

An Application of Interacting Boson Model-2 (IBM-2) Configuration Mixing in Tin nuclei

Abstract

Using IBM-2 configuration mixing calculations, the normal and intruder 2p-2h bands in even-even tin isotopes are examined. The states of the normal and intruder bands were computed separately and then mixed using a basic band-mixing Hamiltonian. The experimental data for energy levels and electronic transition probability from current and past investigations are compared.

Key Words: Interacting Boson Model, Configuration Mixing, Sn nuclei

1. Introduction

The structure of even-even Sn nuclei has previously been investigated using a variety of models, including the BCS approach with neutron two-quasiparticle excitation [1], broken-pair or generalized seniority schemes [2], and others. In the $Z=50$ zone, the presence of a collective band commencing at 0^+ level [3] is a common occurrence. Excitations over the proton shell closure at $Z=50$ are the source of these intruder states [4]. Wenes *et al.*, [5] used a model that included both pure quadrupole vibrational excitations of doubly even nuclei and proton 2p-2h configurations paired with quadrupole vibrational excitations to examine these collective bands in even-even Sn isotopes. The lowest 2_1^+ state in $^{116,118,120}\text{Sn}$ has been defined as vibrational states by Ring and Schuck [6]. The levels 2_2^+ , 2_3^+ and 4_1^+ exhibit a vibrational property, according to a Coulomb excitation study in even Sn isotopes [7]. The measured value of $B(E2; 4_1^+ \rightarrow 2_1^+)$ for $^{116,118}\text{Sn}$, for example, is the same as the anticipated value $2B(E2; 2_1^+ \rightarrow 0_1^+)$ for a two phonon vibrational state. The $B(E2; 2_2^+ \rightarrow 0_1^+)$ and $B(E2; 2_3^+ \rightarrow 0_1^+)$ crossover transitions are likewise severely delayed. Wenes *et al.*, viewed the low-lying structure of Sn isotopes as evidence of extensive mixing between the vibrational and rotational structures. The low-lying levels of $^{115,117,119}\text{Sn}$ isotopes were recently explained in terms of the $U^{BF}(5) \otimes SU^F(4)$ dynamical symmetry $U(6/20)$ super Lie group [8]. The intruder bands in neighboring Cd [9] and Te [10] nuclei have been well described in IBM-2 using a configuration mixing analysis. The regular and intruder configurations are strongly intermingled, according to these findings. The experimental electric transition rate estimates of several Sn isotope transitions also point to a significant mixing of the two coexisting forms.

The goal of this research is to investigate the ground-state band up to two phonon triplets and the collective bands in even-even $^{112,114, 116,118}\text{Sn}$ nuclei using IBM-2 mixing configuration calculations in which the 2p-2h band is linked to the ground band's anharmonic quadrupole vibration. In the context of I spin, it's also important to consider the implications for Sn isotopes [11].

2. The Interacting Boson Model-2 (IBM-2)

In this work, the neutron-proton version of the Interacting Boson Model (IBM-2) is employed, which distinguishes between neutron (ν) and proton (π) bosons; a detailed explanation of IBM-2 may be found in [12]. In IBM-2, there are three terms to the Hamiltonian operator: one for proton bosons, one for neutron bosons, and one for interactions between unlike bosons. The approach of Duval and Barrett [12] is used in this calculation. The IBM-2 Hamiltonian is defined as follows::

$$H = \varepsilon_d (d_{\pi}^+ d_{\pi}^- + d_{\nu}^+ d_{\nu}^-) + \kappa(Q_{\pi}^{\wedge} \cdot Q_{\nu}^{\wedge}) + V_{\pi\pi} + V_{\nu\nu} + M_{\pi\nu} \dots\dots\dots (1)$$

Where

$$V_{\rho\rho} = \frac{1}{2} \sum_{J=0,2,4} (2L+1)^{1/2} C_{L\rho} \left[(d_{\rho}^+ \times d_{\rho}^+)_{\pi}^{(L)} (d_{\rho}^- \times d_{\rho}^-)^{(L)} \right]^{(0)} \dots\dots\dots (2)$$

refers to the interaction between bosons that are similar. The proton (π) or neutron (ν) bosons are represented by the letter ρ .

