M-Theory and F-Theory over Theoretical analysis on Cosmic strings and Calabi-Yau manifolds subject to Conifold singularity with Randall-Sundrum model

Abstract – String theory always comes with heavy mathematical rigor as it questions the most significant and impossible attempt to make a scale-invariant phenomenology between general relativity and quantum theory. Thus, steps have been taken to simplify the theory a bit thereby making it accessible to general yet enthusiastic readers of physics. However, as there is numerous mathematics involved in the modeling of this theory, thus, any chance to make a purely non-mathematical approach towards strings would prove vacuous and intimidating making the pathway of this marvelous theory chocked with unnecessary assumptions resulting in false analogies (or hopes) relating to this theory. Thus, where it's almost impossible to proceed without any equations, we have given a few just to wipe out some logical confusion arising to the first readers of strings. Few necessary diagrams are included along with intense theory and least mathematics for making this significant approach of theoretical physicists accessible to general learners or readers.

Topics – Bosonic string theory, supersymmetric string theory, M – theory, F – theory, dualities and interconnectedness, viability, Randall – Sundrum model for tackling the hierarchy problem of particle physics, conifold singularities, Branes, Bulks, Extremal black holes, Ekpyrotic cosmology, topological aspects of Calabi – Yau (CY) manifolds, A and B models, Mirror Symmetries; AdS/CFT, cosmic strings, all in a way accessible to every reader.

Methods – Theoretical analogies, deductions, principles behind the origin, development along with the probable conclusion of this theory, the roots of its origin, the necessary difficulty for detecting those strings, and approaches done by theorists to work out the pathway of achieving Einstein's dream of unification irrespective of several hindrances.

Results – String theory itself is not a complete theory. Rather it's in the process of further development through the increment of time resulting in more applications of mathematics by developing or incorporating them in due needs. Thus, without stating any concrete results, the theory has been tackled in this paper with a viable hypothesis based on the current understanding, and previous attempts are stated to have been made for its success.

Keywords – String Theory; Brane Cosmology; Supergravity; Randall – Sundrum Model; Mirror Symmetries; AdS/CFT.

1 Overview

One of the exotic branches of differential geometry and algebraic topology is the symplectic geometry that studies differentiable manifolds of closed, nondegenerate 2-form. Those symplectic manifolds have an underlying structure of differential manifolds where the latter can be expressed as the differential analysis of the smooth manifolds including the Riemann-Christoffel curvature tensor for the analysis of several properties related to topological and geometric spaces and their curvatures. Classical mechanics with a Hamiltonian formulation made its first appearance in symplectic geometry to study manifolds. As the topic is vast and pervasive encompassing several branches of mathematics and mathematical physics including the topological aspects of string theory, therefore we will try our best to include as much material as needed to give a proper notion of this geometry.

The subject of mathematical physics is too complex to understand in a normal notion. Because it contains several abstractions that people usually imagine as rather nuisance because of their inabilities to probe deeper inside those theories. However, this can be safely said that no subject in this world is as enriched as the mathematics that governs the physical phenomena of nature. Therefore, as nature is beyond a certain limit, either in very large scale structures like that of a supermassive black hole or in very small scale structures like the Planck's scale, due to the extreme difficulty of the observations either through the physical eye or through experiments, the best fit tool to study them has evolved from the basic norms of mathematics, which to create a concrete scenario fulfilling the human imagination, has been getting much and much stronger as evolved through time. One such example which baffled scientists for more than fifty years or so, is the process of a 'Theory of Everything', although particle physics somehow provides a grand unified model by combining the three out of four fundamental forces of nature, namely the electromagnetic force, strong nuclear force and weak nuclear force in a simple Lie algebraic notion reads as;

$$U(1) \otimes SU(2) \otimes SU(3) \subset SU(5) \subset SO(10) \subset E(8)$$

Where $E(8)$ is the most prominent fundamental group known as the exceptional Lie group. However, unlike the dream of many scientists, most notable Stephen William Hawking, gravity couldn't be included in the standard model because of the complexity of the quantum mechanics, that the physicists faced when incorporating such a weak force, which is immensely weaker than, even the weak nuclear force by $\sim 10^{25}$units of magnitude. Therefore, any attempt to include it will eventually lead to Ultra-Violet divergences that need to be renormalized. So, Physicists came up with a solution to the above problem and find out that string theory might be a suitable candidate for this unification. Still, the attempt was futile. However, Edward Witten came up with a completely new theory that made the 2^{nd} string revolution known as M-Theory which includes an extra theory called 11-D Supersymmetric gravity (SUGRA) beside the other 5 old theories, viz., Type I, Type II(A), Type II(B), Heterotic SO(32) and Heterotic E(8) × E(8) where all of them got connected using two special dualities, viz., T-Duality (Topological duality relates to winding numbers in orders r and $1/r$) and S-Duality (Strong/Weak duality relating to the coupling constant). Apart from that, many other separate approaches have been provided by physicists across the world like ADD-Model with 'large extra dimensions', 'Randall-Sundrum-Model (RS^1 and RS^2)' with 'TeV and Planck Brane scenario' to remove the hierarchy problem, but, no theory is mathematically enriched and consistent like the M-Theory which is so pervasive that, despite technical and observational hindrances, physicists are trying hard to get a complete approach of the long-awaited 'Theory of Everything'.

As we know that 'Lorentz generators' have already shown us that there exist 10 space-time dimensions where the notion of compactification has been adapted to the extra 6 dimensions that got curled up to resemble a special type of complex manifold with a vanishing Ricci curvature of K$\ddot{a}$hler class known as Calabi-Yau (CY) manifold based on the Calabi and Yau's conjectures. Further advancements made about the Type II(B) string theory which is both T and S-Dual to itself and termed as F-Theory by Vafa with 12 space-time dimensions by including additional time dimensions. Both of these theories assume CY manifolds to be the ideal space for hiding the higher dimensions. Therefore, to understand its notion, it's necessary to dive into the norms of Symplectic geometry along with its various applications both inside and outside the string analogy. However, to avoid the rigors of these notions, we will approach more lightly, slowly, and smoothly.

2 Roadway

Those who won't understand call the supersymmetric string theory/ M-Theory, F-Theory – mathematical garbage but they can't realize that this is the only way to solve the Lambda-CDM Problem, the problem of shadow particles, Boson/Fermion cancellations, topological modules of K-3, Kählar, CY, CY-4 manifolds, the 3-genus vibrations, dimensional pervasiveness, restricting UV divergences, renormalizable approaches to non - perturbative reality, preciseness of cosmological constant problem, proof of Nambu-Goto actions, proof of Polchinski equation, proof of Ekpyrotic cosmology in cohomological manifolds, the existence of beyond Planck's epoch with segregated spacetime, de Rham cohomological manifolds, Chan - Paton factors, the existence of TeV - Planck Branes, solutions to the hierarchy problem, dualistic interconnectedness with topological (Winding number/momenta) and strong-weak (coupling strength) and many more!

