

# Analysis of Water Based Carbon Nanotubes on an inclined Porous Sheet with Variable Magnetic Field

## Abstract

In this research, I investigated an unsteady carbon nanotubes in a 2D mixed convection flow made of water on a porous inclined expansible surface with angle in terms of the horizontal axis. When the magnetic field changes, the surface stretches linearly with velocity  $u$ . The Parametric Continuation Method is used to examine unknown physical quantities (PMC). The PDEs and boundary conditions are integrated using similarity transformations to generate a set of coupled ODEs. Physical quantities such as flow behavior, thermal characteristics, thermal variation, skin friction and Nusselt Number are examined using graphical and tabulated results for physical purposes.

**Keywords:** Nanotube, Porous Plate, Magnetic field, Thermal Variation, Nusselt Number, Skin Friction, Similarity Transformations

## 1 Introduction

So far, in the development of water based carbon nanotube, an enormous research work has been made. The notion of coupling a simple fluid with nanoparticles was first proposed by Stephen et al. [1]. They had been given a name to this fluid as "nanofluid." In comparison between a simple fluid and a nanofluid, nanofluid is having several thermo physical effects. Some of the effects include heat conductivity, viscosity, and heat diffusivity. The focus of the study was on internal heat generation in a permeable circular enclosure and the impact of viscous dissipation. Both external and internal circular walls are stored at steady temperature  $T_0$ , although the other walls have been done by Dongonchi and Ganji [2]. On a vertical stretched sheet, they investigated heat transfer and flow of a nanofluid. They analyzed nusselt number, temperature and velocity field. Freidoonimehr et al. [3] had taken heat transfer and 2-D (two dimensional) stagnation point nanofluid into consideration. They analyzed Temperature distribution and velocity. Afridi et al. [4] had studied viscous dissipation, heat transfer with the impact of magnetic field over an inclined non-permeable stretching surface and 2-D (two dimensional) boundary layer flow. Researchers looked at their investigation's numerical answer as well as the surface temperature. Bilal et al. [5] investigate the radiative Maxwell fluid in three dimensions on an inclined stretched surface under the convective boundary condition. They analyzed mass transfer and Heat under thermophoresis effects. They used similarity transformations to convert PDE's (Partial Differential Equations) into ODE's (Ordinary Differential Equations). During their investigation, velocity, temperature and concentration have been analyzed. On an angled stretched sheet, he examined Oldroyd-B non-Newtonian fluid flow. They analyzed 3-D (three dimensional) mixed convection Maxwell fluid under thermophoresis effects on a stretched surface that is inclined. Dhani et al. [6] came upon various solutions for a definite range of physical parameters like velocity slip, solutal slip and thermal slip. They used a mixed convection flow to investigate Nano fluids on an inclined stretched sheet. [5]. Hayat et al. [7] have worked on burger's flow. In the research, the burger's flow was investigated using two variables: heat transfer and mass transfer. This was carried out over an inclined stretching sheet. Their study also included several graph plotting of  $\theta$ -curves against other variable such as temperature and velocity. Many researcher have been focusing on the heat transfer problems that rise especially in the field of energy and several engineering solution were put forward. In the field of heat and fluid flow, Bejan et al. [8] were one of the first scientists to have successfully calculated the value of entropy generation. Qing et al. [9] examined Casson Nano fluid entropy production (Non-Newtonian) across a stretching sheet. The impact of chemical reactions and nonlinear thermal radiation were also considered. They analyzed momentum equation, nanoparticles concentration and energy equation. Aly [10] evaluated 4 water based nanofluids, including  $TiO_2$ ,  $Ag$ ,  $Al_2O_3$  and  $Cu$  nanoparticles, in a study. To report the existence of several solutions, an exact solution was used. Mahanthesh et al. [11] investigated the flow and heat transmission of a three-dimensional MHD nanofluid over a nonlinear bidirectional stretching surface. Water-based nanofluids are having  $Cu$ ,  $Al_2O_3$ , and  $TiO_2$  were used as the working fluid once again. Ashraf et al. [12] examined the non-Newtonian Oldroyd-B fluid flow over an inclined stretched surface. The impact of thermal radiation on flow and heat transfer features was the focus of the discussion. Analytically, the problem was addressed by using the method of homotopy analysis. Nanofluid flow on inclined stretched surfaces has garnered a lot of interest, in addition to mixed convection flow of simple fluids. The flow of a non-Newtonian Casson nanofluid across an angled exponential stretching surface was studied by Bala and Reddy [13]. Solutal slip, thermal slip and Velocity slip were all considered in the numerical study. In another investigation, Dalir et al. [14] looked at Jeffrey flow of nanofluid over a stretched sheet entropy generation due the effect of Brownian motion, thermophoresis and magnetic field. Sajjad et al. [15] analysed the entropy generation of a viscoelastic MHD nanofluid flow across a stretching sheet. On a linear stretched