Where Q_{π}, Q_{ν} as:

$$Q_{\rho}^{(2)} = (s_{\rho}^+ \times d_{\rho}^- + d_{\rho}^+ \times s_{\rho})^{(2)} + \chi_{\rho} (d_{\rho}^+ \times d_{\rho}^-)^{(2)} \dots\dots\dots (3)$$

presented the neutron and proton bosons quadrupole-quadrupole interaction.

$$M_{\pi\nu} = \frac{1}{2} \xi_2 (s_{\pi}^+ \times d_{\nu}^+ - d_{\pi}^+ \times s_{\nu}^+) \cdot (s_{\pi} \times d_{\nu}^- - d_{\pi}^- \times s_{\nu}) - \sum_{K=1,3} \xi_K \left[(d_{\pi}^+ \times d_{\nu}^+)^{(K)} \cdot (d_{\pi}^- \times d_{\nu}^-)^{(K)} \right] \dots (4)$$

is the Majorana force. Both the normal and intruder configurations are computed separately using the above Hamiltonian. The two configurations are then combined using the mixing operator.

$$V_{mix} = \alpha (s_{\pi}^+ s_{\pi}^+ + s_{\pi}^- s_{\pi}^-)^{(0)} + \beta (d_{\pi}^+ d_{\pi}^+ + d_{\pi}^- d_{\pi}^-) \dots\dots\dots (5)$$

Where α and β are strength parameters for the interaction between the two configurations that can be modified In our approach, $H + V_{mix}$ was diagonalized using the basis consisting of the lowest four eigenstates of each configuration. The entire mixing Hamiltonian is then calculated as follows:

$$H_{mix} = H_1 + H_2 + V_{mix} \dots\dots\dots (6)$$

where $H_1(H_2)$ is the IBM-2 Hamiltonian for the first (second) configuration, as defined by Eq. (6), and the energies of the second configuration have been increased by an amount V_{mix} .

The energy gap parameter Δ between the two configurations is calculated using the relation [12];

$$\Delta = B.E(Z, N) - B.E(Z-2, N) - B.E(Z+2, N) + B.E(Z, N) + 4V_{ph} \dots\dots\dots (7)$$

$B.E$ is the ground-state binding energy, and V_{ph} is the particle-hole interaction energy.

Reduced electric transition probability values of transitions and electric quadrupole moments are calculated using mixed wave functions. The E2 transition operator is defined as follows:

$$T^{(E2)} = e_0(e_{\pi 0}Q_{\pi 0} + e_{\nu 0}Q_{\nu 0}) + e_2(e_{\pi 2}Q_{\pi 2} + e_{\nu 2}Q_{\nu 2}) \dots\dots\dots (8)$$

where Q_ρ is defined by Eq.(3) and e_j and $e_{j\rho}$ ($j = 0,2$) are variables that can be changed. The prefixes 0 and 2 respectively correspond to the normal and intruder setups. The reduced electric transition probability $B(E2)$ is written as:

$$B(E2; J_i^+ \rightarrow J_f^+) = \frac{1}{2J_i + 1} \left| \langle J_f^+ || T^{(E2)} || J_i^+ \rangle \right|^2 \dots\dots\dots (9)$$

3. Results and Discussion

3.1 Energy Levels

In Sn isotopes, distinguishing the high-energy members of the usual ground-state band is particularly difficult. In these isotopes, even the two phonon triplets mix with the 2p-2h band and the neutron two quasiparticle levels. In our calculations, we used levels up to the lowest 4_1^+ of the regular band and up to the condition of the invading band. Only terms in the Hamiltonian involving neutron bosons will contribute to the estimation of the energy value because the Sn isotopes' typical configuration lacks proton bosons. The three parameters used in the normal band calculation are ε_d , $C_{0\pi}$, and $C_{2\pi}$, and their values are presented in Table (1).