There are many versions of the superstring theory but M-Theory unifies those versions into a single structural framework. M in M-Theory stands for 'Magic', 'Mystery', or 'Membrane'. Some physicists sarcastically called it a 'Monstrous theory'. M-Theory is the mother of all strings. The most crucial problem in modern physics is the understanding of quantum gravity. Newtonian gravity is superseded by Einstein's theory of gravity in General Relativity. However, quantum mechanics explains the other non-gravitational forces like electromagnetic force, strong and weak nuclear force. So, a quantum theory of gravity is needed to merge relativistic gravity with quantum mechanics and M-Theory is the only way to achieve this.

String theory assumes that the fundamental constituents of nature are not point particles rather they are 1-dimensional strings. String duality is a notion of string theory that assumes that the theories of strings are linked to each other using T-duality and S-duality. They're assumed to be six versions of String Theory.

2.1 Types

1. Bosonic – Only Boson, no Fermions, consists of both open and closed strings and a hypothetical particle of imaginary mass called Tachyon traveling faster than light is believed to be existent in this theory. It takes into account 26 spacetime dimensions.
2. Type I – The concept of supersymmetry is there and is consistent with both open and closed strings in a symmetry of SO (32). It consists of 10-dimensions.
3. Type IIA – The concept of supersymmetry is there between Boson and Fermion and a further extension of the strings called D-Brane of 2 dimensions are there in which open strings are attached. Closed strings are also a part of this theory.
4. Type IIB – Supersymmetry is there between Bosons and Fermions and consists of both open and closed strings attached to D-Branes which takes into account a maximum dimension of 10.
5. Type Heterotic SO(32) – Non-Tachyonic supersymmetric string theory takes into account of 10-dimensions consisting of both closed strings and the maximum spacetime dimensions being 10 with different notions between right and left moving strings.
6. Heterotic E8×E8 – Supersymmetry between forces and matter with closed strings is there and takes a maximum dimension of 10 with a symmetry group of E8×E8 (E stands for exceptional and 8 stands for dimensions).

2.2 Viability

Many people say that string theory is a mathematical fantasy or to some extreme, it is mathematical garbage. Here are 8 examples to show why string theory is a valid theory:

1. Heisenberg's Uncertainty Principle: $< \Psi | \, [\Delta X, \Delta P] \, | \Psi > \sim \hbar$. Position and momentum never commute. So, if the Position is 0-dimension as a point particle then its momentum needs to be infinite. But, to provide the necessary means of cutting off from infinity we need to increase the Position from 0D particles to 1D strings.
2. Mesons: A meson is held by quarks and anti quarks together. But what's that glue substance that has opposite charges that stretches them infinitely apart. This might be strings.
3. Condensate and Supersymmetry: Energy scale > 125 GeV; Higgs are detected around 125 GeV. But to increase further into Planck's mass which is just a particle of dust, we need to increase the energy. The more the energy, the more the frequency. But there is a gap between the GeV and TeV. Something must be present between them. They are the supersymmetric (SUSY) Particles that extend into strings for further amplification of energy. After that, there is a black hole.
4. To provide an accelerator greater than that of LHC: We need to make a particle accelerator 8 light minutes which is about 8,000,000,000,0 miles. After that, we need to achieve Planck's mass into the size of the billionth of a photon which is 1/1000000000000000000 of a photon. It's impossible for humans to discover strings. (Measurements are approx.)
5. We are living in a Brane cosmology with 10 Spatial and 1 temporal dimension. If SUSY is true then, there must be a closed string that penetrates through every dimension, especially the curled-up compactified ones. Ours is a 4D Brane scenario.
6. String theory provides a pixelated theory of the universe in which the AdS is a holographic spacetime near the black hole. It provides the information loss paradox of a black hole where information conservation is protected.
7. Every particle is elementary but not fundamental. So, there must be some unique elements that give rise to the variety of particles. And that particles are extended into 1D strings where every type of resonance is giving distinguishing properties of particles.
8. Strings are high energetic ultraviolet Branes where 2 strings collide to form a black hole. On the other hand, nuclear physics is dealt with Infrared Branes where 2 particles collide to form quark-gluon plasma. Both UV - Branes at high energies is symmetrical to IR - Branes at low energies.

3 Details

The dimensions of D-Branes about IIA are ODD that is, 1,3,5,7,9 while the dimensions of D-Branes about IIB are 0,2,6,4,8 which is EVEN. The closed strings can move freely while the open strings are attached to the D-Branes with their two open ends. Apart from the Bosonic theory, 5 string theories can't be regarded as the candidate for the Theory of Everything (TOE). This theory accounts for 10 + 1 (space + time) dimensions where there are 3 + 1 (space + time) dimensions and the rest 6-dimensions (spatial) are compactified to a very small curled-up structure of a Calabi – Yau manifold (CY). These 5 String theories can be called supersymmetric string theories because of the notation of supersymmetry which again can be unified by a single unified theory called the 'M'-Theory taking into account the supergravity of 11 dimensions. Two theories when are completely different, then they can be linked with the concept of T and S duality thereby making the different theories into a single unified framework by the option of one theory being DUAL to the other by the incorporation of the DUALISTIC Principles. In string theories there are 10 spacetime dimensions which means there are 9-dimensions of

space along with 1-dimensions of time. Now if 1-dimensions can be assumed as the circumference of the circle then one can return from the point where they started.

A string when traveling around the circle then there exist different states of momentum and energy in addition, the strings are wrapped around thereby forming a tension force of stretching and these wrapping modules are called the winding number. Now, the momentum of the strings in curled-up dimensions of radius R is similar to the winding number of the strings in a circle of radius 1/R and vice versa, which makes the strings inter-related to each other by the notion of this duality principle. The large distance of momentum varies with the small distance of winding number and the small distance of winding number varies with the large distance of momentum. In any way, the 5 different string theories can be interrelated by a maximum of 2 to 3 theories.

[1] T-duality relates Type IIA string theory with Type IIB string theory and compactifies their dimensions curled up in a circle and if a worldsheet is considered then they get compactified into a cylinder where the large radius is inherently linked with a small radius and the momentum is related with the winding number. The notion of large and the notion of small became the same, their properties inseparable from one another. They got to wind up in higher dimensions. A string can be either left moving or right moving with the center of the string momenta; is considered as the sum of the right and left fields whereas the stretch in the middle is considered as the difference.

