Probabilities and Probable Solutions of a Modified KdV Type Nonlinear Partial Differential Equation

ABSTRACT

The goal of this work is not only the search for the solutions of a nonlinear partial differential equation, but how to locate and choose a form of solution verifying the nonlinear partial differential equation. In this work, we use the probabilities of appearance of the pairs (n,m) linked to iB-functions for which certain terms of the range of coefficients equations are grouped together to locate and then determine the solutions of the partial differential equation of the KdV type. The pairs (n,m) when identified, indicate with precision the iB-function which will choose from the start as the solution function which we want to build. The probabilities here are essential data to select the analytical sequences of the solutions to be investigated.

Keywords: KdV equation, iB-functions, solitary wave, range equations, , probabilities of the pairs,

1. INTRODUCTION

Most often the fundamental difficulty encountered in physical science is that of finding solutions to equations which model the dynamics of physical systems. These equations in the majority are nonlinear and even strongly nonlinear [1-14]. When the integral method is limited in the resolution, one proceeds by searching for the forced solutions[15-25]. But what is even more difficult to do is to make the choices of the solution function to introduce into the considered equation.
Within the framework of the resolution of certain types of nonlinear and dispersive partial differential equations, we have shown that the use of the iB-function [26-30] was very appropriate in this case.
But this function being multiple, that is to say varying according to three characteristic parameters n, m and α, choose with precision the values of n, m and α so that the iB-function is solution of the nonlinear partial differential equation to solve is also difficult. We realized that the most probable solution to verify the partial differential equation considered depends on the number of times that the pair (n,m) considered favors the grouping of the terms in the range equation that is a probability of appearance of the pairs (n,m) in the total number of pairs which are at the origin of the regroupings [31]. In this work, we go through the probabilities of appearance of the pairs to locate the solutions of modified nonlinear KdV equation type. The principle consists in injecting into the modified KdV equation, the solution of the form $aJ_{n,m}(\alpha x-\alpha_0 t)$, where a, α, α_0, n and m are arbitrary

constants, x the independent variable, t the temporal variable, and then listing all the pairs (n,m) for which certain terms of the coefficient range equation are grouped together and finally identify the most favorable pairs for obtaining solutions. We use the concept of probability in the choice of pairs (n,m) simply because, it makes it possible to avoid the hazardous choices of the forms of solutions to be constructed.
We organize this work in three main sections which are: Some notions on the iB-function, results and discussion which has for subsections : obtaining the coefficient range equation, the probabilities of the possibilities of grouping, the calculations of the coefficients of the terms of the range equation, the search for implicit solutions, the deduction of trigonometric solutions and the conclusion.

2. IB-FUNCTIONS

iB- implicit functions are generally defined by

$$J_{n,m}\left(\sum_{i=0}^{p}\alpha_i x_i\right)=\sinh^m\left(\sum_{i=0}^{p}\alpha_i x_i\right)/\cosh^n\left(\sum_{i=0}^{p}\alpha_i x_i\right), \tag{1}$$

where $J_{n,m}\left(\sum_{i=0}^{p}\alpha_i x_i\right)$ represents the implicit form of the function, $\sinh^m\left(\sum_{i=0}^{p}\alpha_i x_i\right)/\cosh^n\left(\sum_{i=0}^{p}\alpha_i x_i\right)$ the explicit form of the function, α_i $(i=0,1,2,...,p)$ represent the parameters associated with the independent variables x_i $(i=0,1,2,...,p)$, the pair $(n,m)\in R^2$ indicates the power of the function. More precisely, n is the power of $\cosh\left(\sum_{i=0}^{p}\alpha_i x_i\right)$ and m the power of $\sinh\left(\sum_{i=0}^{p}\alpha_i x_i\right)$. This function, as defined in relation (1), is also called the iB-functions of several variables and any derivative operation undertaken in this case is partial.
The iB-functions of a single variable is defined by

$$J_{n,m}(\alpha x)=\sinh^m(\alpha x)/\cosh^n(\alpha x), \tag{2}$$

where $J_{n,m}(\alpha x)$ represents the implicit form of the function, α represents the parameter associated with the independent variable x, the pair $(n,m)\in R^2$ indicates the power of the function.
some important transformations are given by