A gamma soft structure is expected for the intruder configuration. For this calculation, the intruder band was chosen because it has an $E(4_1^+)/E(2_1^+)$ ratio of around 2.0, which is close to the SU(5) limit. The values of the parameter ε_d adopted show a smooth variation with the neutron number, peaking at mid-shell. The value of κ increases from $N = 62$ to $N = 64$, then gradually decreases as the neutron number increases. A similar form of neutron number dependency has been reported for the parameter κ in the computation of Te isotopes. In all intruder bands of Sn isotopes, the value of χ_π has been kept constant. The χ_ν parameter in the intruder configuration has been altered. The adopted values for this parameter are somewhat less than zero. Calculations on adjacent Cd isotopes yielded the Majorana force parameters ξ_1 , ξ_2 , and ξ_3 [9]. The V_{mix} mixing operator admixes the two separately determined configurations. According to Jolie and Lehmann [13], there is only one free parameter for a U(5)-O(6) mixing using Eq.(5) because the matrix components of the two terms containing the two types of bosons are not independent.

As a result, we simply employed one parameter, α , and we didn't vary it based on the mass number. The energy-gap parameter Δ between the two configurations was

calculated using Eq.(7). The $Z = 50$ region the V_{ph} value was found to be 2.540 MeV [14]. The required experimental binding energy values were calculated using the Ref. [15]. The remaining options are all set to zero.

Table (1a): The IBM-2 Hamiltonian Parameters in (MeV units) for Normal Configuration ($N_{\pi} = 0$).

Parameters	^{112}Sn	^{114}Sn	^{116}Sn	^{118}Sn
\mathcal{E}_d	1.25	1.30	1.25	1.20
κ	-0.167	-0.140	-0.147	-0.158
χ_v	0.73	0.65	1.90	0.85
χ_{π}	0.40	0.40	0.40	0.40
C_{0v}	-0.31	-0.31	-0.35	-0.31
C_{2v}	-0.15	-0.15	-0.20	-0.12
C_{4v}	-0.05	0.0	0.0	0.0
ξ_2	0.04	0.04	0.04	0.04

$$\xi_1 = \xi_3 = 0.24 \text{ MeV}, C_{0\pi} = C_{2\pi} = C_{4\pi} = 0.0 \text{ MeV}$$

Table (1b): The IBM-2 Hamiltonian Parameters in (MeV units) for Intruder Configuration ($N_{\pi} = 2$).

Parameters	^{112}Sn	^{114}Sn	^{116}Sn	^{118}Sn
\mathcal{E}_d	0.64	0.62	0.59	0.63
κ	-0.167	-0.140	-0.147	-0.158
χ_v	-0.21	-0.17	-0.12	-0.07
χ_{π}	0.40	0.40	0.40	0.40
C_{0v}	0.0	-0.30	0.0	-0.30
C_{2v}	-0.15	-0.15	-0.20	-0.12
C_{4v}	-0.05	0.0	0.0	0.0
ξ_2	0.04	0.04	0.04	0.04

$$\xi_1 = \xi_3 = 0.24 \text{ MeV}, C_{0\pi} = C_{2\pi} = C_{4\pi} = 0.0 \text{ MeV}$$

The energy levels of $^{112-118}\text{Sn}$ even-even isotopes were compared to experiment values [16,17,18,19] are listed in Table (2). The estimated and actual energy spectra of ^{112}Sn and ^{114}Sn nuclei have also been shown to agree. The wave function has little effect on e energy values. As a result, the transition probabilities must be calculated, which are highly dependent on the system's wave function.

Table (2): Comparison between experimental and IBM-2 energy levels for Sn isotopes (in MeV units)