[2] There comes a second concept of S-duality which takes into account a particular type of aspect called COUPLING CONSTANT which means the ability of the string to emit or absorb other strings within itself. Strings carry mass and electric charge and they may do the process of emitting and absorbing other strings or attach and decay to other strings by a method called PERTURBATION THEORY. String theories have a coupling constant which follows the phenomena of perturbation theory which depends upon the mode of the oscillation of the strings having the coupling constant small or large. If two strings are linked by S-Duality, then a string with a weak coupling constant is linked with a strong coupling constant and by this process the two string theories can be linked with each other. Using S-duality, Type I is related to Heterotic SO (32) and Type IIB theory with itself. Further type IIA is the strong coupling constant and behaves as an 11-dimensional theory with 10 spacetime dimensions and 1 time dimension which incorporates the 11-dimensional M-theory. In the case of T-Duality, the momenta take on discrete values which means if the energy of one string is 1-Joule and the other string is 3-Joule then there can't be any values in between them. The values are always discrete. The existence of these dualities is very important in superstring theories which helps to unify the theories into a single structural framework of M-Theory.

In the case of the DUALITY, the geometry of the spacetime breaks into Planck's scale physics of 1.6×10^{-35} meters and T-duality is closely related to another concept of symmetry called MIRROR SYMMETRY which is a part of the ENUMERATIVE ALGEBRAIC GEOMETRY. The concept of duality makes the two different theories to be dual in a nontrivial way such that the properties of one are dual to the other. In particle physics, particles are of 0-dimensional points which can be further extended to 1-dimensional strings. The mathematics of the string theory can be studied in various higher dimensions apart from the 3 spatial dimensions of length, breadth, and height that is up/down, left/right, and backward/forward. Indeed it takes 1 more dimension of now/then called the time-dimensions along with the other 6-dimensions compactified and curled up in circles. The idea of these curled-up circles comes from the Kaluza-Klein theory of the 5^{th} dimension. If one watches a wire from its near, then the thin wire appears to be of a cylindrical object. But, if he moves away further and further from that wire then the wire will become like a 1-dimensional string or rod rather than a 3-dimensional wire. So, an ant walking over the wire will see the wire as a straight line from its point of view, but from your point of view the wire is 3-dimensional. But the ant can't recognize it. This is because the radius of the wire is getting smaller as it is moving farther from you and ultimately got curled up into a 1-dimensional object but from the viewpoint of the ant, the wire is just flat. So, the higher dimensions are so curled up that we can't notice that just like the ant if we stayed on that wire.

A winding number is a number by which a curve circles a plane from a given point counter-clockwise. If each string is closed without any endpoints, then the winding number is the number of turns or curves the string makes. If the counter-clockwise rotation is 8 times and clockwise rotation is 4 times, then the winding number is equal to 8-4 which is 4 turns. The counter-clockwise turns behave as negative while the clockwise turns behave as positive. Their difference will result in each winding number. The momenta of the strings are quantized and the theories can be simultaneously replaced by the changing of momentum and winding number. The duality of the strings is the equivalence principle with the total energy remaining unchanged.

If string theory takes into account 10-dimensions, then the 4 are the spatial dimensions whereas the other 6-dimensions are compactified into a single curled-up dimension forming a structure called the CY manifold. The strings propagate over these manifolds and vibrate in a chaotic order with ends either open or closed. It was eventually found that a single CY manifold doesn't determine the property of

the string analogy rather two distinct CY manifolds represent the same analogy by a process called Mirror Symmetry. This is an important duality in string theory because they mirror one another and help to solve many complicated problems in string analogy. If the CY manifold is dissected into two pieces, then also the Mirror Symmetry can be observed by T-Duality. A torus can be called the simplest manifestation of the CY manifold which can be treated as the product of two circles. The circles are organized within the torus space and the torus itself acts as a space between the circles. In this case, the Mirror Symmetry can be viewed as the T-Duality acting itself on the longitudinal circles of the torus. T-Duality is called TOPOLOGICAL DUALITY whereas S-Duality is called STRONG-WEAK DUALITY.

In String theory, the 0-dimensional particles are replaced by 1-dimensional vibrating strings having mass and charge and they propagate throughout spacetime freely without any restrictions they are very chaotic, these strings can be open as a curved line or closed as a loop but when the strings are viewed from the macro perspective they can be represented by particles which are several in nature depends upon the distinct vibrational patterns. Two small strings combine to form a large string and one large string can get split up into two small strings.

String theory needs extra dimensions of about 26 for mathematical simplicity and the extra dimensions are compacted to form comparatively curled-up lower dimensions and incorporate an idea of supersymmetry. According to this supersymmetry, every Bosons has a Fermionic partner and every Fermion has a Bosonic partner. They are of different spins. It is the spin that makes the BOSONIC STRINGS identical to the FERMEONIC STRINGS. Some Examples Are, The supersymmetry partner of photons are photinos, gluons are gluinos, Fermions are sfermions, electrons are selectrons, and Higgs is Higgsinos. The most important is the SUPERGRAVITY which incorporates GRAVITY as a closed string of spin 2 with GRAVITINO of half-integer spin 3/2 behaving as the Fermions.

String models have been developed in which strings represent the high energy physics and so for this, the extra dimensions must be compactified to lower dimensions for simplicity. Or in other ways, the extra dimensions can be restricted. When CY manifolds have become a notion for compactifying the extra dimensions than two versions of the string theory can be compactified into two distinct features of the CY manifolds of Type IIA and Type IIB thereby producing a Mirror Symmetry. Mirror Symmetry states that two different models called A-Model and B-Model are equivalent in the same way such that there is a duality between them. These two CY manifold gives rise to the same physics by incorporating Mirror Symmetry. The application of Mirror Symmetry belongs to the branch of mathematics called Enumerative Geometry which raises the question of counting the number of solutions to a geometric question. Calculations of the Mirror Symmetry of the B-Models are much easier than the A-Models. Gauge theory is also related to this type of Symmetry.

When the notion of strings is expressed in higher dimensions then there comes the notion of P-Branes, P being a variable incorporating many dimensions. The world Brane comes from the "membrane" which represents 2-dimensional Branes. D-Brane is an important part of the Branes that arises when one considers the open strings whose endpoints are attached to the Branes. Whereas closed strings can move freely from one brane to another or one dimension to another. Just as graviton is an example of closed strings, the photons are examples of open strings, so, as the end point of photons are attached to the Branes, they can only vibrate within the boundary area of the Brane but not away from it. The letter 'D' in D-Brane refers to Dirichlet boundary conditions. Sub manifolds are manifolds that rest within a manifold. A submanifold is a surface embedded inside the CY manifolds and D-Branes are just like the submanifolds but the endpoint of the strings contains the charges.