$$J_{n+1,m+1}=J_{n,m}J_{1,1}, \tag{3}$$

$$J_{n+2,m+2}=J_{n,m}J_{2,2}, \tag{4}$$

$$J_{n+p,m+p}=J_{n,m}J_{p,p}, \tag{5}$$

$$J_{n-p,m-p}=J_{n,m}J_{-p,-p}, \tag{6}$$

$$J_{n+p,m+p}=J_{2p,2p}J_{n-p,m-p}, \tag{7}$$

$$J_{n,m}.J_{n',m'} = J_{n+n',m+m'},$$

$$J_{n,m}.J_{m,n} = J_{m+n,m+n}, \quad (8)$$

$$J_{m_1,n_1}.J_{m_2,n_2}...J_{m_p,n_p} = J_{m_1+m_2+...+m_p,n_1+n_2+...+n_p}, \quad (9)$$

$$\frac{dJ_{n,m}}{dx} = m\alpha J_{n-1,m-1} - n\alpha J_{n+1,m+1}, \quad (10)$$

and

$$J_{n,m}(i\alpha x) = (i)^m T_{n,m}(\alpha x), i^2 = -1. \quad (11)$$

Some of properties in its compact forms which facilitate the addition and multiplication of expressions are given by the following formulas

$$J_{n,m}(x+y) = \frac{\sum_{k'=0}^{m} C_m^{k'} J_{-k',m-k'}(x) J_{-m+k',k'}(y)}{\sum_{k=0}^{n} C_n^{k} J_{-n+k,k}(x) J_{-n+k,k}(y)}, \quad (12)$$

By matching $y \to -y$, we obtain

$$J_{n,m}(x-y) = \frac{\sum_{k'=0}^{m} (-1)^{k'} C_m^{k'} J_{-k',m-k'}(x) J_{-m+k',k'}(y)}{\sum_{k=0}^{n} (-1)^{k} C_n^{k} J_{-n+k,k}(x) J_{-n+k,k}(y)}, \quad (13)$$

The compact trigonometric formulas which result from formulas (12) and (13) are given by

$$J_{n,m}[i(x+y)] = \frac{\sum_{k'=0}^{m} C_m^{k'} J_{-k',m-k'}(ix) J_{-m+k',k'}(iy)}{\sum_{k=0}^{n} C_n^{k} J_{-n+k,k}(ix) J_{-n+k,k}(iy)}, i^2 = -1, \quad (14)$$

and

$$J_{n,m}[i(x-y)] = \frac{\sum_{k'=0}^{m} (-1)^{k'} C_m^{k'} J_{-k',m-k'}(ix) J_{-m+k',k'}(iy)}{\sum_{k=0}^{n} (-1)^{k} C_n^{k} J_{-n+k,k}(ix) J_{-n+k,k}(iy)}, i^2 = -1. \quad (15)$$

3. RESULTS AND DISCUSSION

3.1. OBTAINING THE MAIN RANGE EQUATION

The modified KdV equation chosen to solve is of the form [32]

$$U_t + \alpha U^2 U_x + \beta U_x U_{xx} + \gamma U U_{xxx} + U_{xxxxx} = 0, \quad (16)$$

where α , β and γ are nonlinear coefficients. We assign arbitrary coefficients to the above equation to obtain its following generalized form.

$$n_0U_t + n_1U^2U_x + n_2U_xU_{xx} + n_3UU_{xxx} + n_4U_{xxxxx} = 0. \quad (17)$$

The goal being not only the resolution of the equation, but also the obtaining of the relations of constraints linking the coefficients $n_i\left(i=0,...,4\right)$ favoring the widening of the field of analysis of the solutions. In order to build solutions in the form

$$U\left(x,t\right) = aJ_{n,m}\left(\alpha x - \alpha_0 t\right), \quad (18)$$

where a, α and α_0 are arbitrary constants, n and m the indicators of the iB-function, we set the change of variable $\xi = \alpha x - \alpha_0 t$ and Eq. 17 becomes

$$-n_0\alpha_0U_\xi + n_1\alpha U^2U_\xi + n_2\alpha^3U_\xi U_{\xi\xi} + n_3\alpha^3UU_{\xi\xi\xi} + n_4\alpha^5U_{\xi\xi\xi\xi\xi} = 0. \quad (19)$$