Levels	¹¹² Sn			¹¹⁴ Sn		
	Exp. [16]	IBM-2		Exp. [17]	IBM-2	
		$N_{\pi} = 0$	$N_{\pi} = 2$		$N_{\pi} = 0$	$N_{\pi} = 2$
0_1^+	0.0	0.0	0.0	0.0	0.0	0.0
2_1^+	1.256	1.256	1.255	1.299	1.299	1.301
4_1^+	2.247	2.244	2.787	2.187	2.200	2.521
6_1^+	2.549	2.611	2.891	3.149	3.183	3.542
8_1^+	4.770	4.801	5.011	3.871	3.901	4.341
10_1^+	4.880	4.971	5.189	4.139	4.220	4.341
2_2^+	2.151	2.220	2.592	2.238	2.267	2.890
0_2^+	2.190	2.258	2.491	1.953	1.843	2.225
4_2^+	2.521	2.618	3.782	2.614	2.724	3.110
3_1^+	2.913	3.141	3.543	3.025	3.125	3.321
0_3^+	2.618	2.731	3.109	2.156	2.254	2.980
2_3^+	2.476	2.510	3.311	2.2454	2.280	2.897
Levels	¹¹⁶ Sn			¹¹⁸ Sn		
	Exp. [18]	IBM-2		Exp. [19]	IBM-2	
		$N_{\pi} = 0$	$N_{\pi} = 2$		$N_{\pi} = 0$	$N_{\pi} = 2$
0_1^+	0.0	0.0	0.0	0.0	0.0	0.0
2_1^+	1.293	1.297	1.345	1.229	1.250	1.341
4_1^+	2.390	2.397	2.619	2.280	2.289	2.543
6_1^+	3.032	3.110	3.431	2.999	2.311	3.761
8_1^+	3.492	3.500	3.754	2.889	2.956	3.650
10_1^+	3.547	3.521	3.750	3.108	3.209	3.761
2_2^+	2.112	2.211	2.311	2.042	2.152	2.675
0_2^+	1.756	1.987	2.118	1.758	1.950	2.530
4_2^+	2.529	2.610	2.980	2.408	2.507	3.329
3_1^+	2.996	3.001	3.622	2.725	2.825	3.761
0_3^+	2.027	2.110	2.563	2.058	2.118	2.581
2_3^+	2.225	2.229	2.462	2.120	2.129	2.619

3.2 Electric Transition Probability

To calculate the electric operator, we relied on Eq.(8). The identification of proton and neutron bosons effective charges e_{π} and e_{ν} is crucial for an E2 transition. These isotopes lying in SU(5) limit (vibrational symmetry), therefore, the relationship between (e_{π}, e_{ν}) and the reduced transition probability B(E2) for vibrational limit is given in the form [20]:

$$B(E2; 2_1^+ \rightarrow 0_1^+) = \frac{5(e_\pi N_\pi + e_\nu N_\nu)^2}{N} \dots\dots\dots (10)$$

where $B(E2; 2_1^+ \rightarrow 0_1^+)$ is the experimental reduced transition probability from the first excited states 2_1^+ to the ground state 0_1^+ , N is the total number of bosons. The relation (10) was used to estimate the effective boson charges for proton and neutron bosons. In this calculation we use the following criteria to determine the effective charges. $e_\pi = 0.12$ $e.b$ is a constant throughout the whole isotopic chain and the $e_\nu = 0.09, 0.085, 0.070$ and 0.075 $e.b$ for $^{112,114,116,118}\text{Sn}$ respectively and the ration ratio $e_2 / e_0 = 1.2$.

Between the states, $B(E2)$ values were obtained by evaluating matrix members of the $T^{(E2)}$ operator. Only the neutron component of the first term of the $T^{(E2)}$ operator contributes to the typical configuration of Sn isotopes. Only e_ν and χ_ν are used to evaluate the matrix elements of the first term of the $T^{(E2)}$ operator. The parameters χ_ν and χ_π are the same as in the computation of the energy value. The effective neutron bosonic charge values used exhibit a gradually changed with neutron number, becoming minimum for $N = 66$. In the calculations on neighboring Te isotopes [10], a similar type of variance was detected. For both configurations, the effective bosonic charge for neutron bosons has remained constant. The ratio of the parameters e_2 / e_0 , which has been kept constant at 1.2 for all $^{112-118}\text{Sn}$ isotopes, has a large impact on the E2 matrix elements.

Table (3) compares experimental and computed $B(E2)$ values for the isotopes $^{112-118}\text{Sn}$. Some of the discrepancies between experimental findings and theoretical predictions are discussed. The E2 transition probability of the $2_3^+ \rightarrow 2_1^+$ and $0_3^+ \rightarrow 2_1^+$ transitions in ^{116}Sn are over-predicted. This suggests that the current model may have overestimated the intruder band's contributions to the 2_3^+ and 0_3^+ states. There aren't a lot of experimental data on $^{112,114}\text{Sn}$ isotopes. The experimental value of the transition $4_1^+ \rightarrow 2_1^+$ is surprisingly tiny when compared to surrounding $^{116,118}\text{Sn}$ isotopes, and cannot be explained using the current model. One reason could be that there is a strong contribution from the neutron two quasiparticle structures in this 4_1^+ state,

Table (3): IBM-2 and Experimental Electric Transition Probability for $^{112-118}\text{Sn}$ isotopes in $e^2 b^2$ units

