Black Holes, Gravitational Waves and Quantum Gravity

Abstract

Loop Quantum Gravity is a theory that attempts to describe the quantum mechanics of the gravitational field based on the canonical quantization of General Relativity. According to Loop Quantum Gravity, in a gravitational field, geometric quantities such as area and volume are quantized in terms of the Planck length. In this paper we present the basic ideas for a future, mathematically more rigorous, attempt to combine black holes and gravitational waves using the quantization of geometric quantities introduced by Loop Quantum Gravity.

1 Introduction

A black hole is a region of the universe where the causal structure of the spacetime is so deformed by gravity that even light can not scape from this region. The existence of black holes in the universe is one of the implications of Einstein´s field equations for General Relativity (GR). Today, indirect as well as direct evidence has accumulated confirming the existence of black holes. The simplest black hole occurs in the Schwarzschild spacetime. The Schwarzschild spacetime is a exact, spherically symmetric, solution of the Einstein equations describing the gravitational field in the exterior region of a mass M with no electric charge and no angular momentum. The geometric structure of the Schwarzschild spacetime is characterized by the existence of a particular value R_S for the radial coordinate below which it becomes impossible to scape form the gravitational attraction of the black hole. The spherical surface defined by R_S is called the event horizon of the black hole.

In 1975 Stephen Hawking [1], using the methods of quantum field theory in curved spacetime, showed that black holes are not really black because, as a consequence of quantum vacuum fluctuations near the event horizon, a black hole can emit quantum particles in the thermal spectrum. As a consequence of this thermal radiation a black hole has a temperature and an associated entropy and can ultimately evaporate. A fundamental open conceptual problem associated with the Hawking thermal radiation is the so-called Information Loss Paradox. Energy is carried away by the Hawking radiation, so that the black hole eventually evaporates away entirely, leaving a future with the causal structure of Minkowski space. Information that falls past the event horizon, for instance the black hole mass, appears to be lost. For a review see refs. [2,3,4,5,6]. In the literature a black hole with Hawking radiation is termed a *semi-classical black hole*, to distinguish it from the classical black hole described by GR. The Hawking radiation is a tiny effect for most black holes. For example, a black hole with 15 times the mass of our Sun has a temperature of $4,1 \times 10^{-9} K$ and the time for the black hole to evaporate all of its mass by means of Hawking radiation is given by $2,2 \times 10^{78} s$ which is about 60 orders of magnitude larger than the age of the Universe [7].

In quantum field theory in curved spacetime, one treats gravitation classically, as in the framework of GR. Thus, spacetime structure is described by a manifold M, on which is defined a classical, Lorentz signature metric $g_{\mu\nu}$. One thereby avoids confronting the fundamental difficulty of how to formulate quantum field theory without a classical background metrical (and causal) structure of spacetime. One expects that quantum field theory in curved spacetime should have only a limited range of validity [4]. In particular, it certainly should break down, and be replaced by a quantum theory of gravitation coupled to matter, when the spacetime curvature approaches Planck scales [4]. In this paper we suggest the possible existence of another mechanism for the emission of quantum radiation by a black hole. Here we are interested in a mechanism of emission of gravitational radiation by a *quantum black hole*. The basic concept that supports the ideas presented in this paper is that of the Planck scale. Let us briefly review the Planck scale.

In 1899 Max Planck [8] noticed that by combining three of the fundamental constants of physics, the Newtonian gravitational constant G, the speed of light c and Planck´s constant ħ in a unique way, he could define a fundamental scale of length, time and mass. Today this fundamental scale is called the Planck scale. It is given by

1) the Planck length

$$L_P = \sqrt{\frac{\hbar G}{c^3}} = 1,62 \times 10^{-35} m \tag{1}$$

2) the Planck time

$$T_P = \sqrt{\frac{\hbar G}{c^5}} = 5,40 \times 10^{-44} s \tag{2}$$

3) the Planck mass

$$M_P = \sqrt{\frac{\hbar c}{G}} = 2,17 \times 10^{-5} g \tag{3}$$

In this paper we will adopt a particular interpretation of the Planck scale. This particular interpretation can be justified as follows. First, observe that the Planck length is the distance that light travels during the Planck time. Therefore the existence of the Planck length L_P, together with the constancy of the speed of light c, automatically defines the Planck time $T_P = L_P/c$. Observe also that the Planck mass M_P can be written in terms of L_P as

$$M_P = \frac{c^2}{G} L_P \tag{4}$$

The above observations can be interpreted as indications that the Planck length L_P is the minimum length in our universe and that the Planck time T_P and the Planck mass M_P can be obtained from this minimum length using the fundamental constants c and G. It is this particular interpretation of the Planck scale that we will adopt in this paper.