With regard to the terms of Eq.19 , we note that progress in the search for the desired solution requires to calculate the successive derivatives which constitute the equation. So the most imposing terms give

$$U^2U_\xi = a^3mJ_{3n-1,3m-1} - a^3nJ_{3n+1,3m+1}, \quad (20)$$

$$U_\xi U_{\xi\xi} = A_1J_{2n-3,2m-3} - A_2J_{2n-1,2m-1} + A_3J_{2n+1,2m+1} - A_4J_{2n+3,2m+3}, \quad (21)$$

with

$$A_1 = a^2m^2\left(m-1\right), \quad (22)$$

$$A_2 = a^2nm\left(m-1\right) + a^2m^2\left(n-1\right) + a^2nm\left(m+1\right), \quad (23)$$

$$A_3 = a^2nm\left(n+1\right) + a^2nm\left(n-1\right) + a^2n^2\left(m+1\right), \quad (24)$$

and

$$A_4 = a^2n^2\left(n+1\right), \quad (25)$$

$$UU_{\xi\xi\xi} = B_1J_{2n-3,2m-3} - B_2J_{2n-1,2m-1} + B_3J_{2n+1,2m+1} - B_4J_{2n+3,2m+3}, \quad (26)$$

with

$$B_1 = a^2m\left(m-1\right)\left(m-2\right), \quad (27)$$

$$B_2 = a^2m\left(m-1\right)\left(n-2\right) + a^2m^2\left(n-1\right) + a^2nm\left(m+1\right), \quad (28)$$

$$B_3 = a^2nm\left(n-1\right) + a^2n^2\left(m+1\right) + a^2n\left(n+1\right)\left(m+2\right), \quad (29)$$

and

$$B_4 = a^2n\left(n+1\right)\left(n+2\right), \quad (30)$$

$$U_{\xi\xi\xi\xi\xi} = C_1J_{n-5,m-5} - C_2J_{n-3,m-3} + C_3J_{n-1,m-1} - C_4J_{n+1,m+1} + C_5J_{n+3,m+3} - C_6J_{n+5,m+5}, \quad (31)$$

with

$$C_1 = am\left(m-1\right)\left(m-2\right)\left(m-3\right)\left(m-4\right), \quad (32)$$

$$C_2 = am\left(m-1\right)\left(m-2\right)\left(m-3\right)\left(n-4\right) + am\left(m-1\right)\left(m-2\right)^2\left(n-3\right)$$
$$+am\left(m-1\right)\left(n-2\right)\left(m-1\right)\left(m-2\right) + am^2\left(n-1\right)\left(m-2\right)^2 + anm\left(m+1\right)\left(m-1\right)\left(m-2\right), \quad (33)$$

$$C_3 = am\left(m-1\right)\left(m-2\right)\left(n-3\right)\left(n-2\right) + am\left(m-1\right)^2\left(n-2\right)^2 + am^2\left(n-1\right)\left(m-2\right)\left(n-2\right)$$
$$+amn\left(m+1\right)\left(m-1\right)\left(n-2\right) + am^2\left(m-1\right)\left(n-2\right)\left(n-1\right) + am^3\left(n-1\right)^2 + am^2n\left(m+1\right)\left(n-1\right)$$
$$+anm^2\left(n-1\right)\left(m+1\right) + amn^2\left(m+1\right)^2 + anm\left(n+1\right)\left(m+2\right)\left(m+1\right),$$

(34)

$$C_4 = am(m-1)(n-2)(n-1)n + am^2(n-1)^2 n + amn^2(m+1)(n-1)$$
$$+an^2 m(n-1)(m+1) + an^3(m+1)^2 + an^2(n+1)(m+2)(m+1) + anm(n-1)(n+1)(m+2)$$
$$+an^2(m+1)(n+1)(m+2) + an(n+1)^2(m+2)^2 + an(n+1)(n+2)(m+3)(m+2),$$

(35)

$$C_5 = anm(n-1)(n+1)(n+2) + an^2(m+1)(n+1)(n+2) + an(n+1)^2(m+2)(n+2)$$
$$+an(n+1)(n+2)^2(m+3) + an(n+1)(n+2)(n+3)(m+4),$$

