AN INVENTORY MODEL WITH THREE PARAMETER WEIBULL DISTRIBUTION FOR DETERIORATING ITEMS UNDER INFLATION

ABSTRACT

This study explores a predetermined inventory systemfor deteriorated items with a Weibull distribution, taking into consideration nonlinear demand and trade credit under inflation and allowing for shortages and partial backlogs. The reason for this study is to find the appropriate order and replenishmentto actually reduce relevant costs.The model is built under two scenarios; **case 1:** $\left(0 \le M \le t_2\right)$ The consumer will be charged interest on the outstanding debt if he does not pay the provider by time M. and **case 2:** $\left(0 \le t_1 \le M\right)$ if the consumer gets to sell all of his commodity as well as earns interest on the revenue until the account is settled.To demonstrate the use and performance of the model, numerical approach and sensitivity analysis are actually given. As a result, this model will assist retailer in determining the optimal replenishment cycles in a variety of situations, as well as provide an innovative management insight that will aid the industry reduce relevant cost.

Keywords

EOQ, Quadratic Demand, Weibull Distribution, Shortages, Trade Credit, Inflation.

Introduction

Inventory modeling is an important part of operation research which is used in solving variety of warehousing and storing problems. The primary purpose of the inventory modeling is to develop policies that will achieve an optimal inventory investment. It plays a significant role in production and operations function of supply chain management in order to make it applicable and flexible in real life situation and also in the control of inventories of deteriorating items ever since the theories on Economic Order Quantity were first introduced.

Harris [1], who investigated the Economic Order quantity (EOQ) model to find the optimum order quantity that minimizes the total cost, became the first to apply mathematical modeling to inventory control. One of the key assumptions of the EOQ model is that their products have an infinite life i.e. their value does not change. Deterioration of commodities in the form of immediate spoiling or progressive physical degradation in the course of items is a natural occurrence in several inventory systems, and it should be taken into account during inventory modeling. Wee [2], described a deteriorated item as one that has rotted, been damaged, evaporated, expired, become invalid, or devalued over time. Many researchers have studied inventory challenges for deteriorating items in depth.Whitin[3], was the first to investigate how fashion items deterioratedat the completion of their storage span. Ghare and Schrader [4]devised an EOQ that deteriorated at a steady rate.Nevertheless, for a few goods such as, fruits, vegetables, the amount of deterioration increased over time. Furthermore, it has been empirically discoveredthat many items' failure and life expectancy can be represented in terms of the Weibull distribution. Berrotoni[5]for instance, discovered that both leakage failure of dry batteries and life expectancy of ethical pharmaceuticals may be stated in terms of Weibull distributions when considering the challenge of fitting empirical data to mathematical distribution. When applied to economic order quantities, Weibull distribution will yields a probability density function that indicates the time for the order to deteriorate. Covert and Philip [6]expanding on the work of Ghare and Schrader [4], employed two – parameter Weibull distribution with no shortage to show the distribution of time to deterioration.In practice however, the two parameter Weibull distribution deterioration may not be useful because some items start deteriorating after a given time of storage, but not at an early stage.Rinne[7], proposed a three-parameter approachfor some items that do not begin to deteriorate instantly but rather after a set amount of time called item's lifespan which varies vary from item to item. By incorporating three parameters of Weibull deterioration, no shortages and a constant demand,Philip [8]expanded the model of Covert and Philip [6].

Various inventory items such as milk, meat, vegetables cannot be stored for a lengthy time due to their shelf life. Also some items, that such as radioactive materials, volatile liquids are constantly losing value due to their chemical qualities or other intrinsic conditions. As a result, the assumption of a constant demand rate may not always be valid for many inventory items, as inventory age has a negative influence on demand due to the loss of consumer confidence in the quality of such products as well as actual material loss.Wee [2]proposed a deteriorating inventory in which demand falls exponentially over time.Covert and Philip [6]model was reviewed byJalan et al [9]who extended it to include a time-dependent demand rate and inventory shortages. However, the majority of the models discussed above are based on time-varying demands such as linear and exponential demand.Ghosh and Chaudhuri[10]investigated an economic order quantity (EOQ) model over a finite time horizon for a deteriorating item with a quadratic, time-dependent demand, allowing inventory shortages.Amutha and Chandrasekaran[11]investigated an EOQ model for deteriorating items using quadratic demand andtime dependentholding cost.Mishra [12]developed an inventory model for an item with a two-parameter Weibull distribution and quadratic demand, with holding costs as a linear function of time.Smaila and Chukwu[13]proposed an EOQ model with Weibull,quadratic demand and shortages with three parameters.

Other components, such as enabling shortages, are necessary during inventory management in addition to demand and deterioration rate. Shortages generally happen in one of two ways: when the shortage products are completely backlogged, or when they are partially backlogged. Various inventory models with entire backlog were described by; Amutha and Chandrasekaran[14]investigated a three-parameter Weibull inventory model for degrading items with price dependent demand, and defined various inventory models including the complete backlog.Chaudhary and Sharma[15]examined Weibull distribution and time-dependent demand inventory model for deteriorating items. Rai and Sharma [16]propose an inventory model for deteriorating items with a non-linear price-dependent demand rate and changing holding costs. However, in real-life situations, a customer's willingness to wait for items to decrease as the length of the wait increases. Chang and Dye [17]explored the EOQ model for deteriorating items with time-varying and partial backlog. Sana [18]investigated an ideal selling price and lot size with time varying deterioration and partial backlog. Roy et al [19]investigated an Economic Order Quantity Model of Imperfect Quality Items with Partial Backlogging.

Most researchers believe that the customer must pay for the things purchased as soon as they are received in traditional inventory economical order quantity (EOQ) models. The most usual approach is for the supplier to provide a credit period to the buyer so that he can finish his bill before the deadline. In terms of finance, inventory is a capital investment that must compete with other assets for a company's limited capital funds. Because most people believe that inflation will have no impact on inventory policy, the consequences of inflation are rarely considered while examining an inventory. The majority of studies have yielded helpful results for the inventory model for deteriorating items with trade credit under inflation without giving due considerationshortages, which would be more accurate. Based on studies in this area of research;Buzacott[20]developed an EOQ model that included inflation and other pricing regimes.