The existence of gravitational waves was also one of the predictions of Einstein field equations for GR. Gravitational waves were finally detected in 2015. Contrary to black holes, which are associated with strong gravitational fields, gravitational waves are usually associated with weak gravitational fields. The most direct way to describe the propagation of gravitational waves is to expand the curved spacetime metric $g_{\mu\nu}$ around the flat spacetime metric $\eta_{\mu\nu}$ and retain only the linear order terms in the expansion. This procedure leads to the linearized Einstein equations. The difficulty of this approach is to separate the physical from the unphysical degrees of freedom described by the gravitational wave. In this paper we describe in detail the procedure for removing the unphysical degrees of freedom contained in the metric that describes a gravitational wave. This is a necessary step to give further support to the quantization of the energy of a gravitational wave described in [9]. It will also support the construction of a quantum gravitational wave described in this paper. Here we will explain how to relate black holes to gravitational waves using the quantization of geometric quantities discovered in the quantum theory of gravity called Loop Quantum Gravity (LQG).

LQG [10,11,12] is a theory that attempts to describe the quantum mechanics of the gravitational field based on the canonical quantization of GR. The construction of LQG only became possible after 1986, when Ashtekar [13] introduced a new set of canonical variables for describing GR. Instead of the traditional metric tensor field $g_{\mu\nu}$ of GR as the configuration variable, Ashtekar introduced an $SU(2)$ connection A^i_μ as the configuration variable. This allowed the description of GR as a constrained Hamiltonian system with first-class constraints [14] only. The most striking result of LQG is that, in a gravitational field, geometric quantities such as area and volume are quantized in terms of the Planck length L_P given in equation (1).

The paper is organized as follows. In section two we review the Shwarzschild solution to the field equations of GR and the basic equations of black hole Thermodynamics. In section three we review in detail the description of the propagation of gravitational waves in the transverse-traceless gauge. In section four we present our contribution to these subjects. We explain how we can construct a quantum gravitational wave and a quantum equation for the Schwarzschild black hole entropy. Finally we explain how a quantum black hole can convert all of its mass into quantum gravitational radiation. We present our conclusions in section five.

2 The Schwarzschild black hole

Einstein equations can be derived from the Einstein-Hilbert action

$$S = \frac{c^3}{16\pi G}\int d^4x\sqrt{[-\det g_{\mu\nu}]}R + S_M \tag{5}$$

where S_M is the matter action. The energy-momentum tensor of matter, $T^{\mu\nu}$, is defined from the variation of the matter action under a change of the spacetime metric $g_{\mu\nu} \to g_{\mu\nu} + \delta g_{\mu\nu}$ according to

$$\delta S_M = \frac{1}{2c}\int d^4x\sqrt{[-\det g_{\mu\nu}]}T^{\mu\nu}\delta g_{\mu\nu} \tag{6}$$

We must now define the relevant geometric quantities obtained from the spacetime metric $g_{\mu\nu}$. The first of these are the Christoffel symbols

$$\Gamma^{\rho}_{\mu\nu} = \frac{1}{2}g^{\rho\sigma}\left(\partial_\mu g_{\sigma\nu} + \partial_\nu g_{\sigma\mu} - \partial_\sigma g_{\mu\nu}\right) \tag{7}$$

In this equation $g^{\rho\sigma}$ is the inverse metric of $g_{\rho\sigma}$. From the Christoffel symbols we define the Riemann curvature tensor

$$R^{\mu}_{\nu\rho\sigma} = \partial_\rho\Gamma^{\mu}_{\nu\sigma} - \partial_\sigma\Gamma^{\mu}_{\nu\rho} + \Gamma^{\mu}_{\alpha\rho}\Gamma^{\alpha}_{\nu\sigma} - \Gamma^{\mu}_{\alpha\sigma}\Gamma^{\alpha}_{\nu\rho} \tag{8}$$

Contracting the Riemann tensor we obtain the Ricci tensor

$$R_{\mu\nu} = R^{\alpha}_{\mu\alpha\nu} \tag{9}$$

Contracting again we obtain the Ricci scalar or scalar curvature

$$R = g^{\mu\nu}R_{\mu\nu} \tag{10}$$

Varying the Einstein-Hilbert action (5) with respect to $g_{\mu\nu}$ we obtain the field equations of GR

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \frac{8\pi G}{c^4}T_{\mu\nu} \tag{11}$$

$T_{\mu\nu}$ describes the flow of energy and momentum through a given point in spacetime.

