathrm{A(t)} - \lambda I| = 0$ gives

$$\lambda_1 = 0.0863, \ \lambda_2 = 0.3714$$

And the corresponding eigenvectors for $k_1 = 1.0044$ and $k_2 = -0.1644$

Any vector of the form $k_1 = \begin{pmatrix} 1.0044 \\ -0.1644 \end{pmatrix} = \begin{pmatrix} 1.0044c \\ -0.1644c \end{pmatrix}$

Any vector of the form:

$$k_2 = \begin{pmatrix} 6.1085 \\ 0.9946 \end{pmatrix} = \begin{pmatrix} 6.1085c \\ 0.9946c \end{pmatrix}$$

Say is an eigenvector corresponding to $\lambda_2 = 0.3714$

$k_1' k_1 = 0.0863$

$\begin{pmatrix} 1.0044c & -0.1644c \end{pmatrix} \begin{pmatrix} 1.0044c \\ -0.1644c \end{pmatrix} = 0.0863, 1.00881936c^2 + 0.02702736c^2 = 0.0863$

$1.03584672c^2 = 0.0863, \ c = 0.2886, \ e_1 = (0.2886) \begin{pmatrix} 1.0044 \\ -0.1644 \end{pmatrix} = \begin{pmatrix} 0.2899 \\ -0.04756 \end{pmatrix}$

$k_2' k_2 = 0.3714$

$\begin{pmatrix} 6.1085c & 0.9946c \end{pmatrix} \begin{pmatrix} 6.1085c \\ 0.9946c \end{pmatrix} = 0.3714, \ 37.31377225c^2 + 0.98922916c^2 = 0.3714$

$38.30300166c^2 = 0.3714, \ c = 0.09847, \ e_2 = (0.09847) \begin{pmatrix} 6.1085 \\ 0.9946 \end{pmatrix} = \begin{pmatrix} 0.6015 \\ 0.0979 \end{pmatrix}$

The first Principal Component:

$$\left.\begin{aligned} Y_1 &= e_1'k_1 = 0.2899k_1 - 0.04756k_2 \\ Y_2 &= e_2'k_2 = 0.6015k_1 + 0.0979k_2 \end{aligned}\right\} \tag{10}$$

To calculate principal component Analysis accounted for:

$$\lambda_1 = 0.0863,\ \lambda_2 = 0.3714 \quad ,\Rightarrow \frac{\lambda_1}{\lambda_1 + \lambda_2} = \frac{0.0863}{0.4577}, \ = 0.1886$$

The proportion of total variance accounted by first principal component is 18% . The two eigen-values describe the level of variance accounted for by the associated principal component.

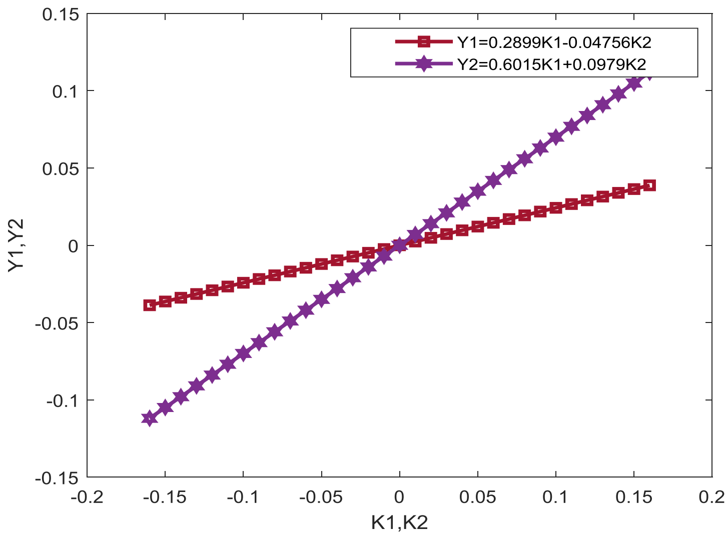

Figure 1: The first Principal Component Analysis of stock market prices

Figure 1 shows the linear combinations of first principal component analysis of stock market prices. It also attests to the proportion of total variance accounted by first principal component. They are uncorrelated and hence measure the overall performance in the financial market.

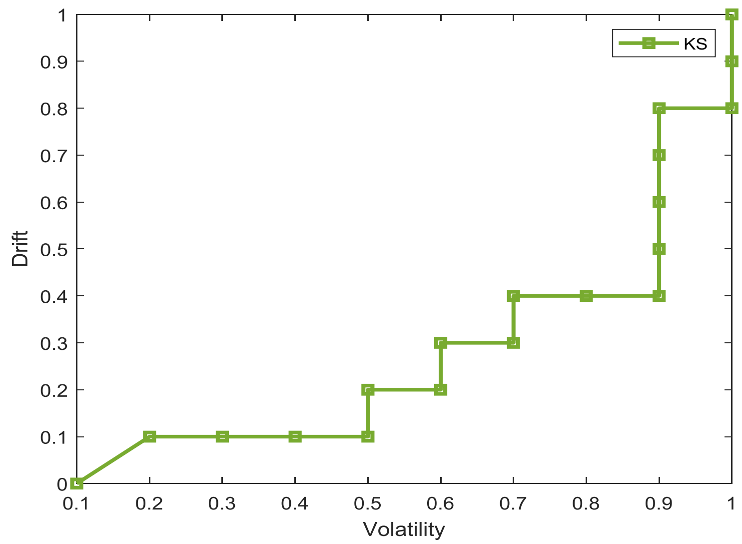

Figure 2: Graphical representation of KS test for Volatility and Drift stock variables

In Figure 2, shows levels of stair-case functions between Volatility and Drift, it portrays significance levels of the stock variables since both of them do not have same characteristics in stock market business.