Misra[21]established a discount cost model that takes into account both inflation and the time value of money.

Chandra and Bahner[22]developed a model to study the effects of inflation and the time value of money on optimal order policies.

Goyal[23]devised an economic order quantity in which the supplier agrees to a certain period of time to settle the amount owing to him.

Aggarwal and Jaggi[24]devised a methodology to calculate the optimal order quantity for deteriorating items with a reasonable payment delay.

Chang et al. [25]studied an inventory model with a variable rate of deterioration and a condition of acceptable in payment, with the restricted assumption of constant and a linear demand trend. Chang and Dye [26]proposed an inventory model for deteriorating items with a permissible payment delay, taking into account the phenomena of physical goods deterioration and a vendor who may provide clients a specified credit time to clear the account. Singh and Panda [27] investigated an inventory model for generalized Weibull deteriorating items with price-dependent demand and allowable payment delays under inflation.

In this study, an inventory model with a three-parameter Weibull distribution under inflation was developed, taking into account quadratic demand and trade creditand partially backlogged.

2. Assumptions

The followings are the assumptions used in deriving the model.

- The rate of demand is a quadratic function of time that is deterministic.
- The lead time is zero.
- The rate of deterioration is represented by a three parameter weibull distribution.
- Replenishment is instantaneous and infinite.
- The rate of inflation is constant.
- Shortage were allowed and partially backlogged.
- During the cycle, no deteriorating items will be repaired or replaced.
- The account is not cleared during this time, and the buyer pays off all sold units and continues making payments on the products in stock.

3. Notations

The followings notations are used in the model's development:

1. A : Ordering cost of inventory per order.
2. $R(t)$: The nonlinear demand rate, i.e., $R(t) = a + bt + ct^2, a > 0, b \neq 0, c \neq 0$ where a, b and c are the initial demand rate , increasing demand rate and change demand rate respectively .
3. $Z(t) = \alpha\beta(t-\gamma)^{\beta-1}, 0 < \alpha < 1, \beta > 0 \,\&\, 0 < \gamma < 1$. Here $\alpha, \beta \,\&\, \gamma$ are called the scale parameter, the shape parameter and the location parameter respectively.
4. $B(t)$: The backlogging rate which was given as $B(t) = e^{-\delta(T-t)}, \delta > 0$ where δ is called the backlogging parameter.
5. δ : The constant backlogging parameter where $0 \leq \delta < 1$.

6. R : Inflation rate

7. M : Allowable time for the account to be settled.

8. T : The predetermined length of each ordering cycle:

9. $I_1(t)$: On-hand inventory at time t when $t \ge 0$.

10. C_p : The purchase cost per unit.

11. C_h : Total holding cost per cycle.

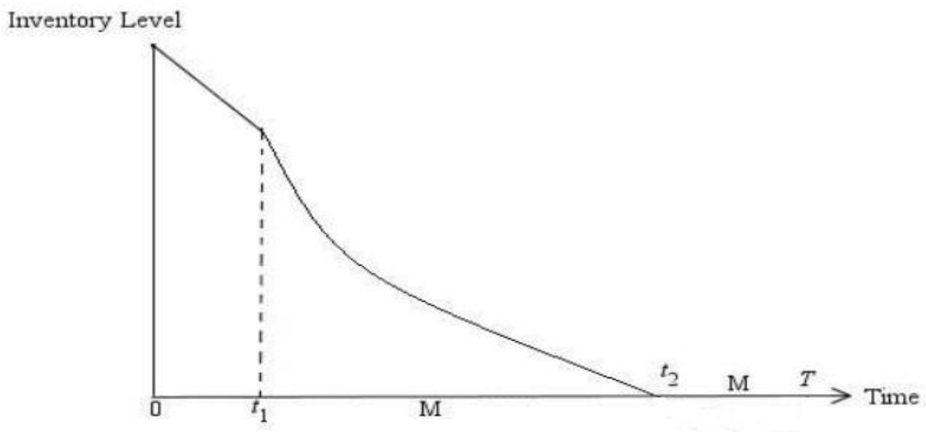
12. t_1 : The point at which inventory drops to zero.

13. TRC : The overall cost per unit of time for all relevant cost.

14.Q: Ordering quantity per cycle.

4. Model formulation

The inventory I(t) at time t $(0 \le t \le T)$, is describe as shown in the figure below.



The inventory system works as follow: at $t = 0$, a certain lots size, certain units are entered into the system. In the interval $[0, t_1]$, the inventory level continually drops due to demand during the interval and evaporates at time $t = t_1$. Then,during the interval shortages are enabled to exist $[t_1, T]$ and shortages decline in part owing to deterioration, and all the demand during the shortage period $[t_1, T]$ is partially backlogged. As a result, the differential equations which explain the instantaneous states of $I(t)$ at any time t during the period $[0, t_1]$ are

$$\frac{dI_1(t)}{dt} = -\left(a + bt + ct^2\right), 0 \le t \le t_1 \quad (3.0)$$

with the boundary condition $I_1(0) = Q_1$

The solution of equation (3.0) becomes

$$I_1(t) = Q_1 - (at + \frac{bt^2}{2} + \frac{ct^3}{3}),\ 0 \le t \le t_1 \quad (3.1)$$

The differential equation that explains the decline inventory level as a result of the demand and deterioration was given by

$$\frac{dI_2(t)}{dt} + Z(t)I_2(t) = -R(t), t_1 \le t \le t_2 \quad (3.2)$$

with boundary conditions at $I(t_2) = 0$ and $I(T) = -Q_2$

The solution of equation (3.2) becomes

$$I_2(t) = K - \alpha\left(a(t_2 - t) + \frac{b(t_2{}^2 - t^2)}{2} + \frac{c(t_2{}^3 - t^3)}{3}\right)(t - \gamma)^\beta, \quad (3.3)$$

considering the continuity of $I(t)$ at $t = t_1$, it follows from the equation (3.0) and (3.2)

$$I_1(t_1) = I_2(t_1)$$

$$\Rightarrow Q_1-(at_1+\frac{bt_1^{\ 2}}{2}+\frac{ct_1^{\ 3}}{3})=K-\alpha\left(a(t_2-t_1)+\frac{b(t_2^{\ 2}-t_1^{\ 2})}{2}+\frac{c(t_2^{\ 3}-t_1^{\ 3})}{3}\right)(t_1-\gamma)^{\beta} \quad (3.4)$$