Isotopes	Transitions	Exp.	IBM-2
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^{112}Sn	$2_1^+ \rightarrow 0_1^+$	0.0512(12)	0.0050
	$2_2^+ \rightarrow 0_1^+$	0.00013(4)	0.00018
	$0_2^+ \rightarrow 2_1^+$	≤ 0.029	0.031
	$2_2^+ \rightarrow 2_1^+$	0.037(150)	0.038
	$2_3^+ \rightarrow 0_1^+$	< 0.00019	0.00022
	$2_3^+ \rightarrow 2_1^+$	< 0.0014	0.0019
	$4_1^+ \rightarrow 2_1^+$	0.018(30)	0.020
^{114}Sn	$2_1^+ \rightarrow 0_1^+$	0.05(100)	0.055
	$2_2^+ \rightarrow 0_1^+$	0.07(300)	0.074
	$0_2^+ \rightarrow 2_1^+$	0.0003	0.00041
	$2_2^+ \rightarrow 2_1^+$	0.016	0.018
	$2_3^+ \rightarrow 0_1^+$	0.013	0.0142
	$2_3^+ \rightarrow 2_1^+$	0.021(60)	0.022
	$4_1^+ \rightarrow 2_1^+$	< 0.01	0.011
	$4_2^+ \rightarrow 2_1^+$	0.25	0.028
^{116}Sn	$2_1^+ \rightarrow 0_1^+$	0.0417(13)	0.0420
	$2_2^+ \rightarrow 0_1^+$	0.0002(8)	0.00025
	$0_2^+ \rightarrow 2_1^+$	0.0605(101)	0.066
	$2_2^+ \rightarrow 2_1^+$	0.0131(50)	0.0135
	$2_3^+ \rightarrow 0_1^+$	0.00017(10)	0.0002
	$2_3^+ \rightarrow 2_1^+$	0.0101(54)	0.012
	$4_1^+ \rightarrow 2_1^+$	0.1277(706)	0.129
	$4_2^+ \rightarrow 2_1^+$	$> 5 \times 10^{-6}$	5.5×10^{-6}
	$4_2^+ \rightarrow 2_2^+$	> 0.0403	0.047
^{118}Sn	$2_1^+ \rightarrow 0_1^+$	0.0416(17)	0.044
	$2_2^+ \rightarrow 0_1^+$	0.00026(4)	0.0003
	$0_2^+ \rightarrow 2_1^+$	0.0653(103)	0.071
	$2_2^+ \rightarrow 2_1^+$	0.0237(34)	0.033
	$0_3^+ \rightarrow 2_1^+$	> 0.00072	0.00088
	$4_1^+ \rightarrow 2_1^+$	0.0585(103)	0.061
	$4_2^+ \rightarrow 2_1^+$	< 0.0096	0.0107

Experimental data are taken from Refs. [16,17,18,19]

resulting in a lower transition probability. The experimental value of the transition $B(E2; 0_2^+ \rightarrow 2_1^+)$ for ^{112}Sn isotope is approximately half that of $^{114,116,118}\text{Sn}$ isotopes, a fact that our calculation does not recreate. Except for these discrepancies, practically

every other transition can be described using this simple model. One of the biggest sources of mistake, we believe, may be traced back to the omission of the contributions of two quasiparticle states.

The electric quadrupole moment of the first excited state was calculated based on the following equation:

$$Q_J = \left[\frac{16\pi}{5} \right]^{1/2} \begin{bmatrix} J & 2 & J \\ -J & 0 & J \end{bmatrix} \langle J_J \| T^{(E2)} \| J_J \rangle \dots\dots (11)$$

We estimated the electric quadrupole moment of the first 2_1^+ state in addition to the transition probabilities. In Table (4), the IBM-2 results are compared to the experimental results. Obviously, the $Q(2_1^+)$ values are negative, which means these isotopes have an oblate shape in this first excited state.

