

A 3-Person Non-Zero-Sum Game for Sachet Water Companies

Abstract

Sachet water (popularly called pure water in Nigeria) businesses are often competitive due to its high demand. This is so because sachet water is the most affordable form of pure drinking water in Nigeria. Therefore, sachet water firms that aim at excelling in terms of results when facing an ever-competitive market in which competition gets tougher everyday need the knowledge of game theory to identify that strategy which will yield better profit independent of the strategy adopted by other competitors. This paper is aimed to investigate and determine the equilibrium point for three sachet water firms using the Nash equilibrium method as it provides a systematic approach for deciding the best strategy in competitive situation. The result showed two Nash equilibria (promo, promo) and (stay-put, stay-put) with the payoffs (82; 82; 82) and (147; 147; 147) respectively. Proof reading...?? Sentences are not clear.

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Keywords: *Game Theory, Sachet Water, 3-Person, Non-zero Sum, Best Response, Nash Equilibrium, Pareto-Optimal, Pareto-Dominated*

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Introduction

The founding fathers of games theory are considered to be John Von Neumann and Oscar Morgenstern year ?. They authored the book "Theory of Games and Economic Behavior" and discussed in it that a game is a situation in which two or more players participated in the pursuit of competing goals. Games theory is the study of mathematical models of strategic interaction among rationalized decision makers. According to Nyor *et al.* (2019), it is a tool that can model any situation in which there are people that interact by taking decisions, making moves, etc. in order to attain certain goals.

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In other words Games theory is a type of decision theory in which one's choice is determined after considering potential options available to the rivals playing the game. In Ordinary decisions under uncertainty, the decision maker is has faced with only a random process. The decision maker and also they have not only to analyze alternative course of action available to him but also consider all the possible goals, strategies and choices of the competitor. A solution to a game describes the optimal decisions of the players, who may have similar, opposed or mixed interest and the outcome that may result from these decisions.

In the business sector, strategic behavior is common among executives, managers, and investors. They must determine whether to enter new markets, develop new products, invest now or miss out on the opportunity to invest, and set pricing and purchasing policies. Game theoretical models

have a lot of possibilities when it comes to assessing business decisions. In game theory models, each player ~~is-has~~ advised to consider the actions of others when deciding on a strategy, as the player ~~may-have~~ decide to react to the moves of his opponents. It is ~~more-very~~ advantageous ~~to~~ ~~afor~~ decision maker (Geckil and Anderson, 2010).

Justin *et al.* (2014) ~~cited-opined~~ that sachet water is an important primary source of drinking water in West Africa and explores the relationship between local perceptions of brand quality and bacteriologic quality after controlling ~~for~~-characteristics of vending environment. The results indicates substantial progress in sachet water regulation and quality control. However,

Sachet water sold in Nigeria and Ghana samples were showed to be pathogenic contamination of as little as (5%) among the (60 ~~to~~ 70%) of the sample tested. Obiri-Danso *et al.* (2003), Mgbakor *et al.* (2011), and Oyelude & Ahenkorah (2012). Demographic and health sector classify sachet water sold in Nigeria as unimproved probably due to its low pricing according to Sridhar *et al.* (2017).

Literature Review

Many literature cited world wide , the Game theory is popular and well known as a vital tool in various fields (cite literature ????). The development of game theory ~~is-has~~ largely expanded. The discussion of game theory ~~is-was~~ noticed ed to initiate with a letter written in 1713 by James 'Waldegrave' in which Waldegrave obtained solution to a two person card game with mixed strategy. James Madison has made what is now known as a game theoretical examination of how states are likely to act under various system of taxation. The study of a dupopoly by Antoine Cournot in 1838 is the most recent example of a formal game theoretical analysis. In his paper, "Researches into the Mathematical Principles of the Theory of Wealth" he provides ed a solution ~~that is a-and~~ constrains on ed case of the Nash equilibrium Crider (2012). In the years 1921 ~~the~~ mathematician 'Prof. Emile Borel' proposed a formal theory ~~ofon~~ games, which was expanded by the John Von Neumann in a 1928 paper titled "theory of parlor games". The Theory of Games and Economic Behavior was published in 1944 by John Von Neumann and Oskar Morgenstern. It established the economic and mathematical foundations for what is now known as "game theory." This book contains a lot of the basic vocabulary and problem-solving techniques that are still used today.

Von Neumann and Morgenstern pioneered the idea that economic and social problems can be modeled mathematically as appropriate strategy games. This is confirmed when Nyor *et al.* (2019)

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used game theory to model Student's Unrest on Nigerian Tertiary Institutions. Their research ~~was~~ examined four popular strategies to manage students' crises which provided scientific information to the authorities of Nigerian Higher Institutions in curtailing students' riots on campuses.