For each cycle, the maximum inventory level is determined by.

$$Q_1=(at_1+\frac{bt_1^{\ 2}}{2}+\frac{ct_1^{\ 3}}{3})+K-\alpha\left(a(t_2-t_1)+\frac{b(t_2^{\ 2}-t_1^{\ 2})}{2}+\frac{c(t_2^{\ 3}-t_1^{\ 3})}{3}\right)(t_1-\gamma)^{\beta} \quad (3.5)$$

Putting equation (3.5) into equation (3.2), we have

$$I_1(t)=\begin{bmatrix}(at_1+\frac{bt_1^{\ 2}}{2}+\frac{ct_1^{\ 3}}{3})+K \\ -\alpha\left(a(t_2-t_1)+\frac{b(t_2^{\ 2}-t_1^{\ 2})}{2}+\frac{c(t_2^{\ 3}-t_1^{\ 3})}{3}\right)(t_1-\gamma)^{\beta} \\ -(at+\frac{bt^2}{2}+\frac{ct^3}{3})\end{bmatrix} 0\le t\le t_1 \quad (3.6)$$

During the shortage interval $[t_1,T]$, the demand at time $'t'$ is partially backlogged at rate

$e^{-\delta(T-t)}$. The differential equation governs that the amount of demand backlog is given by

$$\frac{dI_3(t)}{dt}=-e^{-\delta(T-t)}\left(a+bt+ct^2\right), t_2\le t\le T \quad (3.7)$$

with boundary conditions at $I(t_2)=0$ and $I(T)=-Q_2$

The solution of equation (3.7) becomes

$$I_3(t)=\begin{bmatrix}\left(a(t_2-t)+\frac{b(t_2^{\ 2}-t^2)}{2}+\frac{c(t_2^{\ 3}-t^3)}{3}\right)-a\delta\left(T(t_2-t)-\frac{(t_2^{\ 2}-t^2)}{2}\right) \\ -b\delta\left(T\frac{(t_2^{\ 2}-t^2)}{2}-\frac{(t_2^{\ 3}-t^3)}{3}\right)-c\delta\left(T\frac{(t_2^{\ 3}-t^3)}{3}-\frac{(t_2^{\ 4}-t^4)}{4}\right)\end{bmatrix}, t_2\le t\le T \quad (3.8)$$

The maximum amount of demand backlogged every cycle calculated by Substituting $t=T$ in equation (3.8).

$$Q_2=\begin{bmatrix}\left(a(t_2-T)+\dfrac{b(t_2{}^2-T^2)}{2}+\dfrac{c(t_2{}^3-T^3)}{3}\right)-a\delta\left(T(t_2-T)-\dfrac{(t_2{}^2-T^2)}{2}\right)\\ -b\delta\left(T\dfrac{(t_2{}^2-T^2)}{2}-\dfrac{(t_2{}^3-T^3)}{3}\right)-c\delta\left(T\dfrac{(t_2{}^3-T^3)}{3}-\dfrac{(t_2{}^4-T^4)}{4}\right)\end{bmatrix}$$

$$=\begin{bmatrix}a(1-\delta T)(t_2-T)+\dfrac{1}{2}(b+a\delta-b\delta T)(t_2{}^2-T^2)\\ +\dfrac{1}{3}(c+b\delta-c\delta T)(t_2{}^3-T^3)+\dfrac{1}{4}c\delta(t_2{}^4-T^4)\end{bmatrix} \quad (3.9)$$

By adding (3.5) and (3.9), the total order quantity Q every cycle was obtained as

$$Q=(at_1+\frac{bt_1{}^2}{2}+\frac{ct_1{}^3}{3})+K-\alpha\left(a(t_2-t_1)+\frac{b(t_2{}^2-t_1{}^2)}{2}+\frac{c(t_2{}^3-t_1{}^3)}{3}\right)(t_1-\gamma)^{\beta}$$

$$+\begin{bmatrix}a(1-\delta T)(t_2-T)+\dfrac{1}{2}(b+a\delta-b\delta T)(t_2{}^2-T^2)\\ +\dfrac{1}{3}(c+b\delta-c\delta T)(t_2{}^3-T^3)+\dfrac{1}{4}c\delta(t_2{}^4-T^4)\end{bmatrix} \quad (4.0)$$

The following are the costs associated with the inventory system:

- Ordering cost
- Holding cost
- Shortage cost
- Deterioration cost

We derive the associated cost as follows:

Ordering Cost $\left(OC\right)=A$ (4.1)

Holding cost $\left(HC\right)$:

$$HC=\int_0^{t_2} I(t)e^{-Rt}dt=\int_0^{t_1} I_1(t)e^{-Rt}dt+\int_{t_1}^{t_2} I_2(t)e^{-Rt}dt \quad \text{(See appendix)} \tag{4.2}$$

Shortages Cost:

$$SC=-c_2\int_{t_2}^{T} I(t)e^{-Rt}dt=-c_2\int_{t_2}^{T} I_3(t)e^{-Rt}dt$$

$$=-c_2\int_{t_2}^{T}\left[\left[\begin{array}{l}\left(a(t_2-t)+\dfrac{b(t_2{}^2-t^2)}{2}+\dfrac{c(t_2{}^3-t^3)}{3}\right)-a\delta\left(T(t_2-t)-\dfrac{(t_2{}^2-t^2)}{2}\right)\\ -b\delta\left(T\dfrac{(t_2{}^2-t^2)}{2}-\dfrac{(t_2{}^3-t^3)}{3}\right)-c\delta\left(T\dfrac{(t_2{}^3-t^3)}{3}-\dfrac{(t_2{}^4-t^4)}{4}\right)\end{array}\right]\right]e^{-Rt}dt$$