As mentioned in the introduction, the Schwarzschild spacetime describes the gravitational field in the outside region of a mass M with no electric charge and no angular momentum. In this case the Einstein equations become the vacuum equations

$$R_{\mu\nu} = 0 \tag{12}$$

with the Schwarzschild metric

$$ds^2 = -\left(1 - \frac{2GM}{c^2 r}\right) dt^2 + \left(1 - \frac{2GM}{c^2 r}\right)^{-1} dr^2 + r^2(d\theta^2 + sen^2\theta d\varphi^2) \quad (13)$$

as the only exact solution. Notice that the first term on the right hand side vanishes and the second becomes divergent when $r = R_S = 2GM/c^2$. This value for r is called the Schwarzschild radius and the spherical surface associated with it is called the event horizon for the Schwarzschild black hole. The event horizon acts as a one-way membrane: matter and energy can go in but, once inside, can never go out. For details see [7,15].

The mass M of the central gravitating body is related to the horizon area A by

$$M = \sqrt{\frac{c^3 A}{16\pi G^2}} \quad (14)$$

Advanced methods in field theory that consider quantum vacuum fluctuations that occur in the vicinity of the event horizon find the result that the Schwarzschild black hole can emit quantum particles in the thermal spectrum. This thermal emission is called the Hawking radiation. The distribution of energies emitted by the black hole as Hawking radiation is equivalent to that of a blackbody with a temperature proportional to the surface gravity of the black hole. Specifically, the black hole temperature is given by

$$T = \frac{\hbar c^3}{8\pi k_B GM} \quad (15)$$

where k_B is the Boltzman constant. The temperature (15) is proportional to $\hbar$ so it is a quantum effect that vanishes in the $\hbar \to 0$ classical limit. As a consequence of the temperature (15) the black hole has an entropy given by

$$S = \frac{c^3 k_B}{4\hbar G} A = \frac{k_B A}{4L_P^2} \quad (16)$$

We now briefly describe how equation (16) for the black hole entropy emerges in the framework of quantum gravity. In LQG the area of a surface in a gravitational field is quantized. The area A of the event horizon of a black hole is given by [10]

$$\begin{aligned} A &= 8\pi\gamma \frac{\hbar G}{c^3} \Sigma_i \sqrt{j_i(j_i+1)} \\ &= 8\pi\gamma L_P^2 \Sigma_i \sqrt{j_i(j_i+1)} \end{aligned} \quad (17)$$

where γ is the Immirzi parameter [16], used to fix the exact scale of the quantum theory, and $j_i = j_1, ..., j_n$ are the spins of the links intersecting the event horizon surface. The black hole entropy is given by

$$S = k_B \ln N(A) \quad (18)$$

where $N(A)$ is the number of states that the geometry of a surface with area A can have. The possible states are obtained by considering all sets of j_i that give the area A and, for each set, the dimension of $\otimes_i H_i$ where H_i is the representation space of the spin j_i.

In LQG it was first assumed [17] that the number of possible states is dominated by the case $j = 1/2$. In this case the quantum of area is given by

$$A_{\frac{1}{2}} = 4\pi\sqrt{3}\gamma L_P^2 \tag{19}$$

Hence there are

$$n = \frac{A}{A_{\frac{1}{2}}} = \frac{A}{4\pi\sqrt{3}\gamma L_P^2} \tag{20}$$

intersections and the dimension of $H_{1/2} = 2\frac{1}{2} + 1 = 2$. So the number of quantum states of the event horizon area A is

$$N(A) = 2^n = 2^{A/4\pi\sqrt{3}\gamma L_P^2} \tag{21}$$

and the black hole entropy is

$$S = \frac{\ln 2}{4\pi\sqrt{3}\gamma}\frac{k_B A}{L_P^2} \tag{22}$$

which agrees with equation (16) if we choose

$$\gamma = \frac{\ln 2}{\pi\sqrt{3}} \tag{23}$$

Later, it was realized [18,19] that the number of possible states could instead be dominated by the case $j = 1$ and a similar calculation was performed for the spin $j = 1$. Now the black hole entropy given by equation (16) is again reproduced provided we choose the Immirzi parameter γ to be

$$\gamma = \frac{\ln 3}{2\pi\sqrt{2}} \tag{24}$$

which in turn fixes the minimal quantum of area to be

$$A_1 = 4(\ln 3)L_P^2 \tag{25}$$

The same calculation can be performed with higher spins $j_i = \frac{3}{2}, 2, ...$ and equation (16) for the black hole entropy will always be obtained provided we choose the appropriate value for the Immirzi parameter γ.