In 1950s, John Nash, Harold Kuhn, John Harsanyi and many others developed game theory intensely. John Nash improved the tools and ideas of game theory, according to Von Stengel (2008), introducing universal "cooperative bargaining and non-cooperative theories." In 1951, he established what is now known as the 'Nash equilibrium' of a tactical game by demonstrated ~~ing~~ that every finite game has an equilibrium point at all times, ~~at-on which~~ each competitor selects move that are vital knowing fully well the choices of other competitors. Game theory was furthered theoretically and used to evaluate war-like, political and philosophical in the 1950s and 1960s.

Dixit and Nalebuff (1993) ~~have defined~~ game theory as the branch of social science that studies strategic decision making. Another definition is by Hutton (1996) ~~who defines~~ game theory as an intellectual framework for examining what various parties to decision should do given their possession of inadequate information and different objectives. Shoham and Leyton-Brown (2008) states that: game theory is the mathematical study of interaction among independent, self-interested agents.

Bhuiyan (2016) studied game theory and highlighted some applications of game theory in business and economics, politics, philosophy etc. Author pointed out that in business world game theory is applied for determining different strategies. It offers valuable tools for solving strategy problems. Many business strategies are short or long-term plans to achieve sustainable profitability. A business can often successfully position in the market with right strategy and a business will suffer in the long run with wrong strategy.

Issah *et al.* (2020) discussed the solution methods and techniques for solving games. The Nash equilibrium method, maximin-minimax method, dominance method, arithmetic method, matrix method, graphical method and linear programming method where fully discussed.

William (2015) used Microsoft excel in solving a three person game for both total and partial conflict games. Author mentioned that: these solutions find any pure strategy solutions for the players playing alone and without communication, and then every combination of coalitions between players is found and solved.

According to Cigdem and Bulent (2015), game theory ~~is~~ is not possible to talk about 3-player games for zero-sum games. In three player games, there are not only two opponents, but a third party is also involved in the game. Therefore, even if the profit is distributed equally, the loss of one party will be higher than the other parties' gain.

Definition of Terms used in game theory

Strategy:

is the list of all possible action (move, decision alternative, courses of action) that are likely to adopted by a player for every outcome.

Optimal Strategy:

is a course of action that puts any player in the most preferred position irrespective of the course of action adopted by the other player

Player:

Each participant (interested party) is called a player.

Payoff: is the outcome of playing the game.

Game Theory:

is the study of mathematical models of strategic interaction among rational decision makers.

Three Person Game:

The game with only three players, say A, B and C is called a Three Person Game.

Simultaneous Games:

~~a~~ game in which each player has only one move and that all moves are made simultaneously.

Nash Equilibrium

÷ An outcome is called a pure Nash equilibrium provided no player can gain a higher payoff by deviating from the move, when all other players stick to their choices.

Pareto Dominance

÷ A Nash equilibrium is Pareto-dominated by another Nash equilibrium if every player's payoff in the first one is smaller or the same as in the second one.

Pareto Optimal

÷ A Nash equilibrium is Pareto-optimal if it is not Pareto-dominated by any other Nash equilibrium, except maybe by some having exactly the same payoffs.

Non Zero Sum Game

~~It~~ is a game in which the sum of the winnings and losses of the various players can be less than or more than zero.

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Best Response

✦ In a three player game, the strategy that yields the highest payoff for player A against particular strategies say 'S' and 'R' adopted by the other players is called best response to strategy 'S' and 'R' for player B and C respectively.

Methodology

Formulation of the Problem

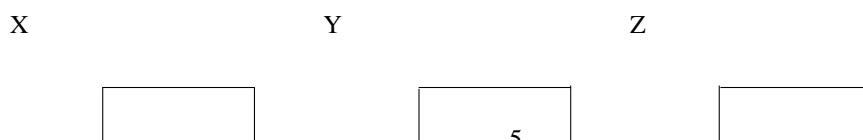
Three sachet water firms playing game with two strategies: promo and stay-put available to each firm. Such game can best be presented in two 2 by 2 matrices; where the first player chooses the matrix, the second player takes choosen the row and the third player goes opted by the column. The first values in each cell are payoffs for first player, middle values for second and last values for the third player respectively.

We considered random variable Let X, Y and Z be three firms (players) in a game with two available strategies Promo (P) and Stay-Put (S) for each player. And let 'p' and 's' represented the payoff for strategies of 'P' and 'S' respectively, then the payoff matrices presented in the following manner is as shown below.