$$=\frac{(T-t_2)^2c_2}{40}\left[\begin{array}{l}\dfrac{-10}{3}Rc\delta t_2^4+\left(\dfrac{-8}{3}Rc\delta T+(-4Rb+8c)\delta-4Rc\right)t_2^3\\ +(3c\delta RT^2+((-4c-3Rb)\delta-8Rc)T+(10b-5Ra)\delta-5Rb+10c)t_2^2\\ +\left(\begin{array}{l}2c\delta RT^3+\left(\left(\dfrac{14}{3}Rb-\dfrac{8}{3}c\right)\delta-\dfrac{16}{3}Rc\right)T^2\\ +\left(\left(-\dfrac{20}{3}b-\dfrac{10}{3}Ra\right)\delta+\dfrac{20}{3}c-10Rb\right)T-\dfrac{20}{3}Ra+\dfrac{40}{3}b+\dfrac{40}{3}a\delta\end{array}\right)t_2\\ +c\delta RT^4+\left(\left(\dfrac{7}{3}Rb-\dfrac{4}{3}c\right)\delta-\dfrac{8}{3}Rc\right)T^3+\left(\left(\dfrac{25}{3}Ra-\dfrac{10}{3}b\right)\delta+\dfrac{10}{3}c-5Rb\right)T^2\\ +\left(-\dfrac{40}{3}a\delta+\dfrac{20}{3}b-\dfrac{40}{3}Ra\right)T+20a\end{array}\right] \tag{4.3}$$

Deterioration Cost:

$$DC = c_p\left[Q_1 - \int_0^{t_2}(a+bt+ct^2)e^{-Rt}dt\right] = c_p\left[Q_1 - \int_0^{t_1}(a+bt+ct^2)e^{-Rt}dt - \int_{t_1}^{t_2}(a+bt+ct^2)e^{-Rt}dt\right]$$

$$= c_p\left[\begin{array}{l} a\left((1-\alpha t_1^{\beta})t_2 + \alpha t_1^{\beta+1}\right) + \frac{1}{2}b\left((1-\alpha t_1^{\beta})t_2^2 + \alpha t_1^{\beta+2}\right) \\ \frac{1}{3}c\left((1-\alpha t_1^{\beta})t_2^3 + \alpha t_1^{\beta+3}\right) + \frac{a\alpha(t_2^{\beta+1} - t_1^{\beta+1})}{\beta+1} + \frac{b\alpha(t_2^{\beta+2} - t_1^{\beta+2})}{\beta+2} \\ + \frac{c\alpha(t_2^{\beta+3} - t_1^{\beta+3})}{\beta+3} + \frac{1}{4}cRt_1^4 - \frac{1}{3}(-bR+c)t_1^3 - \frac{1}{2}(-aR+b)t_1^2 \\ -at_1 + \frac{1}{4}cR(t_2^4 - t_1^4) - \frac{1}{3}(-bR+c)(t_2^3 - t_1^3) \\ -\frac{1}{2}(-aR+b)(t_2^2 - t_1^2) - a(t_2 - t_1) \end{array}\right] \quad (4.4)$$

In this study, we considered two cases; interest paid and interest earned. Case I: $(0 \le M \le t_2)$ and Case II: $(0 \le t_2 \le M)$.

CaseI: $(0 \le M \le t_2)$: The retailers earns interest on sales revenue up to M. interest is earned during the period M to t_2. The interest earned was obtained as;

Interest earned per cycle:

$$IE_1 = pI_e \int_0^{M} t\,\mathrm{e}^{-Rt}\left(a+bt+ct^2\right)dt$$

$$= pI_e\left[\frac{-1}{5}cRM^5 + \frac{1}{4}(-bR+c)M^4 + \frac{1}{3}(-aR+b)M^3 + \frac{1}{2}aM^2\right] \quad (4.5)$$

Interest payable per cycle after the due period M:

$$IP_1 = cI_p \int_M^{t_2} I(t)\,\mathrm{e}^{-Rt}\,dt \text{ (See appendix)} \qquad (2.6)$$

During a cycle, $C_1(t_2,T)$ the overall cost per unit time consists of the following:

$$C_1(t_2,T)=\frac{1}{T}\left[OC+HC+DC+SC+IP_1-IE_1\right] \quad (4.7)$$

The total cost for case I was calculated by substituting equations (4.2-4.6) into equation (4.7).

Equation (18) is differentiated with respect to T and t_1 and equates to zero:

$$\frac{\partial C_1(t_2,T)}{\partial T}=0,\ \frac{\partial C_1(t_2,T)}{\partial t_1}=0 \quad (4.8)$$

The optimal cycle length was found as $T=T^*$ and $t_2=t_2^*$ by calculating equation (4.8) for T and t_2, assuming that it satisfied the equation

$$\frac{\partial^2 C_1(t_2,T)}{\partial T^2}>0,\ \frac{\partial^2 C_1(t_2,T)}{\partial {t_1}^2}>0 \text{ and } \frac{\partial^2 C_1(t_2,T)}{\partial T^2}\left[\frac{\partial^2 C_1(t_2,T)}{\partial {t_1}^2}\right]-\left[\frac{\partial^2 C_1(t_2,T)}{\partial T\ \partial t_1}\right]>0 \quad (4.9)$$

Case II: $(0\le t_1\le M)$ Interest is accrued until the allowable delay time is reached, after which no interest is due.

Interest accrued until the allowable delay time:

$$IE_2=pI_e\left[\int_0^{t_1} t\,\mathrm{e}^{-Rt}\left(a+bt+ct^2\right)dt+\int_{t_1}^{t_2} t\,\mathrm{e}^{-Rt}\left(a+bt+ct^2\right)dt+\left(a+bt+ct^2\right)t_2(M-t_2)\right]$$

$$=pI_e\begin{bmatrix}\frac{-1}{6}cR{t_1}^5+\frac{1}{5}(-bR+c){t_1}^5+\frac{1}{4}(-aR+b){t_1}^4+\frac{1}{3}a{t_1}^3\\ \frac{-1}{5}cR({t_2}^5-{t_1}^5)+\frac{1}{4}(-bR+c)({t_2}^4-{t_1}^4)+\frac{1}{3}(-aR+b)({t_2}^3-{t_1}^3)\\ \frac{1}{2}a({t_2}^2-{t_1}^2)+\left(a+bt+ct_2^2\right)t_2(M-t_2)\end{bmatrix} \quad (5.0)$$

The total cost per unit during a cycle, $C_2(t_2,T)$ consist of the following:

$$C_2(t_2,T)=\frac{1}{T}\left[OC+HC+DC+SC-IE_2\right] \quad (5.1)$$

Substituting the equations (4.2, 4.3, 4.4, and 5.0) into equation (5.1)the total cost for case II was obtained.