In A-model D Branes can also be viewed as submanifolds within the CY manifolds but their length, breadth, and heights are minimized to half. A torus is made up of infinitely many circles which are capable of decomposing the Torus. These circles PARAMETRIZE the circles when they are decomposed meaning that there is a correspondence between the circles and the points. A Torus is the union of these two circles and the circles lay down one after the other to form the torus. This auxiliary space plays a very important role in SYZ or STROMINGER—YAU—ZASLOV CONJECTURE. The CY manifold has 6-Dimensions. They can be divided into 3 Tori which is a topological manifold of 3-genus or holes and a 3-sphere. In a normal sphere or a 2-Sphere, there is a 2-dimensional boundary over a 3-dimensional sphere but in a 3-Sphere, there is a 3-dimensional boundary of a 4-dimensional Hypersphere. If a Torus represents a spacetime, then the strings can propagate through this spacetime. According to the T-Duality, a string when propagating through spacetime can have momentum as well as a winding number. If the Torus can be split into many circles and the T-Duality can be applied then, a new Torus will be formed which is the mirror symmetry of the CY Manifold.

SUPERGRAVITY is an important aspect of M-Theory which combines the notion of supersymmetry along with General Relativity. It develops the idea of Gravitons, the mediator of Gravity. SUPERGRAVITY is said to be of a 4-dimensional theory which can be further extended to 5 Dimensions that being referred to as Kaluza-Klein Theory as Kaluza-Klein constructed a 5-dimensional gravitational theory that when dimensionally reduced on a circle, the electromagnetism can be coupled as gravity. Therefore, it can be said as a unified field theory of physics.

The CY Manifolds are important in Superstring Theory because of the unseen 6-dimensions of the Calabi - Yau space. They are so small that they are impossible to detect experimentally. The small unseen higher dimensions are an example of the CY manifold. But there is

a controversy regarding the nature of the higher dimensions. In the "Braneworld" model, the Calabi – Yau (CY) are large and we live in the subset of the intersection of the CY with the D-Branes. Different strings vibrate differently and each of their vibrational patterns is associated with the holes of the CY manifolds or space. The low energy string vibrations correspond to the known particles whereas the high energy vibrations correspond to the rare elements found in nature. The string vibrations affect the curled-up CY Space and the intersection of different holes corresponds to the masses of the particles.

AdS is an important aspect of the Superstring theory. It consists of a space or a Lorentzian manifold with a negative curvature just similar to a hyperbolic Plane. Just as Minkowski space is the analog of Euclidean space, AdS is the analog of the spherical space. The AdS corresponds to the CFT or the Conformal Field Theory of the Superstrings. The AdS has a negative energy density and a positive vacuum. De-SITTER Space is related to the Theory of General Relativity relating to a cosmological constant comprising of 3+1 dimensions which are analogous to which we live in, differs largely from the AdS which are embedded in the 5-dimensional spacetimes. The CFT doesn't correspond to the gravitational forces rather it takes into account the other 3 types of forces, the electromagnetic force, the strong and weak nuclear force in 4 spatial dimensions and if string theory is included then it takes into account 1 extra dimension that is of 5-dimensions. It is quite different from the constant scalar curvature where the spacetime remains the same everywhere and from the negative curvature which is like a horse saddle surface which is quite opposite to the surface of a sphere having the positive curvature. A CFT describes the force that works on short distances rather than large distances that's why it ignored Gravitational forces.

A CFT is usually a QFT or Quantum Field theory that is scalar invariant which means the notion will not change with the change in length, breadth, energy, and other variables. spacetime in General Relativity is Energy-Mass equivalent and is connected with space, time, and gravity. Curvature in space is considered in presence of the matter which produces a resultant Gravitational Force. The relativistic equation is predicted by Einstein in 1916 as E=mc2. In GR both the large and small masses produce the curvature. Gravity is usually treated as the 3-dimensional superspace that invokes a 2-dimensional spacetime. But geometrically Gravity is the 5-dimensional superspace of the 4-dimensional world. In GR Gravity bends time as well as space and forms an equilibrium between Gravity and acceleration. Thus it differs from Newtonian Gravity. In the AdS, spacetime is slightly curved like a hyperbolic plane and when the projection is done then it's called the CFT. AdS in string theory take into account the 5 dimensions where all the physical structure of the space can be determined. There exists a negative cosmological constant. The extra 5th dimension is the TIMELIKE Dimension rather than LIGHTLIKE or SPACELIKE in a LIGHT CONE.

4 Supersymmetric Phenomenology

String Theory is a very rigorous task to accomplish as it requires the fulfillment of 4 things together:

[1] Planck's Length is 10^{-15} meters.
[2] Planck's time is 10^{-44} seconds.
[3] Planck's mass is 10^{-35} kilogram.
[4] The energy of 10^{19} Giga electron Volts.

Supersymmetry that is manifested by the introduction of Kappa Symmetry in Polyakov action by combining Bosonic and Fermionic fields that is:

$$S' = -1/2\pi \int \partial^2\sigma\sqrt{(det\ -\hbar)}\ \hbar(\tilde{a}', b)\prod(\hat{u}', \delta)\prod(u, \beta)$$

The Supersymmetric model assumes the Boson with integer Spin with Fermion the half-integer spin as:

[1] Photon (1) - Photino (1/2)
[2] Gluon (0) - Gluino (1/2)
[3] Electron (1/2) - Selection (1)
[4] Graviton (0) - Gravitino (3/2)
[5] Proton (1/2) - Sproton (1)
[6] And many more.

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The universe is Supersymmetric with SUSY Coordinates in a worldsheet of 10 spatial dimensions as:

$$\int (\Theta^{\prime\prime} + \Theta^{\prime\prime}) = \xi^a \xi^b (\sigma, \tau)$$

There are 26 Dimensions with 24 degrees of freedom as 2 of the dimensions have negative norms hence ghost term and eliminated. But this includes only Bosons. If Fermion needs to the included then we have to accept that D = 10. According to Dirac Spinors, there must be $2^{D/2}$ components. So, if D = 10 then we have 32 components. By introducing Majorana-Weyl Spinors we reduced them by half that is 16. Then by using Kappa Symmetry we reduce them by another half which is 8. So, then there are 8 degrees of freedom in 10 dimensions. See 10 – 8 = 2. Same as 24 degrees of freedom in 26 dimensions like 26 – 2 4 =2.