(36)

and

$$C_6 = an(n+1)(n+2)(n+3)(n+4). \quad (37)$$

The taking into account of the Eqns. 20-37(20)-) in the Eq.19 leads to the following equation

$$n_4\alpha^5 C_1 J_{n-5,m-5} - n_4\alpha^5 C_2 J_{n-2,m-3} + \left(n_4\alpha^5 C_3 - n_0\alpha_0 am\right) J_{n-1,m-1} + \left(n_0\alpha_0 an - n_4\alpha^5 C_4\right) J_{n+1,m+1}$$
$$+n_4\alpha^5 C_5 J_{n+3,m+3} - n_4\alpha^5 C_6 J_{n+5,m+5} + \left(n_2\alpha^3 A_1 + n_3\alpha^3 B_1\right) J_{2n-3,2m-3} - \left(n_2\alpha^3 A_2 + n_3\alpha^3 B_2\right) J_{2n-1,2m-1}$$
$$+\left(n_2\alpha^3 A_3 + n_3\alpha^3 B_3\right) J_{2n+1,2m+1} - \left(n_2\alpha^3 A_4 + n_3\alpha^3 B_4\right) J_{2n+3,2m+3} + n_1\alpha ma^3 J_{3n-1,3m-1} - n_1\alpha na^3 J_{3n+1,3m+1} = 0,$$

(38)

Eq. 38 is the main coefficient range equation to analyze. Thus, the different values of n and m favorable to the search for solutions will be given in the following sections.

3.2 FIELD OF POSSIBLE SOLUTIONS

We are looking for the values of n and m for which certain terms of Eq. 38 are grouped together. Thus, to obtain the values of n and m for which certain terms of Eq.38 regroup, we solve the pairs of equations in n and m such that if $\lambda_i J_{n,m}$ and $\lambda_j J_{n',m'}$ (with $i \neq j$) are two terms of Eq.30 and where λ_i and λ_j are constants, we have simultaneously $n = n'$ and $m = m'$.

In this quest, we count a total of 42 pairs of equations which lead to the determination of the pairs (n, m). Such that we have

$$n, m \in \{-8, -6, -4, -3, -2, -1, 0, 1, 2, 3, 4, 6, 8\}. \quad (39)$$

The combination of the values of n and m for which there is a grouping of the terms makes it possible to obtain a table of the possibilities of solutions comprising 169 pairs.

The 169 pairs constitute the extended field of pairs for which the search for solutions must be made. But the probabilities of appearance of pairs make it possible to determine the most probable pairs and thereby reduce the field of possibilities in order to obtain a restricted field of possibilities in which the effective research will be carried out.

3.3. PROBABILITIES OF APPEARANCE OF PAIRS (n, m) AND DOMINANT PAIRS

On the 42 pairs of equations solved in n and m, the probabilities of obtaining pairs (n, m) for which certain terms of the range equation are grouped together are given by

$P(-2,-2)=7/42$, $P(-3,-3)=2/42$, $P(-6,-6)=2/42$, $P(-8,-8)=1/42$, $P(4,4)=4/42$, $P(2,2)=8/42$, $P(0,0)=8/42$, $P(1,1)=2/42$, $P(-4,-4)=2/42$, $P(6,6)=2/42$, $P(-1,-1)=2/42$, $P(3,3)=1/42$, $P(8,8)=1/42$.

$P(n,m)$ represents the fraction of the number of times that the couple (n,m) appears in the grouping possibilities or the probability of obtaining the solution for the pair (n,m). The combination of the values of n and m obtained above, gives the pairs of the main field or extended field of solutions research. With regard to the probabilities of appearance of the pairs, we remark that $P(0,0)=8/42$, $P(-2,-2)=7/42$ and $P(2,2)=8/42$. Then, the pairs ($0,0$),($-2,-2$) and ($2,2$) are the dominant pairs.

A combination of the values of n and m for these dominant pairs forms the restricted field of search for solutions. The following table is the narrow field of the search for solutions such that the pairs which form it will examine in detail to see if they lead to solutions.