The situation in LQG described above leaves no doubts about the validity of equation (16) in describing the entropy of the Schwarzschild black hole. But the true microstate responsible for the entropy has not been determined yet. It can be the quantum of area $A_{\frac{1}{2}}$, or the quantum of area A_1, or any quantum of area we choose, provided we select the appropriate value of the Immirzi parameter γ that reproduces the black hole entropy equation (16). As we will see below, equation (16) for the black hole entropy may not be the end of the story. This is because there is one important information about the Schwarzschild spacetime which was not used in LQG to arrive at the entropy (16). This important information is the spherical symmetry of the event horizon.

3 Gravitational waves

General Relativity is invariant under a huge symmetry group, the group of all possible coordinate transformations

$$x^\mu \to \acute{x}^\mu(x) \tag{26}$$

where $\acute{x}^\mu$ is an arbitrary smooth function of x^μ. Under the transformation (26) the spacetime metric transforms as

$$g_{\mu\nu}(x) \to \acute{g}_{\mu\nu}(\acute{x}) = \frac{\partial x^\rho}{\partial \acute{x}^\mu}\frac{\partial x^\sigma}{\partial \acute{x}^\nu} g_{\rho\sigma}(x) \tag{27}$$

This symmetry is the gauge symmetry of GR.

As a first step toward the description of gravitational waves we must expand Einstein equations around the flat spacetime metric

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \qquad \mid h_{\mu\nu} \mid \ \ll 1 \tag{28}$$

and retain only terms to linear order in $h_{\mu\nu}$. The resulting theory is called the linearized theory [20].

After choosing a frame where equation (28) holds, a residual gauge symmetry remains. Consider the transformation of coordinates

$$x^\mu \to \acute{x}^\mu = x^\mu + \xi^\mu(x) \tag{29}$$

where the derivatives $\mid \partial_\mu \xi_\nu \mid$ are of the same order of smallness as $\mid h_{\mu\nu} \mid$. Using the transformation law of the metric, equation (27), we find that the transformation of $h_{\mu\nu}$, to lowest order, is

$$h_{\mu\nu}(x) \to \acute{h}_{\mu\nu}(\acute{x}) = h_{\mu\nu}(x) - \left(\partial_\mu \xi_\nu + \partial_\nu \xi_\mu\right) \tag{30}$$

If $\mid \partial_\mu \xi_\nu \mid$ are of the same order of smallness as $\mid h_{\mu\nu} \mid$, the condition $\mid h_{\mu\nu} \mid \ll 1$ is preserved. Now we are ready to construct the linearized version of Einstein equations.

To linear order in $h_{\mu\nu}$ the Christoffel symbols are given by

$$\Gamma^\rho_{\mu\nu} = \frac{1}{2}\eta^{\rho\lambda}\left(\partial_\mu h_{\nu\lambda} + \partial_\nu h_{\lambda\mu} - \partial_\lambda h_{\mu\nu}\right) \tag{31}$$

Lowering an index for convenience the Riemann tensor becomes

$$R_{\mu\nu\rho\sigma} = \frac{1}{2}\left(\partial_\rho \partial_\nu h_{\mu\sigma} + \partial_\sigma \partial_\mu h_{\nu\rho} - \partial_\sigma \partial_\nu h_{\mu\rho} - \partial_\rho \partial_\mu h_{\nu\sigma}\right) \tag{32}$$