Table (4): Electric quadrupole moments for first excited states $Q(2_1^+)$ in *eb* units

isotopes	¹¹² Sn		¹¹⁴ Sn	
	Exp.[16]	IBM-2	Exp. [17]	IBM-2
$Q(2_1^+)$	-0.03(11)	-0.033	-	-0.141
isotopes	¹¹⁶ Sn		¹¹⁸ Sn	
	Exp.[18]	IBM-2	Exp.[19]	IBM-2
$Q(2_1^+)$	-0.17(4)	-0.18	-0.05(14)	-0.057

4. Concluding Remarks

In this research, we looked at both regular and intruder configurations using IBM-2 mixed configuration calculation. Both normal and intruder bands are accurately reproduced by this calculation. The differences between experimental B(E2) values and theoretical expectations, particularly in the two phonon triplets in the Sn isotopes, suggest that the low lying states have a far more complex structure. This analysis did not take into account the neutron two-quasiparticle structure, which could have altered the conclusions.

References

- [1] W. F. Van Gunsteren, E. Bocker, and K. Allaart" Nuclear Structure of Sn Nuclei" *Z. Phys.* **267** (1974) 87.
- [2] G. Bonsignori, M. Savoia, K. Allaart, A. Van Egmond, and G. Te Velde
"Microscopic approach to magnetic excitations in IBM-2", *Nucl. Phys.* **A432**
(1985) 389.
- [3] J. Bron *et al.*, "Shape Coexistence and its Symmetries" *Nucl. Phys.* **A318** (1979)
335.
- [4] K. Heyde, P. Van Isacker, M. Waroquier, G. Wenes, and M.
Sambataro" Description of the Low-Lying Levels in $^{112-114}\text{Cd}$ ", *Phys. Rev. C* **25**
(1982) 3160.
- [5] G. Wenes, P. Van Isacker, M. Waroquier, K. Heyde, and J. Van Maldeghem,
"Collective bands in doubly even Sn nuclei: Energy spectra and electromagnetic
decay properties" *Phys. Rev. C* **23** (1981) 2291.
- [6] P. Ring and P. Schuck "The Nuclear Many-body Problem" Springer-Verlag, New
York, (1980) p. 218.
- [7] N. G. Jonsson, A. Backlin, J. Kantele, R. Julin, M. Luontama, and A. Passoja,
" 0^+ states and E0 and E2 Transition rates in Even-Even Sn Nuclei" *Nucl. Phys.*
A371 (1981) 333.
- [8] A. K. Singh, G. Gangopadhyay, and D. Banerjee "U(6/20) Supersymmetry in
 $^{115-119}\text{Sn}$ Isotopes" *Phys. Rev. C* **53** (1996) 2524.
- [9] M. Deleze *et al.*, "Study of the mixed ground-state and intruder bands in
 $^{110,112,114}\text{Cd}$ Systematics" *Nucl. Phys.* **A551** (1993) 269.
- [10] J. Rikowska, N. J. Stone, P. M. Walker, and W. B. Walters, *Nucl. Phys.* **A505**,
(1989) 145.
- [11] C. De Coster, K. Heyde, B. Decroix, P. Van Isacker, J. Jolie, H. Lehmann, and J.
L. Wood "Particle-hole excitations in the Interacting Boson Model (I) General
Structure and Symmetries" *Nucl. Phys.* **A600** (1996) 251.
- [12] P. D. Duval, and B. R. Barrett" Quantitative Description Configuration Mixing In
the Interacting Boson Model", *Nucl. Phys.* **A376** (1982) 213.
- [13] J. Jolie and H. Lehmann" The U(5)-O(6) model : An Analytic Approach to Shape
Coexistence" , *Phys. Lett. B* **342** (1995) 1.
- [14] E. R. Flynn and P. D. Kunz" Absolute E0 and E2 transition rates and Collective
States", *Phys. Lett.* **68B** (1977) 40.
- [15] C. M. Lederer and V. S. Shirley and Wiley "Table of Isotopes", edited by, New
York, 1978.
- [16] D. De Frenne and E. Jacobs, *Nuclear Data Sheets* 79 (1996) 725.
- [17] Jean Blachot, *Nuclear Data Sheets* 97 (2002) 663.
- [18] Jean Blachot, *Nuclear Data Sheets* 111 (2010) 783.
- [19] K. Kito, *Nuclear Data Sheets* 75 (1995) 118.
- [20] P. Van Isacker, K. Heyde, J. Jolie, and A. Sevrin" The F-spin Symmetric Limits
of the Neutron-Proton Interacting Boson Model" *Ann. Phys. (NY)* 171 (1986)
253.