sheet, for boundary layer flow, Crane [16] discovered a closed form similarity solution. Following Crane's study [16], a number of researchers explored the issue, focusing on various elements. Haung et al. [17] investigated heat transmission in a fluid with a high viscosity boundary layer flow induced by an inclined stretched sheet. Qasim et al. [18] investigated the effects of heat radiation on viscoelastic mixed convection flow along an inclined sheet. In order to account for Soret-Dufour occurrences, Sravanthi [19] studied MHD slip flow on an exponentially stretched inclined sheet. Eldahad and Aziz [20] presented mixed convection, heat generation/absorption, and blowing/suction influences on MHD boundary layer flow on an inclined stretched sheet. Kandasamy et al. [21] studied the influence of viscosity and thermophoresis on MHD mixed convective heat and mass transfer across a porous wedge. Rashad et al. [22] studied the creation of internal heat and the effect of a magnetic field on free convection fluid flow in a rectangular container loaded with nanoparticles. The results show that as the Hartmann number or solid volume percentage increases, the average Nusselt number decreases, whereas the inverse is true for magnetic field vector augmentation. They also discovered that raising the Hartmann number reduces the maximum value of the stream function while raising the maximum temperature. Using Brownian motion and a magnetic field, Hsiao [23] studied incompressible micropolar nanofluid flow over a moving plane. According to him, the temperature reduces as the Prandtl number rises. Rashidi et al. [24] investigated Nanofluid flow and the associated heat transfer in an expanding plane. The magnetic parameter is proportional to the temperature distribution along the plane, they showed. Feroz et al. [25] investigated the influence of two types of nanoparticles, multi-walled and single-walled carbon nanotubes used with a water base, on the entropy production for an MHD mixed-conventional-flow (on a stretching surface) with water as the base fluid in recent years. Entropy production, nusselt number variation, skin friction temperature distribution, and velocity were all studied.

## 2 Mathematical Modeling

On a permeable inclined stretched surface with an angle to the horizontal axis, the flow of water-based carbon nanotubes was examined in an unstable 2D mixed convection flow. As the magnetic field changes, the surface extends linearly with velocity  $u$ . Flow is assumed to be Incompressible, Laminar, Velocity field  $U = U(u(x, y, t), v(y, t))$ , Temperature field  $T = T(y, t)$  and Magnetic field  $B = B(b1(x, y, t), b2(y, t))$ .

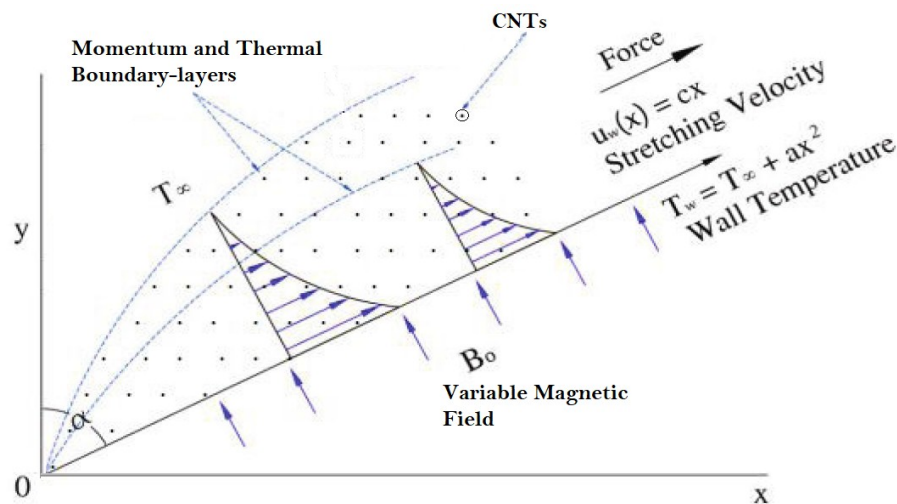


Figure 1

Magnetic field variability

### 2.1 Governing equations and boundary conditions

The governing equations in a rotating frame of reference are:

**Continuity Equation**

$$\nabla \cdot V = 0, \quad (1)$$

**Navier Stoke's Momentum equation**

$$\frac{\partial V}{\partial t} + (V \cdot \nabla)V = \frac{\mu_{nf}}{\rho_{nf}} \nabla \cdot (\nabla V) + g \frac{(\rho\beta)_{nf}}{\rho_{nf}} (T - T_{\infty}) \cos \alpha (V \times B) \times V \quad (2)$$

**Magnetic Equation**

$$\frac{\partial B}{\partial t} = \nabla \times V \times B + \frac{1}{\sigma \mu_e} \nabla^2 B \quad (3)$$

**Energy equation**

$$\frac{\partial T}{\partial t} + (V \cdot \nabla)T = K_{nf} \nabla \cdot (\nabla T) \quad (4)$$

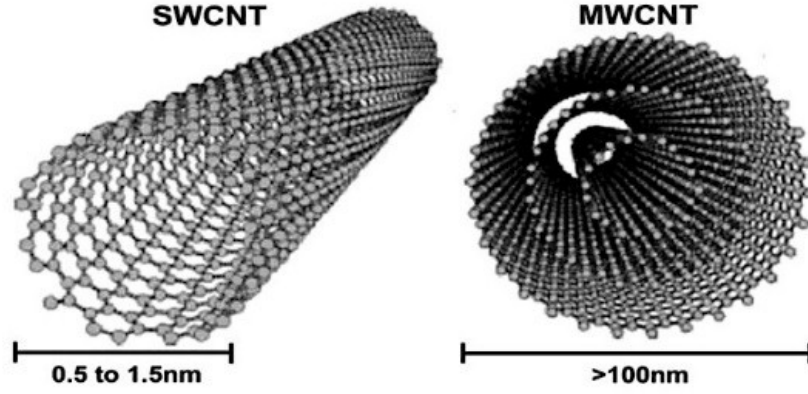


Figure 2 kinematic viscosity

Where  $\rho_{nf}$  represents density of nanofluid,  $V$  represents velocity,  $\nabla$  represents gradient,  $\mu_{nf}$  represents kinematic viscosity of nanofluid,  $B$  represents magnetic field,  $K_{nf}$  denotes thermal conductivity of nanofluid,  $T$  denotes the temperature, while  $g$  denotes the gravitational constant.

The Boundary Conditions are as follows:  $U = u_w(x)$ ,  $v = v_w(y)$ ,  $T = T_w(x)$  at  $y = 0$ ,  
 $u \rightarrow 0$ ,  $T \rightarrow 0$ , as  $y \rightarrow \infty$ ,  
 $U = u_w(x)$  represents stretching velocity in x-direction  
 $v = v_w(y)$  represents stretching velocity in y-direction  
 $y \rightarrow \infty$

Inserting the following transformations in the above mentioned equations,

$$u = \frac{ax}{2(1-\alpha t)} f', \quad v = \frac{-al}{(1-\alpha t)^{0.5}}, \quad \theta = \frac{T - T_h}{T_w - T_h}, \quad \eta = \frac{y}{l\sqrt{(1-\alpha t)}} \quad (5)$$

After using these transformations, the above equations become,

$$f^{iv} = S(1-\phi)^{2.5} \left[ 1 - \phi + \phi \frac{\rho_s}{\rho_f} \right] [\eta f'''' + 2f'' - 2ff'''] + 4\lambda(\rho B)_f \left[ \frac{(1-\phi+\phi(\rho B)_s)}{(\rho B)_f} \right] \theta \cos \alpha + \frac{M\delta M_0^2 m K'}{1 - \phi + \phi \frac{\rho_s}{\rho_f}} \quad (6)$$