Table 1: Payoff Description

Y / Z	Firm X Offers Promo (Adopts strategy P)		Firm X Stays Put (Adopts strategy S)	
	P	S	P	S
P	p; p; p	p; p; s	s; p; p	s; p; s
S	p; s; p	p; s; s	s; s; p	s; s; s

It can also be shown in a diagrammatic form representation as:



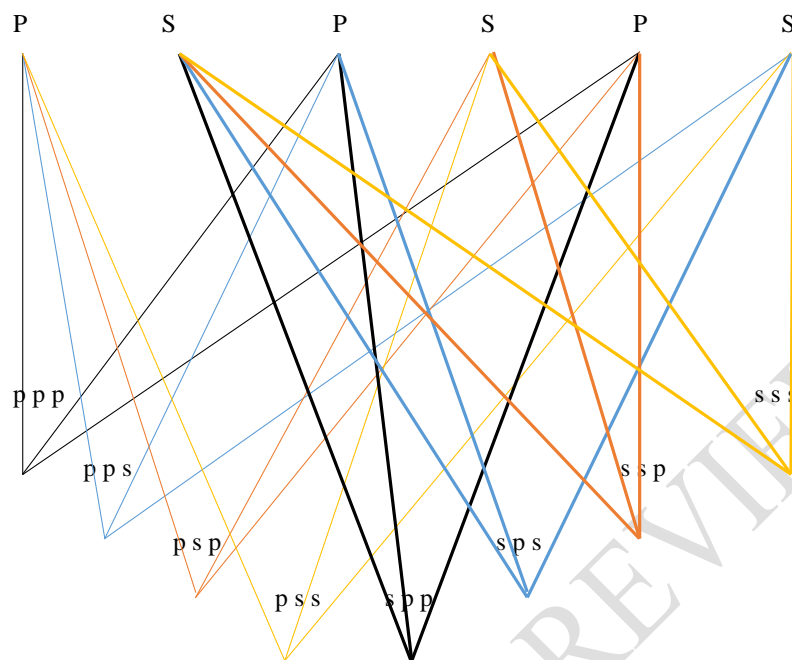


Figure 1:- Showing Strategies and Payoffs Available for Firms X, Y and Z

Algorithm for Best Response Method of Three Person Game

We Considered ~~ing~~ the payoff matrices in tables 1, we ~~look have seen at~~ the best response method to search for Nash equilibrium(s) ~~if any~~ for the sachet water firms. The step by step algorithms of the method ~~is was~~ showed ~~a~~ below.

- We Compared ~~d~~ cells having the same position in the different matrices, and underlined ~~d~~ the highest value player X's payoff (written first in each cell).
- Underlined ~~d~~ in each column the largest middle value in each cell and called it player Y's best response
- In each row, underlined ~~d~~ the biggest value of player Z's payoff (written last in each cell).
- If at all each value in a cell is underlined ~~d~~, then the outcome is called 'Nash equilibrium'.

Description of the Problem

Three sachet (pure) water companies in a particular location share a market and are currently competing for sales. Each company is now to decided ~~d~~ whether to offer a promo of one bag per 10

bags purchased by the retailers. It ~~is was~~ expected that, an average of 8000 bags of pure water are sold per month from each company. Whenever the three firms adapted ~~ed~~ to promo strategy, the market ~~will bewas~~ shared equally and the relative cost of the strategy is in vein. In a case where two firms adapted ~~ed~~ to promo, the third firm loses half of its market shared ~~to the~~ two firms offering promo. Also, in a situation where two firms ~~were~~ decided ~~d~~ not to offer promo, the third firm gains one third from each of their market shares. Research shows that, each firm ~~has signify~~ the capacity of producing 14000 bags per month. ~~We~~ Determined ~~d~~ the optimal strategy for the companies.

Data Collection and Procedure

The data used in this paper was collected in March 2021 through an interview with the manager Tarayya table water. Tarayya, Ideal and Asa table water companies co-exist in the same locality having the same target customers. The aim of the three firms ~~is to provided~~ healthy drinking water and employment opportunity to the people living in Gangara and the neighboring villages. A town located in Giwa L.G.A. of Kaduna State in Nigeria.