Differentiating equation (5.1) with respect to T and t_2 and equate it to zero:

$$\frac{\partial C_2(t_2,T)}{\partial T}=0, \text{ and } \frac{\partial C_2(t_2,T)}{\partial t_2}=0 \quad (5.2)$$

By solving equation (5.2) for T and t_1 , the optimal cycle length was obtained $T=T^*$ and $t_2=t_2^*$

Provided it satisfy the equation

$$\frac{\partial^2 C_2(t_2,T)}{\partial T^2}>0,\ \frac{\partial^2 C_2(t_2,T)}{\partial {t_2}^2}>0 \text{ and } \frac{\partial^2 C_2(t_2,T)}{\partial T^2}\left[\frac{\partial^2 C_2(t_2,T)}{\partial {t_2}^2}\right]-\left[\frac{\partial^2 C_2(t_2,T)}{\partial T\ \partial t_2}\right]>0 \quad (5.3)$$

5. Numerical Example

To illustrate the models numerically, the following parameters and inventory data taken from Raman and Veer [28]were used. A = 100,=25, p =40, I_p =0.15, I_e =0.12, M =0.08 years, a =1000, b =0.05, c =0.01, c_p C_2 = 8, C_3 = 2, R =0.01, backlogging parameter δ= 0.8, and t_1 = 0.05. In addition to the data we let scale parameter α = 0.04, shape parameter β = 2, location parameter γ = 0.5.

Case I:

For this case, the optimal value of t_2^* = 0.6052, T^* =1.0843, the optimal total cost $C_1(t_2,T)$ = 865.1502 and the optimum order quantity Q^* = 301.523 are obtained.

Case II:

For this case, the optimal value of t_2^* = 0.6843, T^* =1.0451, the optimal total cost $C_2(t_2, T)$ * = 624.541 and the optimum order quantity Q^* = 204.21are obtained.

6. Sensitivity Analysis

The sensitivity analysis is done in regard to certain connected parameters.This is accomplished by altering one parameter while keeping the others constant.Only two cases were subjected to a sensitivity analysis.

Case I: $M \leq t_2$

Table 1: Sensitivity Analysis for Case I

Parameters	**%**	**t_2**	**T**	**Cost**	**Q**
p	+50%	0.0532	1.049	844.5	280.6
	+20%	0.1253	1.0729	798.3	295.3
	-20%	0.1556	1.0924	793.6	295.4
	-50%	0.1753	1.1525	678.5	300.4
α	+50%	0.0456	1.0271	890.4	320.4
	+20%	0.1552	1.0294	856.3	302.5
	-20%	0.0765	1.0458	798.6	301.2
	-50%	0.0835	1.0734	769.2	300.7
β	+50%	0.1172	1.0353	672.2	245.3
	+20%	0.1178	1.0247	740.2	265.5
	-20%	0.1182	1.0287	798.5	268.2
	-50%	0.1192	1.0399	845.8	285.3
M	+50%	0.1263	1.1875	709.5	301.2
	+20%	0.1248	1.1801	742.5	315.5
	-20%	0.5130	1.1824	765.4	322.3
	-50%	0.5342	1.1860	780.2	331.7
R	+50%	0.1986	1.1370	824.1	336.5
	+20%	0.1820	1.1401	822.3	325.4
	-20%	0.2462	1.1440	800.3	320.1
	-50%	0.4562	1.4820	794.5	306.3

Case II: M ≥ t_1

Table 2: Sensitivity Analysis for Case II

Parameters	**%**	**t_2**	**T**	**Cost**	**Q**
p	+50%	0.5863	1.8854	709.2	277.3
	+20%	0.4583	1.1712	543.1	286.2
	-20%	0.4355	1.1799	519.4	328.5
	-50%	0.3501	1.5598	511.1	348.0
α	+50%	0.8346	1.1444	611.5.	384.21
	+20%	0.1409	1.3076	618.3	251.5
	-20%	0.5512	1.8333	678.4	299.7
	-50%	0.6065	1.6664	775.2	363.52
β	+50%	0.7067	1.7045	621.8	229.24
	+20%	0.6160	1.1046	770.7	252.15
	-20%	0.0125	1.4254	678.2	363.65
	-50%	0.2745	1.9635	688.4	316.24
M	+50%	0.0255	1.0481	500.8	333.8
	+20%	0.3596	1.9908	528.7	335.69
	-20%	0.7630	1.8240	587.8	348.95
	-50%	0.9197	1.4880	691.1	397.52
R	+50%	0.7033	1.0360	760.2.3	298.14
	+20%	0.8095	1.4061	771.4	307.56
	-20%	0.9253	1.9785	624.8	338.74

	-50%	0.2535	1.8235	569.2	357.62

Observations

From the above results, the followings are observed:

1. When the selling price (p) rises, and the overall system total cost increases it is very sensitive to the selling price inboth scenarios 1 and 2.

2. In both scenarios 1 and 2, as the demand parameter (α) rises, the total cost of the system rises as well.

3. For both scenarios, it was discovered that as the parameter (β) increases, the overall value price drops.

4. For both scenarios, it was revealed that if the value of M increases or falls, the overall price and quantity reduces or increases.

5. An increase in the inflation parameter (R) corresponds to an increase in the total cost.

7. **Conclusion**

In this study, a three-parameter Weibull distribution inventory model for deteriorating items with nonlinear demand and trade credit under inflation was proposed.Shortages were permitted and there was a partial backlog. The sensitivity analysis revealed that the parameters p, α, β, M, & R are more sensitive to changes in t_2, T, Q and total cost

7.1 Recommendation

There are several approaches to expand the presented model. For instance,we can make it more realistic by including non-zero lead time and stochastic demand. Furthermore, the model's parameters can be regarded of as fuzzy variables.