There are spontaneous symmetry breaking in the Minimally Extended Supersymmetric Standard Model (MESSM) which is manifested by the Higgs giving mass to the non-Abelian gauge group of SO(N) with N = 2 and $N^2 = 4$ indicating the number of Gauge fields. But as you see that Z and W± bosons have acquired mass by symmetry breaking whereas the photons are massless hence there is a factor called Chan-Paton Factors indicated by $|D >\times |i, j >$ with the values of coincident D(p)-Branes ranging from $(1,1)(2,2)(1,2)(2,1)$ thereby making the symmetry broken by restricting spontaneous transformation of both translation and rotation to the Póincare groups and as the mass gets squared therefore no negative mass or Tachyonic state exists in SUSY. There is a trade-off between Quadratic divergences and Logarithmic divergences $as\ a/\sqrt{m} \implies 1000\ GeV\ or\ 1\ TeV$.

5 Vibration States

The vibration states of the string correspond to the particle nature as:

1. Left node vibration of $|R >$ sector with left node vibration of $|NS >$ sector give rise to Bosons.
2. Right node vibration of $|R >$ sector with right node vibration of $|NS >$ sector gives rise to Bosons.
3. Right node vibration of $|R >$ sector with left node vibration of $|NS >$ sector give rise to Fermions.
4. Right node vibration of $|NS >$ sector with left node vibration of $|R >$ sector give rise to Fermions.

The conserved charge of a string is given as:

$$d\rho^2/d^2t \ * \ \nabla \cdot D \ = \ 0$$

The string tension is given as:

$$T = 1/2\pi a'$$

6 TeV - Planck (Randall-Sundrum) Phenomenology

This model accesses many important functions in particle physics and orbifold string dimensions using:

- The hierarchy problem
- Anti-De-Sitter /Conformal field theory correspondence (AdS/CFT)
- Lorentzian pseudo-Riemannean manifold
- Brane tension (Planck Brane and TeV Brane)
- The Higgs and spontaneous Gauge coupling with symmetry breaking
- Wrapped up higher dimensions

Yang-Mills Theory takes up the non-Abelian Gauge Group of $U(1) * SU(2) * SU(3)$ as respectively electromagnetic, weak and strong interactions but failed to take account of Gravity. The hierarchy problem arises from the spontaneous symmetry breaking where it has been found that the weak force is 10^{-24} times stronger than gravity. This problem needs serious consideration and here the Braneworld and Bulk cosmology require the fine-tuning of the Brane Tension which arises as an opposite polarity depending upon the coupling constant and dual to the AdS/CFT Correspondence using Einstein's cosmological constant.

Large Extra Dimensions or LDD appears from the space which is Riemannian with a positive coefficient matrix of every signature (M,1) or (1,M) when taken into a negative coefficient of the Pseudo-Riemannian matrix of Lorentz transformations where the metric signature

denotes the (N+1,1) or (1.N+1) {Sign convention} to the negatively curved Anti-De-Sitter Space. The Conformal field theory links the most common particle interactions described by Yang-Mills Theory in 2 Spatial coordinates as a Function of $\partial(\sigma, \tau)$. Here due to the worldsheet analogy, the string theory comes to play.

Summarization in order leads to the conclusions:

- Note that in the Kaluza-Klein theory Gravity and electromagnetism can be united in the fifth dimension where $x^5 \sim x^5 — > 2\pi RW$ where RW is the radius of the orbifold and a torpid twisted dimension with a cylindrical base of E^4 tensor product as $\mathbb{R}^n \implies M^4 \otimes S^1$
- The Lorentzian dimension of 4D Brane can be written as $dx^2 = -cdt^2 + dx^2 + dy^2 + dx^2$.
- Now, in the twisted 5-dimensional analogy this same metric goes like $ds^2 = 1/KW * (\mu_{ab} * dx^a * dx^b) + dy^2$ where W is the winding contribution, K is the $(-,+,+,+)$ Signature and $\mu_{ab} * dx^a * dx^b$ is the 5-D Metric Tensor.
- If there are 2 Branes floating on the Bulk then two formalists arises as RS^1 and RS^2 formalism.
- Gravity can be treated as Graviton (Spin-2) SUSY. As Gravitino is Fermion (3/2) therefore the probabilistic wavefunction can be treated as residing between 2 Branes: the Planck-Brane and TeV-Brane.

In RS^1 model the 2 Branes are side by side expanding with cosmological constant and having Brane Tension. Graviton being a closed string is detached from the Dirichlet and Newmann boundary conditions and are therefore moving From Planck-Brane with low volume low mass lighter to the TeV Brane with high mass high volume and heavier about 10^{-17} where the standard model comes into play. The hierarchy problem can be reduced to a large extent. The RS^2 formalism is on the other way a non-perturbed solution's If AdS/CFT as the TeV Brane is infinitely away from the Planck Brane. The extra dimensions are large and the scalar field potential $\Phi(X^5, d\Phi)$ Graviton can move freely around the Planck Brane to TeV-Weak Brane to reduce the hierarchy problem a large extent.

7 Calabi – Yau (Compact Ricci-Flat Kähler) Manifold

A Calabi-Yau (CY) Manifold when would be a Kähler manifold with the following metrics $\Omega = dx^1 dx^2 * Log(det\ g) = 0$ with vanishing Ricci curvature R_{ij} - Then this satisfies the vacuum solutions of Einstein field equations. The Calabi conjecture from which the famous Calabi-Yau manifold of 6 compactifying spatial dimensions arose with no parity between the right-handed and left-handed spin. This is a serious problem for the weak force. But with applications to supersymmetry, the problem has been solved to some extent.

The Calabi conjecture basically lies with Euler's Polyhydra Equation $F - E + V = 2 \sim -2g$ if genus is there in a first vanishing Chern class Kähler Manifold with a Ricci flatness (Ricci curvature = 0) and a Tangent spaces with a proper Gaussian Plane specifying the requirements of continuous non-oriented ranged spaces with a closed holonomy of neighbourhood differential parallel transport. 3 Genus of Calabi–Yau Manifold gives rise to 3-States of fundamental particles through their resonances.