$$\theta'' = \left[ \frac{1 - \phi + 2\phi \frac{K_s}{K_s+K_f} \ln \frac{K_s+K_f}{2K_s}}{1 - \phi - \phi - 2\phi \frac{K_s}{K_s+K_f} \ln \frac{K_s+K_f}{2K_s}} \right] \delta Pr \left[ 1 - \phi + \phi \frac{(\rho C_p)_s}{(\rho C_p)_f} \right] \left[ \eta \phi' - f \phi' - \frac{Ec}{S(1-\phi)^{2.5}} \cdot \frac{1}{1 - \phi + \phi \frac{(\rho C_p)_s}{(\rho C_p)_f}} f'^2 \right] \quad (7)$$

$$K'' = A\sigma\mu_e l^2 \left[ K + \eta K' + \frac{mf''}{2l} - aK'f' \right] \quad (8)$$

$$m'' = \frac{\sigma u_e l a}{2} [m + K\eta] \quad (9)$$

The boundary conditions are as follows:

$$f(0) = 1, \quad \theta(0) = 1, \quad K(0) = 1, \quad m(0) = 1.$$

$$f'(\infty) = 0, \quad \theta(\infty) = 1, \quad K(\infty) = 0, \quad m(\infty) = 0.$$

### 3 Result and Discussion

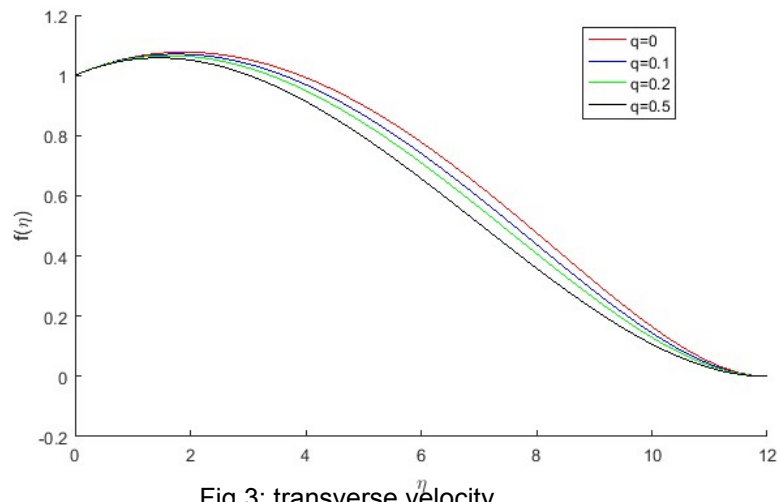
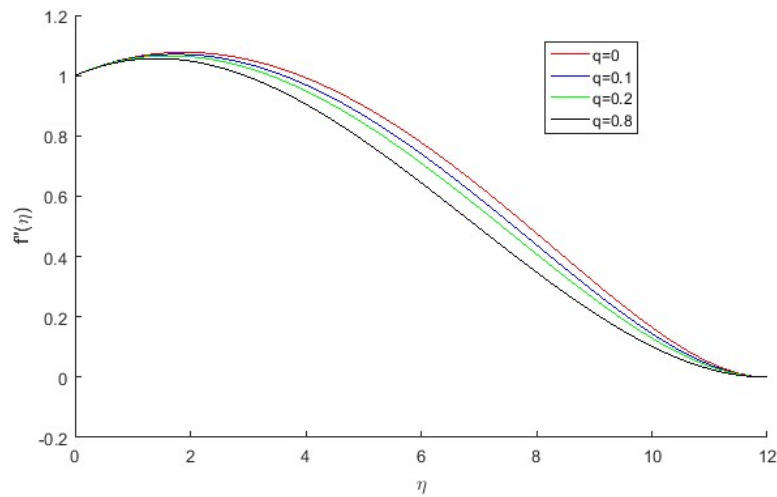