Raw Data

As revealed by the manager, the cost of production are as follows

~~One~~ kg of leather roll cost ~~was~~ ₦1650, a super pack of packing bag (which contains 10 packs and each pack contains 100 pieces of packing bags) costs ~~of~~ ₦6450, an average of 35 bags ~~are~~ ~~were~~ produced per kg, one gallon of gas which cost ~~of~~ ₦1200 is burned by the generator in two hours, two Dingli packing machines produce average of 100 bags per hour each and it takes 30mins to raise the temperature of the machines before packing starts, 1 gallon of petrol which cost ~~of~~ ₦1000 is used in cars for the distribution of 200 bags of sachet water, ₦400 ~~is~~ given as salary for the production of 100 bags of sachet water and ₦600 ~~spent~~ for selling the same number of bags. The cost of maintenance ~~is was~~ not unique hence it ~~is was~~ ~~pre~~assumed to cost the company ₦5 per bag.

Analysis of Data

The table below ~~contains shows~~ the list of materials that constituted ~~d~~ the cost of production of the sachet ~~of~~ water.

Table 2: Showing Analysis of Cost of Production

Production Materials	Initials	Cost per bag(naira)
Leather Roll	L	₦47.15
Packing bag	P	₦6.45
Generator fuel	G	₦8
Distribution Car fuel	C	₦5
Producers Salary	R	₦4
Distributors Salary	D	₦6
Maintenance	M	₦5

Computations for Payoffs

The analyzed data is used to compute the payoffs for each of the firms

- If all the firms adapted to promo, then each produce $(8000 + 800) = 8800$ bags since one bag is given as promo per 10 bags.

Income per Month = price per bag \times bags sold per month

$$100 \times 8000 = 800,000$$

Expenses per Month = $(L + P + G + C + R + D + M) \times$ bags produced per month

$$(47.15 + 6.45 + 5 + 8 + 4 + 6 + 5) \times 8800$$

$$81.6 \times 8800 = 718,080$$

Profit per Month = Income – Expenses

$$800,000 - 718,080 = 81,920$$

- If two firms offer promo, and one firm stay-put then, the two companies each have:

Income per Month = price per bag \times bags sold per month

$$100 \times 10000 = 1,000,000$$

Incurred Expenses per Month:-

Note: 11000 bags are produced (one bag as promo for each 10 bags of 10000 bags)

$$(L + P + G + C + R + D + M) \times \text{bags produced per month}$$

$$(47.15 + 6.45 + 5 + 8 + 4 + 6 + 5) \times 11,000$$

$$81.6 \times 11000 = 897,600$$

The payoff is: $1,000,000 - 897,600 = 102,400$

— Now, for the stay-put company

Income per Month = price per bag \times bags sold per month

$$100 \times 4000 = 400,000$$

Expenses per Month:

$(L + P + G + C + R + D + M) \times$ bags produced per month

$$(47.15 + 6.45 + 5 + 8 + 4 + 6 + 5) \times 4000$$

$$81.6 \times 4000 = 326,400$$

The payoff ~~is~~: $400,000 - 326,400 = 73,600$

c. If only one firm can adapted to the promo strategy, then the firm sales 12,000 bags hence

Income per Month = price per bag \times bags sold per month

$$100 \times 12,000 = 1,200,000$$

Incurred Expenses per Month:

Note: 13,200 bags are produced (since promo of one per ten bags is offered)

$(L + P + G + C + R + D + M) \times$ bags produced per month

$$(47.15 + 6.45 + 5 + 8 + 4 + 6 + 5) \times 13,200$$

$$81.6 \times 13,200 = 1,077,120$$

Profit per Month = Income – Expenses

$$1,200,000 - 1,077,120 = 122,880$$

For the other firms that stay-put

Income per Month = price per bag \times bags sold per month

$$100 \times 6000 = 600,000$$

Expenses per Month = $(L + P + G + C + R + D + M) \times$ bags produced per month

$$(47.15 + 6.45 + 5 + 8 + 4 + 6 + 5) \times 6000$$

$$81.6 \times 6000 = 489,600$$

Profit per Month = Income – Expenses

$$600,000 - 489,600 = 110,400$$

d. Lastly, in a case where all firms decide to stay-put, then

Income per Month = price per bag \times bags sold per month

$$100 \times 8000 = 800,000$$

Expenses per Month = (L + P + G + C + R + D + M) \times bags produced per month

$$(47.15 + 6.45 + 5 + 8 + 4 + 6 + 5) \times 8000$$

$$81.6 \times 8000 = 652,800$$

Profit per Month = Income – Expenses

$$800,000 - 652,800 = 147,200$$

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The Payoff Matrix

The payoffs was computed the above are matrices was used to formulate the two payoff matrices. Tarayya (T) firm choosen s the matrix, Ideal (I) choosen the row and Asa (A) choosen s the column. The payoffs are were approximated to the nearest thousands s

Table 3. Formulated Payoff Matrix

I / A	Firm T Offers Promo (Adopts strategy P)		Firm T Stays Put (Adopts strategy S)	
	P	S	P	S
P	82; 82; 82	102; 102; 73	73; 102; 102;	110; 122; 110
S	102; 73; 102	122; 110; 110	110; 110; 122	147; 147; 147

Result

We first find the best response moved s for the three firms. For firm 'T', we consider the first entries and compare compared the cells having the same position in different matrices, and underlinedd the highest valuess.