7.1 Origin of Gravity from 8D Conifold Singularity

Consider 2 CY(Calabi-Yau)-Manifold. 1 CY is at a distance 'x' from the other CY. Now each CY is of 6 compactified dimensions. Consider in general case that each CY manifold has a mouth opened at point 0 while other end has a mouth opened at infinity. And as infinity is not measured therefore, we can safely say that the CY manifolds other mouth is not there. The dimensions at the infinite other end of the CY fold has noncompactified dimensions of 4 spacetime as we know. The dimensions at the mouth of the 0 end of CY fold contains a torus of cycle $pa + qb$. "p" wraps at "a" directions while "q" wraps at "b" directions having a minimum limit of T. Now, as the cycle wraps in higher dimensions it gets smaller and smaller ultimately at the point when $T = 0$. Then a singularity emerged. This singularity emerges at the ends of 1 CY manifold and at the ends of the other CY manifold (we considered 2 CY manifold at the beginning separated by distance "x"). Now, the wormhole that emerges from the naked singularity is microscopic and is actually a $D(p) - 1$ Brane which is a magnetic monopole. Now, when two such monopolic ends of $D(p) - 1$ Branes comes in contact with each other; 2 things can happen as:

1. If the monopoles are opposite they attached with each other forming a bipolar electromagnetism.
2. If the monopoles are same then they get a repulsive back reaction and crumpled into a circle of closed strings that gets attached to the $T = 0$ singularities. Hence gravity is always compactified as it can travel through dimensions.
3. On the other hand gravity originates from monopoles if back reaction occurs. This also explains that as the volume is shrinked from infinity to 0, the forces of gravity is weaker at the 0 ends.

7.2 KKLT Conifold Singularity

The flux tubes, the Calabi-Yau Spaces with a million holes, the Anti-D Brane that allows a universal repulsion like our expanding universe with a positive cosmological constant with other D-1, D-2, D-3, D-4, D-5 Branes compactified in a Torus and giving the remaining 3-Branes, The D3 Branes residing in the landscape gives rise to a conifold Singularity with many vacua. The M theory considers the strings as membranes with each membrane being compactified in 6 directions resulting in an expansion in other directions similar to a D-5 Brane where the Gravity is being tossed over. That's 11 Dimensional Supergravity.

What's the difference between M theory and String theory?

The ends of D-2 Membranes when get closed together attach to form a closed string of Graviton similar to Type IIA and Type IIB Theory where IIA has D-0, D-2, D-4, D-6, D-8 Branes, and IIB has D-1, D-3, D-5, D-7, D-9 Branes. Each of the Branes is compactified along 6 directions and our world represents a 3-Sphere or 3-Torus; more specifically a 3-Torus as the local dimension is 3 but overall it's 4 dimensional and if you try to lasso a D-1 Brane or string in a torus – it won't slip off. But if you try to lasso a D-1 Brane in the equator of a 3-Sphere it will not lasso up, it will slip off to either of the edges. Now replacing the D-1 with D-3; It's our world flexible to move in 4th dimension time and 7 compactified dimensions including 1 of supersymmetric Gravity.

- Internal Dimensions could be identified as 4, 5, 6, 7 while the complex dimensional plane composes of 8, 9 dimensions. The elliptic fibers and gauge bundle in vector spaces in this context are of 10, and 11 dimensions. Adding up all this 4,5,6,7,8,9,10,11 – one could get a 8-D Calabi-Yau manifold where the 8,9 dimensions are orthogonal truncation and if we consider this as a $D-7$ (p,q) Brane of cycle $pa+qb$ where a and b are the wrap number of the 2-toric manifold then the other 5-D Brane goes away at the Planck-Brane and the intersecting cycles of winding contribution gives a coupling singularity of $1/G_S$ where 3 F-1 Branes intersects. Thus one could get gauge Bosons at the junction point. If the complex τ goes mapping between the Branes as $\frac{a(\tau)+b(\tau)}{c(\tau)+d(\tau)}$, what we get is an infinitely stretched Planck Brane from the M-2 Brane (TeV) Brane and we at the truncation get Gauge Bosons called Gravity with a Lie Group $SL(2,\mathbb{Z})$ where S-Duality is dual to Type IIB Strings. A Non-Perturbed lighter Gravity can be decomposed from F-Theory by the Truncation of K3 4-Folds in Randall – Sundrum Model.

8 Cosmic Strings

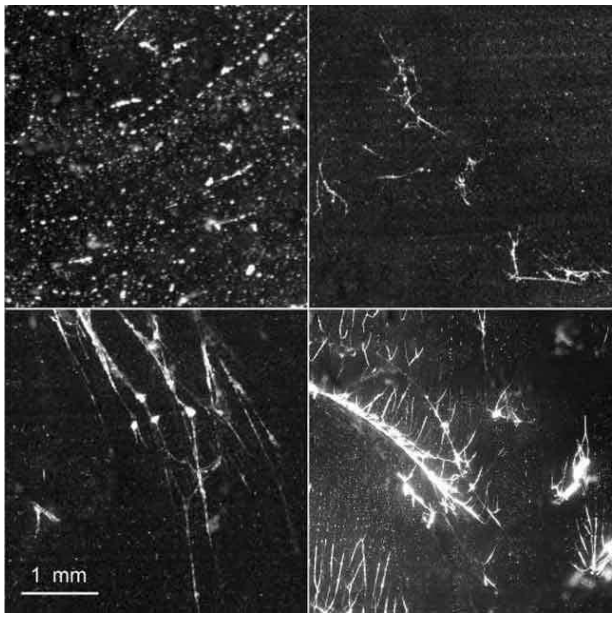
Arising out of topological defects during the phase transitions of the early universe [cosmic strings] are giving new insights into the structure of the universe. The worldsheet coordinates vary from worldline coordinates, and if the string is closed then the loop it forms in space is a cylinder that is continuously oscillating with periodic expansions and contractions in an affine manifold. If 2 or more such strings intersect then this is a tensor product of different cylinders joined together. The worldsheet coordinates could be denoted by $X_{\mu}(\sigma,\tau)$, where:

1. X_{μ} : representing coordinate indices
2. σ : spatial coordinates
3. τ : temporal coordinates

The area swept by a fundamental string can be unified by compactification on a T-2 torus and is the product of (3) Lie groups; Electro-Weak, Strong Force unifications as described in section 1. In cosmic Strings; the pulsating gravity beams they emit while looping in pairs along with their adjoints. Due to the phase transition from low energy to high energy, there exists some high energy trapped zones in the continuum which as a result of the TOPOLOGICAL DEFECTS turned out to be cosmic strings whose span is more than or about the size of an observable universe with a very high tension acts on them making it infinitesimal to about $\gtrapprox 1/10^{9}$ of hydrogen nuclei; still having a heavy mass around them.