Fig 3: transverse velocity

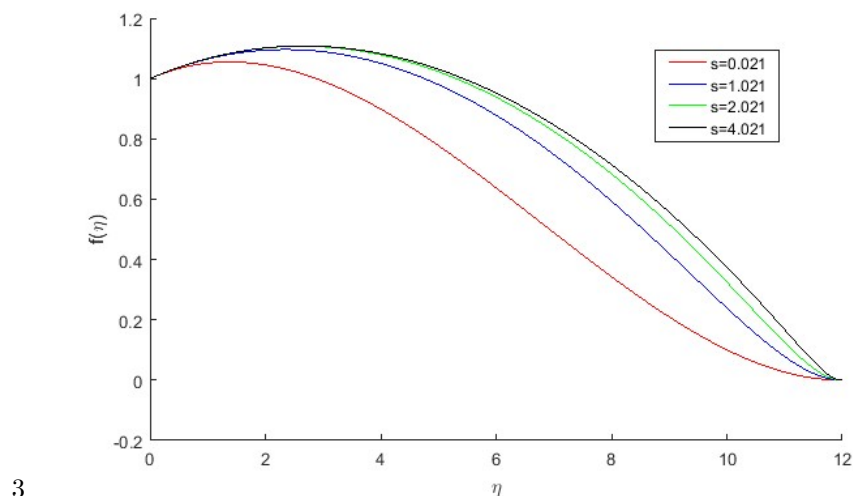
In this figure,  $f(\eta)$  versus  $\eta$  is plotted for the values of Nano-particles parameter  $q = 0, 0.1, 0.2, 0.5$  with other parameters being fixed seen in the graph. The transverse velocity decreases when the value of  $q$  is increased, as depicted in this graph. As  $q$  is increasing, the density of fluid increases that reduces the vertical velocity component.

Fig 4: horizontal velocity



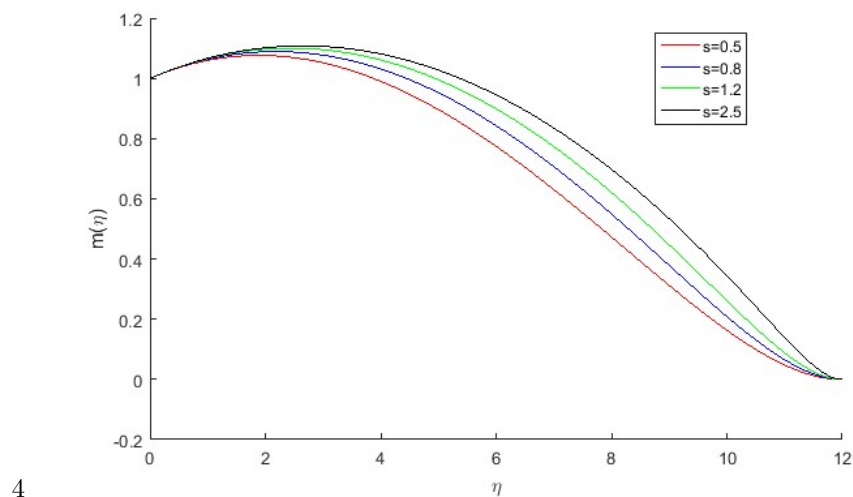
The horizontal velocity  $f'(\eta)$  vs.  $\eta$  for various values of  $q$  is plotted in this figure, with the other parameters held constant. From the figure, it is noticed that as the values of  $q$  is increasing, the horizontal velocity  $u$  decreases due to increase in values of  $q$ . The increase of nanoparticles into the region decreases the vertical velocity. It is due to opposing the other forces, the vertical velocity is decreasing.

Fig 5: transverse velocity  $v$  verses for identical values of suction parameter



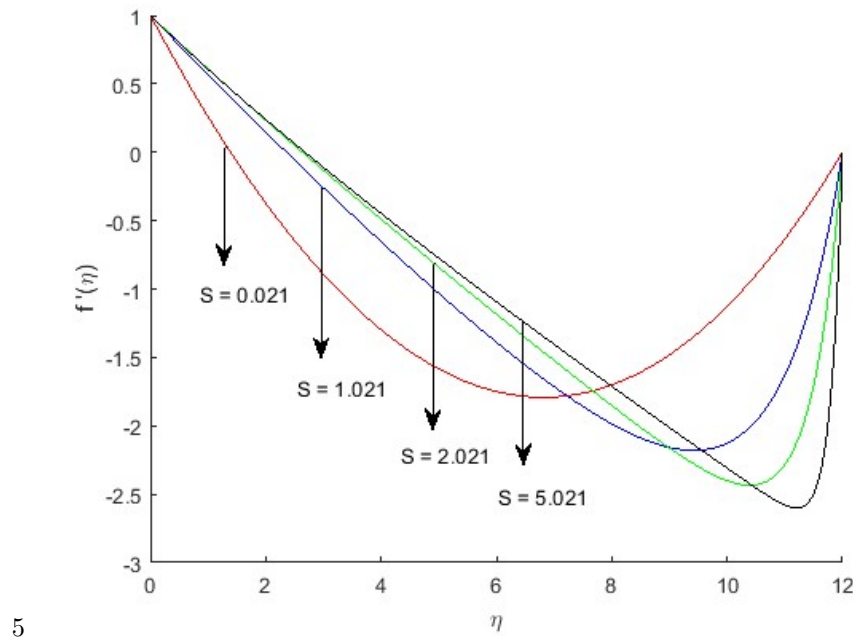
In this figure, the profile of transverse velocity  $v$  verses for identical values of suction parameter  $s$ . In this graph, as the suction parameter is increased, it is observed that, the transverse velocity rises due to the liquid that entered into the region in the same direct direction.