The Ricci tensor is given by

$$R_{\mu\nu} = \frac{1}{2}\left(\partial_\sigma \partial_\nu h^\sigma_\mu + \partial_\sigma \partial_\mu h^\sigma_\nu - \partial_\mu \partial_\nu h - \Box h_{\mu\nu}\right) \tag{33}$$

where we have defined the trace of the perturbation as $h = \eta^{\mu\nu} h_{\mu\nu} = h^{\mu}_{\mu}$ and $\Box = -\frac{1}{c^2}\partial_t^2 + \partial_x^2 + \partial_y^2 + \partial_z^2$. And finally the Ricci scalar is

$$R = \partial_\mu \partial_\nu h^{\mu\nu} - \Box h \tag{34}$$

The linearized equations of motion are written more compactly defining $\bar{h}_{\mu\nu} = h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu} h$. It is a straightforward algebra to compute the linearized Einstein tensor $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R$ and we find that the linearization of the Einstein equations (11) gives

$$\Box \bar{h}_{\mu\nu} + \eta_{\mu\nu} \partial^\rho \partial^\sigma \bar{h}_{\rho\sigma} - \partial^\rho \partial_\nu \bar{h}_{\mu\rho} - \partial^\rho \partial_\mu \bar{h}_{\nu\rho} = -\frac{16\pi G}{c^4} T_{\mu\nu} \tag{35}$$

We can now use the residual gauge freedom (29) to choose the Lorentz gauge

$$\partial^\nu \bar{h}_{\mu\nu} = 0 \tag{36}$$

In this gauge the last three terms on the left-hand side of equation (35) vanish and we get a simple wave equation

$$\Box \bar{h}_{\mu\nu} = -\frac{16\pi G}{c^4} T_{\mu\nu} \tag{37}$$

Observe that the Lorentz gauge gives four conditions, that reduce the 10 independent components of the symmetric 4×4 matrix $h_{\mu\nu}$ to six independent components. Equations (36) and (37) together imply for consistency that

$$\partial^\nu T_{\mu\nu} = 0 \tag{38}$$

which is the conservation of energy-momentum in the linearized theory.

Equation (37) is the basic result for computing the generation of gravitational waves within the linearized theory. To study the propagation of gravitational waves we are rather interested in this equation outside the source, where $T_{\mu\nu} = 0$,

$$\Box \bar{h}_{\mu\nu} = 0 \tag{39}$$

Outside the source we can greatly simplify the form of the metric, observing that equation (36) does not fix the gauge completely. To see this, using the symmetry transformation (30), we can impose the Lorentz gauge (36) and we observe that, in terms of $\bar{h}_{\mu\nu}$, equation (30) becomes

$$\bar{h}_{\mu\nu} \rightarrow \bar{h}'_{\mu\nu} = \bar{h}_{\mu\nu} - \left(\partial_\mu \xi_\nu + \partial_\nu \xi_\mu - \eta_{\mu\nu} \partial_\rho \xi^\rho\right) \tag{40}$$

and therefore

$$\partial^\nu \bar{h}_{\mu\nu} \rightarrow \left(\partial^\nu \bar{h}_{\mu\nu}\right)' = \partial^\nu \bar{h}_{\mu\nu} - \Box \xi_\mu \tag{41}$$

Equation (41) means that the Lorentz gauge condition (36) does not remove all the unphysical degrees of freedom. As we see from (41), we can perform a further coordinate transformation $x^\mu \rightarrow x^\mu + \xi^\mu$ with

$$\Box \xi^\mu = 0 \tag{42}$$

and the Lorentz gauge (36) is not spoiled.

If $\Box\xi_\mu = 0$ then also $\Box\xi_{\mu\nu} = 0$, where

$$\xi_{\mu\nu} = \partial_\mu\xi_\nu + \partial_\nu\xi_\mu - \eta_{\mu\nu}\partial_\rho\xi^\rho \tag{43}$$

since the flat d'Alembertian $\Box$ commutes with ∂_μ. Then equation (40) tells us that, from the six independent components of $\bar{h}_{\mu\nu}$, which satisfy $\Box\bar{h}_{\mu\nu} = 0$, we can subtract the functions $\xi_{\mu\nu}$, which depend on four independent arbitrary functions ξ_μ, and which satisfy the same equation, $\Box\xi_{\mu\nu} = 0$. This means that we can choose the functions ξ_μ so as to impose four conditions on $\bar{h}_{\mu\nu}$. In particular, we can choose ξ^0 such that the trace $\bar{h} = 0$. Note that if $\bar{h} = 0$, then $\bar{h}_{\mu\nu} = h_{\mu\nu}$. The three functions $\xi^i(x)$ are now chosen so that $h^{0i}(x) = 0$. Since $\bar{h}_{\mu\nu} = h_{\mu\nu}$, the Lorentz condition (36) with $\mu = 0$ reads