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APPENDIX

The results below were obtained with the help of a software **Mathematica 11** version 11.3.0.0

Holding Cost:

1. $HC = \int_0^{t_2} I(t)e^{-Rt}dt = \int_0^{t_1} I_1(t)e^{-Rt}dt + \int_{t_1}^{t_2} I_2(t)e^{-Rt}dt$

$$= \int_0^{t_1} \left[\begin{array}{l} (at_1 + \frac{bt_1^{\,2}}{2} + \frac{ct_1^{\,3}}{3}) + a\left((t_2 - t) + \frac{\alpha[(t_2-\gamma)^{\beta+1} - (t-\gamma)^{\beta+1}]}{\beta+1} \right) \\ +b\left(\frac{(t_2^{\,2} - t^2)}{2} + \alpha\left(\frac{t_2[(t_2-\gamma)^{\beta+1} - (t-\gamma)^{\beta+1}]}{\beta+1} - \frac{[(t_2-\gamma)^{\beta+2} - (t-\gamma)^{\beta+2}]}{(\beta+1)(\beta+2)} \right)\right) \\ +c\left(\frac{(t_2^{\,3} - t^3)}{3} + \alpha\left(\frac{t_2^{\,2}[(t_2-\gamma)^{\beta+1} - (t-\gamma)^{\beta+1}]}{\beta+1} \right.\right. \\ \left.\left. -\frac{2}{\beta+1}\left(\frac{t_2[(t_2-\gamma)^{\beta+2} - (t-\gamma)^{\beta+2}]}{(\beta+2)} - \frac{[(t_2-\gamma)^{\beta+3} - (t-\gamma)^{\beta+3}]}{(\beta+2)(\beta+3)} \right)\right)\right) \\ -\alpha\left(a(t_2 - t_1) + \frac{b(t_2^{\,2} - t_1^{\,2})}{2} + \frac{c(t_2^{\,3} - t_1^{\,3})}{3} \right)(t_1-\gamma)^{\beta} \\ -(at + \frac{bt^2}{2} + \frac{ct^3}{3}) \end{array} \right] e^{-Rt}dt$$

$$+\int_{t_1}^{t_2} \left[\begin{array}{l} a\left((t_2 - t) + \frac{\alpha[(t_2-\gamma)^{\beta+\ldots}}{\beta \ldots} \right. \\ +b\left(\frac{(t_2^{\,2} - t^2)}{2} + \alpha\left(\frac{t_2[(t_2 \ldots}{} \right.\right. \\ +c\left(\frac{(t_2^{\,3} - t^3)}{3} + \alpha\left(\frac{t_2^{\,2}[(t \ldots}{} \right.\right. \\ -\frac{2}{\beta+1}\left(\frac{t_2[(t_2-\gamma)^{\beta+2} - \ldots}{(\beta+2 \ldots} \right. \\ -\alpha\left(a(t_2 - t) + \frac{b(t_2^{\,2} - t^2 \ldots}{2} \right. \end{array} \right.$$

$$\frac{\beta^4}{(\beta+1)(\beta+2)(\beta+3)(\beta+4)(\beta+5)(\beta+6)}$$

$$\left[\left[\begin{array}{l} (t_2 - t_1)(\mathrm{t}_1 - \gamma)(\mathrm{R}\, t_1 - 1)\left(\frac{1}{3}ct_1 + (\frac{1}{3}ct_1 + \frac{1}{2}b)t_2 + \frac{1}{3}ct_1 + \frac{1}{2}t_1 b + a \right)\beta^5 \\ + \left(\begin{array}{l} -\frac{19}{3}\left(\frac{18}{19}t_1^2\gamma + \gamma t_1 - \frac{20}{19} \right)ct_2^3 \\ -\frac{19}{2}\left(\frac{18}{19}t_1^2\gamma + \gamma t_1 - \frac{20}{19} \right)bt_2^2 \\ -\frac{19}{2}\left(\frac{18}{19}t_1^2\gamma + \gamma t_1 - \frac{20}{19} \right)at_2 \\ +17\left(\begin{array}{l} \frac{4}{17}Rct_1^4 + \left(\left(\frac{13}{34}c + \frac{7}{17}b \right)R - \frac{13}{51}c \right)t_1^3 \\ + \left(\left(\frac{15}{34}b + \frac{16}{17}a \right)R - \frac{14}{51}c - \frac{15}{34}b \right)t_1^2 \\ + \left(Ra - \frac{8}{17}b - a \right) \end{array} \right) t_1 - \frac{18}{17}a \end{array} \right) t_1 \end{array} \right] \beta^4 \right]$$

$$
+\frac{\beta^3}{(\beta+1)(\beta+2)(\beta+3)(\beta+4)(\beta+5)(\beta+6)}
\left[+\left(\begin{array}{l}
-\frac{137}{3}\left(\frac{121}{137}t_1^2\gamma+\gamma t_1-\frac{155}{137}\right)ct_2^3\\
-\frac{137}{2}\left(\frac{121}{137}t_1^2\gamma+\gamma t_1-\frac{155}{137}\right)bt_2^2\\
-137\left(\frac{121}{137}t_1^2\gamma+\gamma t_1-\frac{155}{137}\right)at_2\\
+104\left(\begin{array}{l}
\frac{49}{312}Rct_1^4+\left(\left(\frac{7}{39}c+\frac{67}{208}b\right)R-\frac{7}{39}c\right)t_1^3\\
+\left(\left(\frac{77}{208}b+\frac{7}{8}a\right)R-\frac{5}{24}c-\frac{77}{208}b\right)t_1^2\\
+\left(-a+Ra-\frac{89}{208}b\right)t_1-\frac{119}{104}a
\end{array}\right)t_1
\end{array}\right)\right]
$$