Fig 6: suction parameter with variable magnetic field



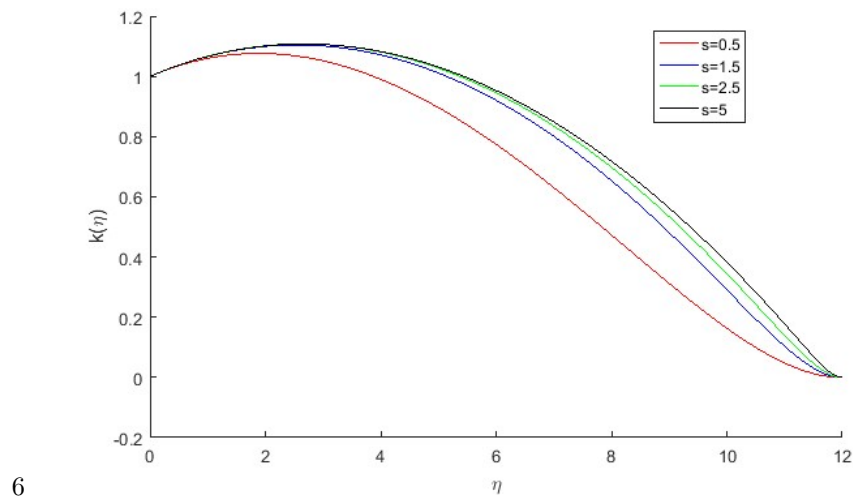
In this figure, variable magnetic field  $m(\eta)$  along x-axis verses  $\eta$  is plotted for identical values of suction parameter  $s = 0.5, 0.8, 1.2, 2.5$  by other parameters as fixed. This graph reveals that by increasing the values of suction parameter  $s$ , variable magnetic field  $m(\eta)$  is also increasing. The reason behind this increase is the collision of nanoparticles and base fluid in the region, when fluid is increased in the region, the collision of nanoparticles and fluid enhance variable magnetic field in the region.

Fig 7:horizontal velocity



In this figure, the profile of horizontal velocity  $u$  versus  $\eta$  for identical values of section parameter  $s$ . It can be seen in this graph that as the section parameter  $s$  is increased, the horizontal velocity rises due to because the increasing section parameter  $s$ . The fluid velocity entered into the region increased that cause to increase the horizontal velocity.

Fig 8:variable magnetic field is increasing as the values



In this figure, shows the graph of variable magnetic field along y-axis for identical values of suction parameter  $s$ . Observation seems that the variable magnetic field is increasing as the values of suction parameter  $s$  is increasing. When more and more fluid is entered into the region, the collision of fluid and nanoparticles enhance the variable magnetic field inside the region.

## 4 Concluding remarks

From this study, the following results were drawn:

- Nusselt number, Skin friction, Velocity field, temperature change and magnetic field were all investigated.
- The transverse velocity decreases as the value of  $q$  increases.
- It is shown that increase in Prandtl number, temperature profile decreases and opposite trend is observed in Concentration profile.
- The horizontal velocity decreases when the values of  $q$  are increases.
- By increasing the suction parameter  $s$  values, variable magnetic field  $m(\eta)$  is also increasing.
- By increasing the suction parameter  $s$  values, variable magnetic field  $k(\eta)$  is increasing.

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