$$\partial^0 h_{00} + \partial^i h_{0i} = 0 \tag{44}$$

Having fixed $h_{0i} = 0$, this simplifies to

$$\partial^0 h_{00} = 0 \tag{45}$$

so h_{00} becomes automatically constant in time. A time-independent term h_{00} corresponds to the static part of the gravitational interaction, that is, to the Newtonian potential of the source which generated the gravitational wave. The gravitational wave itself is the time-dependent part and therefore, as far as the gravitational wave is concerned, $\partial^0 h_{00} = 0$ means that $h_{00} = 0$. So, we have set all four components $h_{0\mu} = 0$ and we are left only with the spatial components h_{ij}, for which the Lorentz gauge condition now reads $\partial^j h_{ij} = 0$, and the condition of vanishing trace becomes $h^i_i = 0$. In conclusion, we have set

$$h_{0\mu} = 0 \qquad h^i_i = 0 \qquad \partial^j h_{ij} = 0 \tag{46}$$

This defines the transverse-traceless gauge, or TT gauge [20]. By imposing the Lorentz gauge, we have reduced the 10 degrees of freedom of the symmetric matrix $h_{\mu\nu}$ to six degrees of freedom, and the residual gauge freedom, associated to the four functions ξ^μ that satisfy equation (43), has further reduced these to just two physical degrees of freedom. This is the same number of physical degrees of freedom in an electromagnetic wave. We will denote the metric in the TT gauge by h^{TT}_{ij}.

Equation (39) has the plane wave solutions

$$h^{TT}_{ij} = e_{ij}(\vec{k})e^{ikx} \tag{47}$$

with $k^\mu = \left(\omega/c, \vec{k}\right)$. The tensor $e_{ij}(\vec{k})$ is called the polarization tensor. We follow the usual convention that the real part is taken at the end of the computation. For a gravitational wave propagating along the z axis we have

$$h^{TT}_{ij}(t,z) = \begin{pmatrix} h_+ & h_\times & 0 \\ h_\times & -h_+ & 0 \\ 0 & 0 & 0 \end{pmatrix}_{ij} \cos[\omega\left(t - \frac{z}{c}\right)] \tag{48}$$

where h_+ and $h_\times$ are called the amplitudes of the *plus* and *cross* polarization of the wave.

4 Black holes, gravitational waves and quantum gravity

In this section we present the basic ideas for a future, mathematically more rigorous, attempt to combine black holes and gravitational waves using the quantization of geometric quantities introduced by LQG.

4.1 Energy spectrum for gravitational waves

As mentioned in the introduction, LQG is a theory that attempts to describe the quantum mechanics of the gravitational field based on the canonical quantization of GR. From the point of view of LQG, GR is a constrained Hamiltonian system with first-class constraints [14]. These first-class constraints generate the gauge symmetries of GR in the Hamiltonian formalism. The most interesting result of LQG is that geometric quantities, such as area and volume, are quantized in terms of the Planck length L_P. However, the equations giving the eigenvalues of the area and volume operators in LQG depend on the choice of an arbitrary parameter γ, the Immirzi [16] parameter, which fixes the precise scale of the quantum theory.

In ref. [9], using the value

$$\gamma = \left(4\pi\sqrt{3}\right)^{-1} \tag{49}$$

it was pointed out that a plane gravitational wave propagating in the quantum space of LQG can only have wavelengths that are integer multiples of the Planck length L_P. Since L_P is the distance that light travels during the Planck time T_P, the period of the gravitational wave must be an integer multiple of the Planck time T_P. Assuming the validity of the above conditions and using the similarities between electromagnetic and gravitational waves, which are: a) both types of wave describe the same number of physical degrees of freedom, b) both types of wave are transverse waves and c) both types of wave propagate at the speed of light, ref. [9] introduced an energy spectrum for gravitational waves. This energy spectrum is given by

$$E_n = h\nu_n = \hbar\omega_n = \frac{2\pi}{n}c^2 M_P \qquad n = 1, 2, 3, ... \tag{50}$$

where M_P is the Planck mass given in equation (3) and the integer n determines the energy and the number of Planck lengths contained in the wavelength of the gravitational wave. In equation (50) $c^2 M_P = 1,22 \times 10^{19}$Gev so gravitational waves with wavelengths containing a small number of Planck lengths can have large energies. As $n \to \infty$ the energy difference between two consecutive energy levels becomes infinitesimal. Therefore in the $n \to \infty$ limit the energy spectrum becomes effectively continuous, as is the case in Hawking´s thermal radiation.