Table 1: Restricted field of possibilities

(n,m)	-2	0	2
-2	$(-2,-2)$	$(-2,0)$	$(-2,2)$
0	$(0,-2)$	$(0,0)$	$(0,2)$
2	$(2,-2)$	$(2,0)$	$(2,2)$

The goal of obtaining the restricted field of pairs aims to verify whether in addition to the dominant pairs, there are other pairs which lead to non-trivial solutions. Thus, in the following lines, we will solve Eq.38 for the different values of the above pairs. But before going to the effective resolution, we will determine the values of $A_i\,(i=1,\dots,4)$, $B_i\,(i=1,\dots,4)$ and $C_i\,(i=1,\dots,6)$ for each pair (n , m) of the restricted field.

3.3. CALCULATION OF COEFFICIENTS OF TERMS

For each couple in the restricted field of possibilities, the values of $A_i\,(i=1,\dots,4)$, $B_i\,(i=1,\dots,4)$ and $C_i\,(i=1,\dots,6)$ are as follows.

- For $(n,m)=(-2,-2)$, we have

$A_1=-12a^2$,

$A_2=-28a^2$, $A_3=-20a^2$, $A_4=-4a^2$, $B_1=-24a^2$, $B_2=-42a^2$, $B_3=-16a^2$, $B_4=0$,

$C_1=-720a$, $C_2=-1488a$, $C_3=-1280a$, $C_4=-272a$, $C_5=0$, $C_6=0$.

- For $(n,m)=(2,2)$, we have

$A_1 = 4a^2$,

$A_2 = 20a^2$, $A_3 = 28a^2$, $A_4 = 12a^2$, $B_1 = 0$, $B_2 = 16a^2$, $B_3 = 40a^2$, $B_4 = 24a^2$, $C_1 = 0$,

$C_2 = 0$, $C_3 = 260a$, $C_4 = 1232a$, $C_5 = 1680a$, $C_6 = 720a$.

- For $(n,m) = (2,0)$, we have

$A_1 = 0$,

$A_2 = 0$, $A_3 = 4a^2$, $A_4 = 12a^2$, $B_1 = 0$, $B_2 = 0$, $B_3 = 16a^2$, $B_4 = 24a^2$, $C_1 = 0$, $C_2 = 0$,

$C_3 = 0$, $C_4 = 272a$, $C_5 = 960a$, $C_6 = 240a$.

- For $(n,m) = (-2,0)$, we have

$A_1 = 0$,

$A_2 = 0$, $A_3 = 4a^2$, $A_4 = -4a^2$, $B_1 = 0$, $B_2 = 0$, $B_3 = 8a^2$, $B_4 = 0$, $C_1 = 0$, $C_2 = 0$, $C_3 = 0$,

$C_4 = -32a$, $C_5 = 4a$, $C_6 = 0$.

- For $(n,m) = (2,-2)$, we have

$A_1 = -12a^2$,

$A_2 = 4a^2$, $A_3 = -20a^2$, $A_4 = 12a^2$, $B_1 = -24a^2$, $B_2 = 8a^2$, $B_3 = -8a^2$, $B_4 = 24a^2$,

$C_1 = -720a$, $C_2 = -224a$, $C_3 = -32a$, $C_4 = 0$, $C_5 = 240a$, $C_6 = -240a$.

- For $(n,m) = (-2,2)$, we have

$A_1 = 4a^2$,

$A_2 = -28a^2$, $A_3 = 28a^2$, $A_4 = -4a^2$, $B_1 = 0$, $B_2 = -32a^2$, $B_3 = 32a^2$, $B_4 = 0$, $C_1 = 0$,

$C_2 = 0$, $C_3 = 464a$, $C_4 = -512a$, $C_5 = 0$, $C_6 = 0$.

- For $(n,m) = (0,-2)$, we have

$A_1 = -12a^2$,

$A_2 = -4a^2$, $A_3 = 0$, $A_4 = 0$, $B_1 = -24a^2$, $B_2 = -16a^2$, $B_3 = 0$, $B_4 = 0$, $C_1 = -720a$,

$C_2 = -400a$, $C_3 = -280a$, $C_4 = 0$, $C_5 = 0$, $C_6 = 0$.