$$
+\frac{\beta^2}{(\beta+1)(\beta+2)(\beta+3)(\beta+4)(\beta+5)(\beta+6)}
\left[+\left(\begin{array}{l}
-\frac{461}{3}\left(\frac{372}{461}t_1^2\gamma+\gamma t_1-\frac{580}{461}\right)ct_2^3\\
-\frac{461}{2}\left(\frac{372}{461}t_1^2\gamma+\gamma t_1-\frac{580}{461}\right)bt_2^2\\
-461\left(\frac{372}{461}t_1^2\gamma+\gamma t_1-\frac{580}{461}\right)at_2\\
+268\left(\begin{array}{l}
\frac{13}{134}Rct_1^4+\left(\left(\frac{63}{268}c+\frac{23}{201}b\right)R-\frac{23}{201}c\right)t_1^3\\
+\left(\left(\frac{153}{536}b+\frac{54}{67}a\right)R-\frac{28}{201}c-\frac{153}{536}b\right)t_1^2\\
+\left(-\frac{97}{268}b-a+Ra\right)t_1-\frac{171}{134}a
\end{array}\right)t_1
\end{array}\right)\right]
$$

$$
+\frac{\alpha t_1^{\beta+2}}{(\beta+1)(\beta+2)(\beta+3)(\beta+4)(\beta+5)(\beta+6)}
\left[+\left(\begin{array}{l}
-234\left(\frac{254}{351}t_1^2\gamma+\gamma t_1-\frac{58}{39}\right)ct_2^3 \\
-351\left(\frac{254}{351}t_1^2\gamma+\gamma t_1-\frac{58}{39}\right)bt_2^2 \\
-702\left(\frac{254}{351}t_1^2\gamma+\gamma t_1-\frac{58}{39}\right)at_2 \\
+240\left(\begin{array}{l}
\frac{1}{18}Rct_1^4+\left(\left(\frac{3}{20}b+\frac{1}{15}c\right)R-\frac{1}{15}c\right)t_1^3 \\
+\left(\left(\frac{3}{16}b+\frac{3}{4}a\right)R-\frac{1}{12}c-\frac{3}{16}b\right)t_1^2 \\
+\left(-a+Ra-\frac{1}{4}b\right)t_1-\frac{3}{2}a
\end{array}\right)t_1 \\
-360\left(\frac{2}{3}t_1^2R+Rt_1\right)t_2\left(\frac{1}{2}bt_2+\frac{1}{3}ct_2^2+a\right)
\end{array}\right)\right]
$$

$$+\frac{2\alpha t_2^{\beta+2}}{(\beta+1)(\beta+2)(\beta+3)(\beta+4)(\beta+5)(\beta+6)}$$

$$\left(t_2+\gamma\right)\left(bt_2+ct_2^2+a\right)\left(t_2R-1\right)\beta^4$$

$$\left[\begin{array}{l} +\left(\begin{array}{l} +\left(\begin{array}{l} 12t_2^4Rc+\left(\left(\frac{27}{2}c+\frac{27}{2}b\right)R-\frac{27}{2}c\right)t_2^3 \\ +\left(\left(15a+15b\right)R-15b-15c\right)t_2^2 \\ +\left(\frac{-33}{2}a-\frac{33}{2}b+\frac{33}{2}Ra\right)t_2-18a \end{array}\right)\beta^3 \\ +\left(\begin{array}{l} 49t_2^4Rc+\left(\left(\frac{123}{2}b+\frac{123}{2}c\right)R-\frac{123}{2}c\right)t_2^3 \\ +\left(\left(77b+78a\right)R-77b-78c\right)t_2^2 \\ +\left(\frac{193}{2}Ra-\frac{193}{2}a+193b\right)t_2-119a \end{array}\right)\beta^2 \\ +\left(\begin{array}{l} 78t_2^4Rc+\left(\left(109c+109b\right)R-109c\right)t_2^3 \\ +\left(\left(164a+153b\right)R-164c-153b\right)t_2^2 \\ +\left(-231Rb-231Ra-231a\right)t_2-342a \end{array}\right)\beta \\ +180\left(\frac{1}{2}bt_2+\frac{1}{3}ct_2^2+a\right)\left(\frac{2}{3}t_2^2+t_2\right)R \end{array}\right) \end{array}\right]$$

$$+\frac{1}{2(\beta+1)(\beta+2)(\beta+3)}$$

$$\left[+\left(\begin{array}{l} -(\beta+2)(\beta+3)\alpha t_1a\left(\frac{2}{3}t_1^2\gamma+\gamma t_1\right)t_1^{\beta+2}-\frac{1}{2}(\beta+3)\,\mathrm{t}_1\,\alpha b\left(\frac{2}{3}t_1^2\gamma+\gamma t_1\right)\beta(\beta+1)t_1^{\beta+2} \\ -\frac{1}{3}(\beta+2)\alpha t_1c\left(\frac{2}{3}t_1^2\gamma+\gamma t_1\right)\beta(\beta+1)t_1^{\beta+3}-(\beta+3)(\beta+2)\,\mathrm{t}_1\,\alpha a\left(\frac{2}{3}t_1^2\gamma+\gamma t_1\right)t_2^{\beta+1} \\ -(\beta+3)\,\mathrm{t}_1\,\alpha b\left(\frac{2}{3}t_1^2\gamma+\gamma t_1\right)(\beta+1)t_2^{\beta+2}-(\beta+2)\,\mathrm{t}_1\,\alpha c\left(\frac{2}{3}t_1^2\gamma+\gamma t_1\right)(\beta+1)t_2^{\beta+3} \end{array}\right)\right]$$

$$+\frac{t^2}{2(\beta+1)(\beta+2)(\beta+3)}$$

$$\left[\left(\begin{array}{l}\left(\frac{2}{3}t_2^2\gamma+\left(\frac{2}{3}t_2^2\gamma+\gamma t_1\right)t_2+\frac{2}{3}t_1^2\gamma++\gamma t_1\right)+\\ \left(\left(bt_2+ct_2^2+a\right)\beta^2+\left(3ct_2^2+4bt_2+5a\right)\beta+2ct_2^2+3bt_2+6a\right)\left(t_1-t_2\right)\alpha\gamma t_2^{\beta}\\ +(\beta+3)(\beta+2)\\ +\left(\begin{array}{l}\left(\frac{2}{3}t_1^2\gamma R+\gamma Rt_1\right)\left(\frac{1}{2}bt_2+\frac{1}{3}ct_2^2+a\right)\alpha t_1^{\beta}\\ -\frac{1}{3}t_2\left(\begin{array}{l}\frac{1}{3}t_2^4Rc+\left(\left(\frac{2}{5}b+\frac{3}{5}c\right)R-\frac{3}{5c}\right)t_2^3\\ +\left(\left(\frac{4}{3}b+\frac{1}{2}a\right)R-\frac{3}{2}c-\frac{3}{2}b\right)t_2^2\\ +\left(2b+Ra-a\right)t_2-3a\end{array}\right)\left(\beta+1\right)\end{array}\right)\end{array}\right)\right]$$