Due to the high energy gradient flowing in them, if 2-strings tend to cross each other, then a 2-Way junction can be formed between them which then tends to separate each other through high energy gravity beams pulsating from them or a huge gravitational discharge, as a result, the gravity of the loops become close to the ambient gravity to some extent by some fractional corrections which ultimately wraps the matter around it and collapse it with the Gravity and carries it to the far away universe homogeneously which is one cause of the "distribution of the stars and galaxies uniformly in the universe"

The cosmic strings being light-years long can be proven by gravitational lensing. If there's a cosmic string between a galaxy and earth, then the light from that galaxy will be disrupted by the high gravitational effects of the cosmic strings: therefore, the resultant image observed from the earth is the 2-Images of the same galaxy, one with a short period after other. Calculations of CMBR/Planck-WMAP data; don't match with the predicted value as the energy spectrum is being shifted from a higher scale to a very lower scale beyond experimental detection. Doppler Shift Curve along the CMBR detects some unusual crests and troughs but if the cosmic string is a viable exposure to the universal physics, then the curve will be of a smooth sinusoidal function. The string analogy of the fundamental strings can be mapped to cosmic string by SUPER-COSMIC STRINGS.



Liquid helium exhibits quantum mechanical defects; phase transitions {comparative analogy}. Image courtesy: DPL Nonlinear Dynamics Lab.

9 Extremal Black Holes

Extremal black holes are a special type of Blackhole which is balancing between the charge and gravity. Generally, black holes have a minimum charge; That means whenever you put a charge in a Black hole, it will neutralize itself by attracting opposite charges from its vicinity. But in the case of Extremal black holes, this thing never occurs because they are the minimum possible mass black holes that exist in this mathematical universe. How do such black holes form?

Segregating the dimensions of the universe as:

$$\Sigma + \Gamma + S + T \Longrightarrow 11$$

Where,

- Σ - 3 Lorentzian dimensions.
- Γ - 5^{th} Kaluza-Klein Dimensions.
- T - 1 Time dimensions.
- S - 6 compactified dimensions on a Calabi-Yau Manifold.

Considering the string warp in the compactified dimensions over and over. This will lead the weight of the compactified dimensions increase to a Planck mass which will eventually collapse to form a Black hole. But apart from that, by giving some wiggling to these strings. – a charge will generate and that charge is equal to the gravity at the Planck's scale to balance the forces and reaches equilibrium after collapsing.

- Such black holes have 0 entropy.
- They don't emit Hawking radiation.
- They are continuously forming in the higher dimensions.

10 Ekpyrotic Models (Cyclic Cosmology)

Cyclic cosmological models of Ekpyrotic theory are based on the advanced concepts of Braneworld scenarios. A metric had been proposed by Randall and Sundrum, that describes 2-Branes fixed at the boundaries of a 5-Dimensional universe, which can move along the 5th dimension through the 11-Dimensional bulk. This is the idea of Ekpyrosis deeply rooted in superstring/M theory (Heterotic). The rest 11 – 1 temporal – 4 spatial = 6 spatial dimensions being compactified on a tiny scale that is irrelevant to the present cosmological scales. Imagine a universe, existing eternally but goes through a cyclic pattern that begins with an initial state characterized by the boundary Dirichlet(5)-Branes in a cold, flat, empty state. The two branes with a scalar field called the radion flux between them come close to each other and collide in such a non-singular way, that, it gives birth to a universe via a "new Ekpyrosis" called the 'big bang'. The energy from the collision creates the hot matter-fluid plasma in the branes, which then moves apart from one another and successively cools down. The radion flux which is a function of the distance between the two boundary 5-branes evolves through a period of slow acceleration, followed by deceleration and contraction. And then, again collides with a non-singular bounce causing the reheating of the universe. Two important problems solved by Ekpyrosis are stated:

1. The homogeneity, that is the universe is the same in all directions, implies the CMBR is the same everywhere, that is, the universe begins with the same initial conditions at all points.
2. The flatness problem could be solved by making the initial state of the branes a vacuum. As being in a state of vacuum, the branes are flat and empty, so, no mysterious fine-tuning is required to make the universe flat. The assumption asserts that the branes, that start in the vacuum force to be flat.

However, while viewing it from a quantum scale, everything is not as exact, as it seems to be. There are brane fluctuations or brane ripples that arise because of the movements of the brane in the bulk. Not, every point in the brane, during 'bounce' collides at the same instant, some may collide after the other, while most will collide at an average time. This shows us, that rather than the whole universe being consistent with a uniform temperature, some regions are cooler (as the branes of that part collided earlier) than the others which are hotter (as the branes collided later times than the former). The seeds of creation lie in this phenomenology to create the large-scale structures like the galaxies of the universe.

The distasteful feature of a universe formation is the singularities which are space-time points where gravitational curvature and temperature blow up to infinity. One such example is the big bang singularity. However, in this model, the singularity is very milder than that of the classical general relativity, because before the branes collide, all ghost perturbations are cut-off at the Plancks scale with a non-singular bounce, that bounces off and returns to the initial positions. The radiation and matter density of the branes are always finite and there are no singular points where, matter, space, and time are sprung off by a magical fiat. The 'singular' behaviors are however observed in the big crunch when the two branes collide thereby disappearing the extra-dimensions between them which later re-emerges when the boundary branes move apart from each other. Of course, it's difficult to believe, that contrary to the normal supposition, space and time are always existent as in the Ekpyrotic models, but, more advanced theories and experiments will conclude the results with an exact precession.

The Ekpyrotic phenomenology also gives a block to the missing puzzle of "where all the matters of the universe come from?" The answer is, however, relatively simple, that is, during the brane to brane collision, a huge amount of thermal energy got created from the kinetic energy of the branes, which in turn condenses to matters via $E = mc^2$.

This cyclic model proposes the fact that,

- The universe is an eternally existent being that runs through repeated cycles of Brane collisions.
- The big bang is not the origin of time.

As an alternative to inflations, this model solves various puzzles of the cosmological phenomena that may be distinguished by observational tests where the gravitational waves are not scale invariant, unlike inflation.