Following the hypothesis testing of Section 2.3 using the Kolmogorov-Smirnov test, P-value $= 0.0486$ and KSSTAT $= 0.4211$ all are greater 0.05 implies that we should accept H_1 and therefore conclude that there is significant difference between Drift and Volatility. That is to say that both stock variables do not belong to a common distribution.

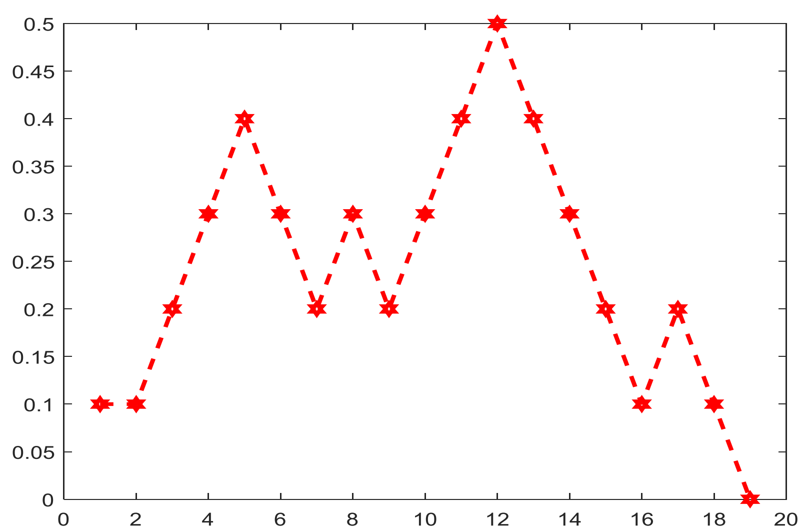

Figure 3: The difference in ranked KS test for Volatility and Drift stock variables

The two stock variables are displaying stock price fluctuations which cause panic buying in real-life trading. They are uncorrelated and follow a normal distribution. This also shows that the two stock variables do not belong to a common distribution, see Figure 3.

4 Conclusions

In this paper, we studied the problem of stock price fluctuations using stochastic differential equations, principal component analysis and KS goodness of fit test. The analytical solutions were detailed; the computational and graphical results were presented and discussed respectively.

The analyses were logically extended to stochastic vector differential equation that would help in predicting different commodity price processes, and the result obtained by exploring the properties of the principal component analysis. Furthermore, the results show the level of proportion accounted by first Principal Component Analysis (PCA) a function of the drift is 18%. To this end, Kolmogorov-Smirnov (KS) was carried out; the test showed that there exist a difference between distributions of volatility and drift as it affects stock market.

We therefore conclude that, this model will help investors, economist, policy makers and opinion leaders who are working assiduously in making sure of maximizing profit and minimizing lost.

REFERENCES

[1] Abodolsadeh, N. and Moslem, P. (2011). Financial modeling by ordinary and stochastic differential equations. *World Applied Sciences Journal*, 13 (11): 2288-2295.

[2] Adeosun, M.E. , Edeki, S.O. and Ugbebor, O.O.(2015).Stochastic Analysis of stock Market price models: A case study of the Nigeria stock exchange. WSEAS transaction on Mathematics,14

[3] Andersen, T. G., Chung, H. J. and Sorensen, B. E. (1999). Efficient method of moments estimation of a stochastic volatility model: A monte Carlo Study. *Journal of Econometrics*. 91,pp 61 – 87.

[4] Bachelier, L. (1964). *Theories de la speculative paris: Gauthier-Villar. Transplanted in cooler.*

[5] Conover, W.J. (1999). *Practical Nonparametric Statistics*. Third Edition, John Wiley and Sons, Inc. New York.

[6] Davies, I. Amadi, I.U and Ndu, R.I(2019).Stability Analysis of Stochastic Model for Stock Market Prices. *International journal of Mathematics and Computational Methods ,4*,79-86.

[7] Dmouj, A.(2006). Stock price modeling: Theory and practice, BIM paper

[8] Fama, E. F. (1965).The behavior of stock-market price. *Journal of Business vol. 38*Issue 1, 34 – 105.

[9] Osu, B. O. (2012). Predicting the value of an option base on an option price. *Journal of mathematical comp*, No.4 1091 – 1100.

[10] Osu, B. O; Amamgbo, O. R. and Adeosun M. E. (2012). Investigating the effect of capital flight on the Economy of a developing nation via the NIG Distribution. *Journal of computation and modeling*, vol. *2*, No. 1, 77 – 92.

[11] Osu, B.O. and Okoroafor, A.C.(2007).On the measurement of random behavior of stock price changes. *Journal of mathematical science Dattapukur,18(2)*,131-141.

[12] Straja, S. R. (2005). Stochastic modeling of stock prices. *PhD Montgomery investment technology*, Inc. 200 Federal street Camden, NJ 08103.www.fintools.com.