2. $IP_1 = c\int_M^{t_2} I\left(t\right)e^{-Rt}dt$

$$=c\int_M^{t_2}\left[\begin{array}{l}a\left((t_2-t)+\frac{\alpha[(t_2-\gamma)^{\beta+1}-(t-\gamma)^{\beta+1}]}{\beta+1}\right)\\ +b\left(\frac{(t_2{}^2-t^2)}{2}+\alpha\left(\frac{t_2[(t_2-\gamma)^{\beta+1}-(t-\gamma)^{\beta+1}]}{\beta+1}-\frac{[(t_2-\gamma)^{\beta+2}-(t-\gamma)^{\beta+2}]}{(\beta+1)(\beta+2)}\right)\right)\\ +c\left(\frac{(t_2{}^3-t^3)}{3}+\alpha\left(\frac{t_2{}^2[(t_2-\gamma)^{\beta+1}-(t-\gamma)^{\beta+1}]}{\beta+1}\right.\right.\\ \left.\left.-\frac{2}{\beta+1}\left(\frac{t_2[(t_2-\gamma)^{\beta+2}-(t-\gamma)^{\beta+2}]}{(\beta+2)}-\frac{[(t_2-\gamma)^{\beta+3}-(t-\gamma)^{\beta+3}]}{(\beta+2)(\beta+3)}\right)\right)\right)\\ -\alpha\left(a(t_2-t)+\frac{b(t_2{}^2-t^2)}{2}+\frac{c(t_2{}^3-t^3)}{3}\right)(t-\gamma)^{\beta}\end{array}\right]e^{-Rt}dt$$

$$
=\frac{c_p I_p (M-t_2)\alpha M^{\gamma(\beta+1)}}{3(\beta+1)(\beta+2)(\beta+3)}
$$

$$
\left[\begin{array}{l}
(RM-1)(M-t_2)\left(ct_2^2+\left(Mc+\frac{3}{2}b\right)t_2+cM^2+\frac{3}{2}Mb3a\right)\beta^4 \\
+\left(\begin{array}{l}
(-13McR+14c)t_2^3+\left(-\frac{39}{2}MbR+21b\right)t_2^2 \\
+(-39MaR+42a)t_2+7M^4cR+\left(-8c+\frac{27}{2}bR\right)M^3 \\
+(-15b+33aR)M^2-36Ma
\end{array}\right)\beta^3 \\
+\left(\begin{array}{l}
(71c-59McR+14c)t_2^3+\left(\frac{213}{2}b-\frac{177}{2}MRb\right)t_2^2 \\
+(-177MaR+213a)t_2+14M^4cR+\left(\frac{69}{2}bR-17c\right)M^3 \\
+\left(114aR-\frac{87}{2}b\right)M^2-141Ma
\end{array}\right)\beta^2 \\
+\left(\begin{array}{l}
(-107McR+154c)t_2^3+\left(-\frac{321}{2}MRb+231b\right)t_2^2 \\
+(462a-321MaR)t_2+8M\left(M^3cR+\left(\frac{45}{16}bR-\frac{5}{4}c\right)M^2+\left(15aR-\frac{15}{4}b\right)M-\frac{45}{2}a\right) \\
-180(-2+RM)t_2\left(a+\frac{1}{2}bt_2+\frac{1}{3}ct_2^2\right)
\end{array}\right)\beta
\end{array}\right]
$$

$$
+\frac{2c_p I_p (M-t_2)\alpha t_2^{\gamma(\beta+1)}}{(\beta+1)(\beta+2)(\beta+3)}
$$

$$
\left[\begin{array}{l}
(bt_2+ct_2^2+a)(-1+Rt_2)\beta^3+\left(\frac{15}{2}ct_2^3R+(9bR-9c)t_2^2+\left(-\frac{21}{2}b+\frac{21}{2}Ra\right)t_2-12a\right)\beta^2 \\
+\left(\frac{33}{2}ct_2^3R+(-24c+23bR)t_2^2+\left(\frac{67}{2}aR-\frac{67}{2}b\right)t_2-47a\right)\beta+30(t_2R-2)\left(a+\frac{1}{2}bt_2+\frac{1}{3}ct_2^2\right)
\end{array}\right]
$$

$$+\frac{c_p I_p (M-t_2)}{15(\beta+1)(\beta+2)(\beta+3)}$$

$$\left[\begin{array}{l} -\frac{15}{2}\left(\left(bt_2+ct_2^2+a\right)\beta+\left(4bt_2+5a+3ct_2^2\right)\beta+3bt_2+6a+2ct_2^2\right)\alpha\gamma\left(t_2R+RM-2\right)t_2^{\beta+2} \\ +\left(\beta+1\right)\left(\beta+2\right)\left(\beta+3\right)(M-t_2) \\ \left(\frac{3}{2}ct_2^3R+\left(-\frac{15}{4}c+3McR\frac{15}{8}bR\right)t_2^2+2M^2cR+\left(-\frac{5}{2}c+\frac{15}{4}bR\right)M+\frac{5}{2}Ra-5b\right)t_2 \\ +M^3cR+\left(\frac{15}{8}bR-\frac{5}{4}c\right)M^2+\left(5aR-\frac{5}{2}b\right)M-\frac{15}{2}a \end{array}\right]$$