11 Wiping off the Confusions

11.1 The Source Code

Amplitudes of scattering of strings are sinusoidal or a combination of Sine and Cosine waves which resembles the particles as a string. Explaining that notion concretely: we all know that for every positive integer the relation suffices to correlate with Gamma functions and factorials as:

$$Z\ldots\ldots\Gamma(Z) \;=\; (Z-1)!$$

For the positive real part of Z which is a complex number,

$$\Gamma(Z) \;=\; \lim_{n\to\infty} \frac{N^z}{Z(1+Z)(1+Z/2)}\ldots\ldots\left(1+\frac{Z}{n}\right)$$

Where in N^z, Z is the complex exponentials with real part positive; thus having,

$$Cos\ N \;+\; Z\ Sine\ N$$

This particular expression $Cos\ N \;+\; Z\ Sine\ N$ shows the Gamma function as a periodic oscillator with $Z = i$ (Complex Number). Leonard Euler combined this Gamma function with the Beta function as,

$$\beta(x,y) = \frac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)}$$

Here in S-matrix or Scattering Matrix – the Space like channels (S) have 4 momenta as,

$$P\ldots\ldots(P1+P2)^2 = (P3+P4)^2$$

Along with the Time like channels (T) with 4 momentum as,

$$P\ is\ (P1-P3)^2 = (P2-P4)^2$$

Veneziano combined these Mandelstam Variables where α is the Regge Trajectory. Thus string theory originated from this 250-year-old equation in 1968.

$$(\mathrm{S},\mathrm{T})\text{is}\ \mathrm{B}(\mathrm{S},\mathrm{T}) \;=\; \frac{\big(1-\alpha\Gamma(\mathrm{S})\big)\big(1-\alpha\Gamma(\mathrm{T})\big)}{2-\alpha\Gamma(\mathrm{S})-\alpha\Gamma(\mathrm{T})}$$

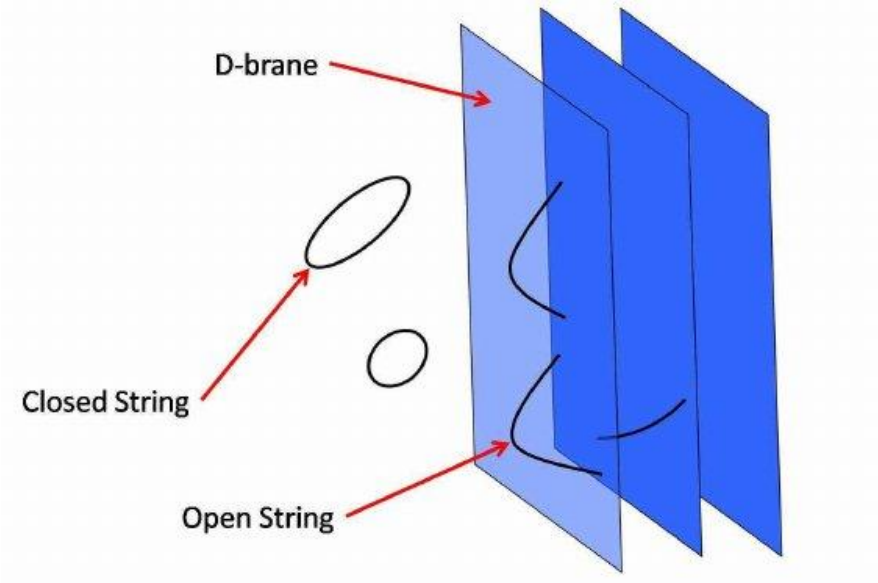
11.2 Dimensional Workout

To obtain the maximum dimensions one has to construct Lorentz-Generators as,

$$[M^{-i}, M^{-j}] = \; -\frac{1}{(p^+)^2}\sum_{n=1}^{\infty}\left(a^i_{-n}a^j_n - a^j_{-n}a^i_n\right)(\Delta_n - n)$$

$$\Delta_n = n\left(\frac{D-2}{8}\right)+\frac{1}{n}\left(2a_{NS} - \frac{D-2}{8}\right)$$

To maintain Lorentz invariance, we must have $[M^{-i}, M^{-j}] \;=\; 0$. This can only be true when $n\ \left(\frac{D-2}{8}\right)$ collapses to n and when it is equal to 1, and $\frac{1}{n}\left(2a_{NS} - \frac{D-2}{8}\right)$ becomes 0 when $\left(2a_{NS} - \frac{D-2}{8}\right)$ is equal to 0. Therefore, $n\left(\frac{D-2}{8}\right) = 1$ or, D = 10 and, $\left(2a_{NS} - \frac{D-2}{8}\right)$ and D = 10 or, $a_{NS} \;=\; \frac{1}{2}$ which is the normal ordering constant of $NS-Sector$ in superstring theory.



Closed strings with no boundary attachments to Branes, hence allowed to move in extra-spacial dimensions. Whereas, open strings being attached to Branes couldn't make inter-dimensional travel and are restricted to oscillating in present dimensions. Those Branes which have a 'radion' fields in them, when contracts then, touches each other making a high explosion giving birth to a new universe, while when they start to move away from each other, then the universe expands which this cycles of Branes getting away from each other, and again coming close to each other is expressed via the 'cyclic cosmology' or 'Ekpyrotic cosmology'. The standard model of particle physics resides in the Branes while the gravity being closed strings are free from any dimensional confinements. Dimensions are there in Branes which remain floating in the Bulks. [Image courtesy: Myers, R. C., and Vázquez, S. E. (2008). Quark soup al dente: applied superstring theory. Classical and Quantum Gravity, 25(11), 114008. https://doi.org/10.1088/0264-9381/25/11/114008]

12 Concluding Remarks

If there exists any such theory in theoretical physics that has dared to ask the most impossible question by probing deeper into the hearts of nature then, without any hesitation this is the string theory, rather the supersymmetric string theory. Not only does it attempts to find the deepest symmetry by peering beyond the horizons of traditional methodologies used in physics, but also it paves and shows humans a mathematical framework or tools for a complete unification resulting in achieving the dreams of almost every physicist – TOE. Enriched with posh mathematical sophistication, the theory is still building up: more complex analogies are getting developed, symmetries and dualities along with interconnectedness being woven out, resulting in picking a particular branch of theory and amplifying it through methods of rigorously achievable computations encompassing the topological aspects and pervading the quantum cosmology, we can still hope for some suitable schemes of unifications achievable through experimentations. Starting with Einstein's relativity, through Kaluza-Klein analogies, then moving forward by analyzing the harmonic oscillators, the approaches of quantum theory without gravitation have been attempted to merge with relativity making a smooth road for a conjugated model of all fundamental forces through the principles of scale invariance, i.e., by making a modular approach through higher dimensions, removing the constraints of energy limits, making topological models, attempts have been made at its best to scale down the difference in gravity from the strong, weak, electromagnetic forces. Although this is a proposal beyond attractiveness enriching with hopes of finding the ultimatum of schemes of unifications, the mid-1970s had shattered the hopes of physicists when the predictions about strong nuclear forces deviated from the results of more-refined experiments. However, few physicists had claimed that experiments might have failed the predictions because of the existence of typical massless particles whose notion had not been encountered by studying the strong forces before. Works are going on, new methodological interpretations are getting built up but, more mathematics is still needed for humans to encounter to achieve the dream of Einstein's unified theory.

13 Declaration of Interest

Author of this paper holds no conflicting interests.

14 Further Readings and References

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