- For $(n,m) = (0,2)$, we have

$A_1 = 4a^2$,

$A_2 = -4a^2$, $A_3 = 0$, $A_4 = 0$, $B_1 = -4a^2$, $B_2 = 0$, $B_3 = 0$, $B_4 = 0$, $C_1 = 0$, $C_2 = 0$, $C_3 = 24a$,

$C_4 = 0$, $C_5 = 0$, $C_6 = 0$.

3.4 SOLVING THE MAIN RANGE EQUATION

When we fix the values of the pairs (n,m), the resulting range equation is the secondary coefficient range equation. Thus, we look for the solutions of Eq. 38 for the pairs of the restricted field of possibilities.

For the pairs $(0,0)$,$(-2,2)$, $(2,-2)$, $(2,0)$, $(0,-2)$,$(-2,0)$ and $(0,2)$ the equation admits trivial solutions so that the search for solutions is reduced only for the pairs $(-2,-2)$ and $(2,2)$.

- Case $(n,m)=(-2,-2)$

Taking into account the pair $(n,m)=(-2,-2)$ in the Eq. 38 we obtain

$$\left[-720an_4\alpha^5-12a^2n_2\alpha^3-24a^2n_3\alpha^3-2n_1\alpha a^3\right]J_{-7,-7}+\left[1488an_4\alpha^5+28a^2n_2\alpha^3+42a^2n_3\alpha^3+2n_1\alpha a^3\right]J_{-5,-5}$$
$$-\left[1280an_4\alpha^5+2n_0\alpha_0 a+20a^2n_2\alpha^3+16a^2n_3\alpha^3\right]J_{-3,-3}-\left[2n_0\alpha_0 a-272an_4\alpha^5-4a^2n_2\alpha^3\right]J_{-1,-1}=0, \quad (40)$$

Eq. 40 is valid for $a\neq 0$ if and only if we have

$$360n_4\alpha^4+\left(6n_2\alpha^2+12n_3\alpha^2\right)a+n_1a^2=0, \quad (41)$$

$$744n_4\alpha^4+\left(14n_2\alpha^2+21n_3\alpha^2\right)a+n_1a^2=0, \quad (42)$$

$$640n_4\alpha^5+n_0\alpha_0+\left(10n_2\alpha^3+8n_3\alpha^3\right)a=0, \quad (43)$$

and

$$n_0\alpha_0-136n_4\alpha^5-2an_5\alpha^3. \quad (44)$$

From Eq.44 we obtain

$$a=\left(n_0\alpha_0-136n_4\alpha^5\right)/2n_2\alpha^3,\ n_2\neq 0,\ \alpha\neq 0. \quad (45)$$

The introduction of Eq.45 in Eqns.41-44 permits to obtain the constraint relation

$$40n_2+61n_3=0. \quad (46)$$

The solution in this case is given by

$$U(\xi)=\left[\left(n_0\alpha_0-136n_4\alpha^5\right)/2n_2\alpha^3\right]J_{-2,-2}(\xi)\Rightarrow U(x,t)=\left[\left(n_0\alpha_0-136n_4\alpha^5\right)/2n_2\alpha^3\right]J_{-2,-2}(\alpha x-\alpha_0 t). \quad (47)$$

- Case $(n,m)=(2,2)$

Taking into account the pair $(n,m)=(2,2)$ in Eq.38 we obtain

$$\left[260an_4\alpha^5-2n_0\alpha_0 a+4a^2n_2\alpha^3\right]J_{1,1}+\left[2n_0\alpha_0 a-1232an_4\alpha^5-20a^2n_2\alpha^3-16a^2n_3\alpha^3\right]J_{3,3}$$
$$+\left[28a^2n_2\alpha^3+40a^2n_3\alpha^3+2n_1\alpha a^3\right]J_{5,5}-\left[720n_4\alpha^5+12a^2n_2\alpha^3+24a^2n_3\alpha^3+2n_1\alpha a^3\right]J_{7,7}=0, \quad (48)$$