4.2 Distant observers

In the classical theory a black hole with vanishing charge and vanishing angular momentum evolves rapidly towards the Schwarzschild solution, by radiating away all excess energy. In the quantum theory, however, the Heisenberg uncertainty relations prevent the black hole from converging exactly to a Schwarzschild metric, and quantum fluctuations may remain [10]. From the point of view of this paper, the logical procedure now would be to describe the quantum mechanical processes of emission and absorption of gravitational radiation by quantum fluctuations of the event horizon. However, at present, a mathematical description of such quantum mechanical processes is still under study. In addition to the technical difficulties, there are also conceptual difficulties. But, as we shall see in the next subsection, the basic ideas behind the physics are very simple. This simplicity of the basic ideas is the motivation for this paper. We will use this subsection to consider two of the conceptual difficulties.

The Schwarzschild spacetime becomes flat at large distances from the gravitational source and the coordinates (t, r, θ, φ) provide a global reference frame only for an observer at infinity. However, physical quantities measured by arbitrary observers are not specified directly by the coordinates but rather must be computed from the metric. Therefore, to measure a time interval for a stationary clock at r we set $dr = d\theta = d\varphi = 0$ in the line element (13) and we use $ds^2 = -c^2 d\tau^2$ to obtain

$$d\tau = \sqrt{\left(1 - \frac{2GM}{c^2 r}\right)} dt \tag{51}$$

In the above equation $d\tau$ is the *proper time* and dt is the *coordinate time*. The physical time interval measured by a local observer is given by the proper time $d\tau$, not by the coordinate time dt. Proper time and coordinate time will be approximately equal only if the effect of the gravitational field is very weak. In the context of this paper, this means that the t coordinate that appears in the Schwarzschild metric (13) can be identified with the t coordinate that appears in the gravitational wave (48) only in the reference frame of a distant observer.

Setting $dt = d\theta = d\varphi = 0$ in the line element (13) gives an interval of radial distance

$$ds = \frac{dr}{\sqrt{\left(1 - \frac{2GM}{c^2 r}\right)}} \tag{52}$$

ds is the *proper distance* and dr is the *coordinate distance*. The physical radial interval measured by a local observer is the proper distance ds, not the coordinate distance dr. The proper distance and coordinate distance will be approximately equal only for distant observers. In addition to this, equation (52) is valid only in an arbitrary fixed direction since, to arrive at it, we required that $d\theta = d\varphi = 0$. But the gravitational wave (48) propagates in an arbitrary z direction. A partial solution for this problem comes from the fact that the linearized version of GR is invariant under global Lorentz transformations [20].

Therefore, for distant observers, before fixing the Lorentz gauge condition (36) we can perform a Lorentz rotation

$$x^{\mu} \to \Lambda^{\mu}_{\nu} x^{\nu} \tag{53}$$

and make the z direction of the distant observer´s coordinate system coincident with the particular direction of the Schwarzschild r coordinate. Rotations never spoil the condition $\mid h_{\mu\nu} \mid \ll 1$. The distant observer will then interpret the gravitational wave as coming directly from the black hole.

4.3 The perspective of quantum gravity

In this subsection we use the reference frame of a distant observer and impose a quantization process on the gravitational wave (48) and on the the event horizon of the Schwarzschild black hole. We will find that the concepts of a quantum gravitational wave and a quantum event horizon can be related to each other using the concept of a quantum of length.

First we recall the usual relations $k = 2\pi/\lambda$ and $\lambda\nu = c$ for a plane gravitational wave. Then we write

$$\omega\left(t - \frac{z}{c}\right) = \omega t - \frac{\omega}{c} z \tag{54}$$

Recalling that the angular frequency of a wave is defined as $\omega = 2\pi\nu$ we can write

$$\lambda\nu = \lambda\frac{\omega}{2\pi} = c \quad \to \quad \frac{\omega}{c} = \frac{2\pi}{\lambda} = k$$

and therefore equation (54) becomes

$$\omega\left(t - \frac{z}{c}\right) = \omega t - kz \tag{55}$$

From the energy spectrum (50) we quantize the angular frequency as

$$\omega_n = \frac{E_n}{\hbar} \tag{56}$$

and introduce the quantized wave number as $k_n = 2\pi/\lambda_n$ with λ_n giving the number of Planck lengths L_P contained in the wavelength of the gravitational wave [9]. Finally we use de Broglie´s relation $p = h/\lambda$ and introduce the quantized momentum variable

$$P_n = \frac{h}{\lambda_n} = \frac{h}{2\pi} \times \frac{2\pi}{\lambda_n} = \hbar k_n \tag{57}$$