Table 4: Showing Best Response for Firm T

I / A	Firm T Offers Promo (Adopts strategy P)		Firm T Stays Put (Adopts strategy S)	
	P	S	P	S
P	<u>82</u> ; 82; 82	102; 102; 73	73; 102; 102; <u>110</u> ; 122; 110	
S	102; 73; 102	122; 110; 110	<u>110</u> ; 110; 122	<u>147</u> ; 147; 147

For firm 'I' we

underlined d the highest value second entry in each column in both matrices

Table 5: Showing Best Response for Firm I

I / A	Firm T Offers Promo (Adopts strategy P)		Firm T Stays Put (Adopts strategy S)	
	P	S	P	S
P	82; <u>82</u> ; 82	102; 102; 73	73; 102; 102; 110; 122; 110	
S	102; 73; 102	122; <u>110</u> ; 110	110; <u>110</u> ; 122	147; <u>147</u> ; 147

Lastly, for firm A, we underlined d the highest value third entry in each row in both matrices.

Table 6: Showing Best Response for Firm A

I / A	Firm T Offers Promo (Adopts strategy P)		Firm T Stays Put (Adopts strategy S)	
	P	S	P	S
P	82; 82; <u>82</u>	102; 102; 73	73; 102; 102; 110; 122; <u>110</u>	
S	102; 73; 102	122; 110; <u>110</u>	110; 110; 122	147; 147; <u>147</u>

Putting the best response moves together, we have

Table 7: Showing Nash Equilibrium

I / A	Firm T Offers Promo (Adopts strategy P)		Firm T Stays Put (Adopts strategy S)	
	P	S	P	S

P	82; 82; 82	102; 102; 73	73; 102; 102; 110; 122; 110
S	102; 73; 102	122; 110; 110	110; 110; 122 147; 147; 147

Discussion of Results

The result showed in table 7 contains showed two Nash equilibria (promo, promo) and (stay-put, stay-put) with the payoffs (82; 82; 82) and (147; 147; 147) respectively. Games with more than one pure Nash equilibria are sometimes called "coordination games". The strategy (promo, promo) is was shown Pareto dominated by (stay-put, stay-put), thus (stay put, stay put) is Pareto optimal. That is to say, both firms should should stay put, but that is was likely not to not happened except if negotiations can take place in advance. Firm 'T' shows fears that firms 'T' and 'A' might to be given promo, if it placed stays put lowering firm 'T's payoff to 73. Also, firm 'T' fears that firms 'T' and 'A' might give promo if it stays put and hence reducing the payoff of firm 'T' to 73. Similarly, firm 'A's payoff reduces to 73 when 'T' and 'T' give promo and it stays put. This is so because the game is symmetric in nature. Therefore, both firms should give promo if the rules of the game do not allow negotiations. Rewrite it is not clear..... ?

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Conclusion and Recommendation

In this paper, we discussed three person game in general; consisting of a tabular and diagrammatic representation of the payoffs, formulating the payoff matrix and step by step algorithm for best response (Nash equilibrium) method. The result shows that, each firm should give promo, except if negotiation is possible in advance, then it is advised that each firm should stay-put??? Mention meaning. Hence, in non-zero sum setting; negotiation, partnership and alliances may yield a win-win for the three firms. Therefore, we may now have concluded that the aim of this project was to determined the optimal strategy has been is achieved.

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Based on the observations and results obtained, author recommend that the three sachet water companies: Tarayya, Ideal and Asa should consider this research and apply the principles of game theory in determining that particular strategy that yields the greatest gain and negotiate if necessary. In fact, business firms should consult and seek the advice of specialist in Operations Research and Particularly Game Theory when situations of competitiveness arise. Rewrite it is not clear..... ?

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Authors have declared that no competing interests exist. The products used for this research are commonly and predominantly use products in our area of research and country. There is absolutely no conflict of interest between the authors and producers of the products because we do not intend to use these products as an avenue for any litigation but for the advancement of knowledge. Also, the research was not funded by the producing company rather it was funded by personal efforts of the authors.

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