Eq.48is valid for $a\neq 0$ if and only if we have

$$130n_4\alpha^5-n_0\alpha_0+2an_2\alpha^3=0, \quad (49)$$

$$14an_2\alpha^2+20an_3\alpha^2+n_1a^2=0, \quad (50)$$

$$360n_4\alpha^4+6an_2\alpha^2+12an_3\alpha^2+n_1a^2=0, \quad (51)$$

and

$$n_0\alpha_0-616n_4\alpha^5-10an_2\alpha^3-8an_3\alpha^3=0. \quad (52)$$

From Eq.52, we obtain

$$a=\left(130n_4\alpha^5-n_0\alpha_0\right)/2n_2\alpha^3,\ n_2\neq 0,\ \alpha\neq 0. \quad (53)$$

The introduction of the Eq.53 in Eqns.49-52permits to obtain the constraint relation

$$213n_2^2 - 117n_2n_3 - 240n_3^2 = 0. \tag{54}$$

The solution in this case is given by

$$U(\xi) = \left[\left(130n_4\alpha^5 - n_0\alpha_0\right)/2n_2\alpha^3\right]J_{2,2}(\xi) \Rightarrow U(x,t) = \left[\left(130n_4\alpha^5 - n_0\alpha_0\right)/2n_2\alpha^3\right]J_{2,2}(\alpha x - \alpha_0 t). \tag{55}$$

3.5 TRIGONOMETRIC SOLUTIONS

One of the great peculiarities of the use of the iB-function is that it facilitates the passage from the hyperbolic form to the trigonometric form and vice versa. When we make the correspondences $\alpha \to i\alpha$ and $\alpha_0 \to i\alpha_0$ with $i^2 = -1$, we obtain respectively from Eqns. 46,47 the following trigonometric solutions

$$U(x,t) = -i\left[\left(n_0\alpha_0 - 136n_4\alpha^5\right)/2n_2\alpha^3\right]\cot an^2(\alpha x - \alpha_0 t), \tag{56}$$

and

$$U(x,t) = -i\left[\left(130n_4\alpha^5 - n_0\alpha_0\right)/2n_2\alpha^3\right]\tan^2(\alpha x - \alpha_0 t). \tag{57}$$

4. CONCLUSION

The objective of this work was to show how, in the impossibility of using the integral methods to solve a nonlinear partial differential equation, one can proceed to choose or know the suitable form of solution. To this end, we decided to use this technique to first locate the forms of solutions and then build them by relying on the modified partial differential equations of the KdV type. For this purpose, we have considered building a solution of the form $aJ_{n,m}(\alpha x - \alpha_0 t)$ where $a, n, m, \alpha, \alpha_0$ are real constants to be determined. But the fixed values of n and m are those which make it possible to indicate with precision the solution functions. It is for this that the work to be done first and foremost consisted in determining the values of n and m for which we suspect the solutions. In this perspective, we have obtained 42 possible (n,m) pairs forming what we have called the extended field of the possibilities of solutions. But of all these pairs, only a few are more favorable to obtaining the solutions. These pairs, called dominant pairs, are identified through a high probability of presence among the pairs for which certain terms of the coefficient range equation are grouped together.

In the case of this study, we have the pairs $(0,0)$, $(-2,-2)$ and $(2,2)$ having respectively for probability $P(0,0) = 8/42$, $P(-2,-2) = 7/42$ and $P(2,2) = 8/42$. These three pairs indicated are those which are more favorable to obtaining the solutions. But in order to detect other particular solutions which do not appear at first glance, we have combined the values of n and m of the dominant pairs to make what we have called a restricted field of the possibilities of obtaining solutions.

This allowed to obtain a total of nine (n,m) pairs that we thoroughly examined in search of possible solutions. Of all these analyzes, only the dominant pairs $(-2,-2)$ and $(2,2)$ made it possible to have non-trivial solutions, the pair $(0,0)$ leading to a trivial solution like

$U(\xi)=a$. We have deduced from the implicit solutions obtained, the trigonometric solutions by making use of the magnificent properties of iB-functions.

These results obtained confirm our predictions, namely, only the $(n,m)\neq(0,0)$ pairs having the greatest probabilities of appearance among the pairs of the field of possibilities are the most favorable to obtaining the solutions.

We can see that unlike the classical KdV equation which has the third-order dispersion term and admits a pulse-type solitary wave solution for (n,m) = (2,0), the modified KdV equation has the term of dispersion of order six and as treated in this article, admits non-trivial solutions just for pairs (-2, -2) and (2,2), which are solitary wave solutions of the kink type.

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