Using the above equations, a quantized gravitational wave propagating in the z direction, in the transverse-traceless gauge can be written as

$$h^{n}_{ij}(t,z) = \begin{bmatrix} h_{+} & h_{\times} & 0 \\ h_{\times} & -h_{+} & 0 \\ 0 & 0 & 0 \end{bmatrix}_{ij} \cos[\frac{1}{\hbar}\left(E_n t - P_n z\right)] \qquad n = 1, 2, 3, ... \tag{58}$$

where n gives the possible energy values, the possible momentum values and the number of Planck lengths L_P contained in the wavelength of the wave. We can use the quantum gravitational wave (58) as a basis for the interpretation of L_P as a quantum of length.

Using the idea that L_P defines a quantum of length we now quantize the Schwarzschild black hole event horizon area using the fact that the event horizon is spherically symmetric. We start by noting that the area of the horizon is $A = 4\pi R_S^2$ and the circumference of the horizon is $L = 2\pi R_S$. Therefore the area of the event horizon is

$$A = \frac{L^2}{\pi} \tag{59}$$

We now impose the quantum condition that the circumference L of the event horizon must contain an integer number N of Planck lengths L_P, that is $L = NL_P$. Inserting this condition into equation (59) we obtain the quantized area of the event horizon

$$A = \frac{1}{\pi} N^2 L_P^2 \tag{60}$$

Inserting equation (60) into equation (16) for the entropy of a classical black hole, we obtain the entropy for a black hole with a quantized event horizon area

$$\begin{aligned} S &= \frac{c^3 k_B}{4\hbar G} A \\ &= \frac{k_B}{4L_P^2} A \\ &= \frac{k_B}{4L_P^2} \frac{1}{\pi} N^2 L_P^2 \\ &= \frac{k_B}{4\pi} N^2 \end{aligned} \tag{61}$$

From equation (61) we see that what gives rise to the Schwarzschild black hole entropy (16) is the number of quantum of length L_P contained in the circumference of the event horizon.

Let us now consider the black hole mass M given by equation (14). Combining equations (14) and (60) we obtain for the mass of the black hole

$$\begin{aligned} M &= \sqrt{\frac{c^3 A}{16\pi G^2}} \\ &= \sqrt{\frac{c^3 N^2 L_P^2}{16\pi^2 G^2}} \\ &= \frac{N}{4\pi\sqrt{c}} \sqrt{\frac{\hbar c}{G}} \\ &= \frac{N}{4\pi\sqrt{c}} M_P \end{aligned} \tag{62}$$

Equation (62) shows that the mass of the black hole is quantized in terms of the Planck mass M_P, given in equation (3). This result confirms the consistency of the ideas we present in this paper. Combining now equation (62) with equation (4) we have

$$M = \frac{\sqrt{c^3}}{4\pi G} N L_P \tag{63}$$

Equation (63) shows that a black hole can increase its mass to a mass

$$M = \frac{\sqrt{c^3}}{4\pi G} \left(N + n\right) L_P \tag{64}$$

by absorbing a quantum gravitational wave of the type (58) with a wavelength containing n quanta of length L_P. In the opposite process, a black hole can decrease its mass to a mass

$$M = \frac{\sqrt{c^3}}{4\pi G} \left(N - n\right) L_P \tag{65}$$

by emitting a quantum gravitational wave of the type (58) with a wavelength containing n quanta of length L_P. From equation (65) we see that a black hole can completely convert its mass into quantum gravitational radiation by emitting N quanta of length L_P. Notice that in this case there is no information loss paradox. The mass of the black hole is converted in quantum gravitational radiation with well-defined polarization, energy and momentum.

4.4 Conclusions

The objective of this paper is to expose the basic ideas that will give support to a future, mathematically more rigorous, study of the emission of quantum gravitational radiation by black holes. For this purpose, in section two we reviewed the Schwarzschild vacuum solution for the Einstein equations of GR. This solution describes a spherically symmetric spacetime with a black hole and an event horizon. We displayed the equation that relates the black hole mass to the area of the event horizon, the equation that gives the black hole temperature due to Hawking´s thermal radiation and the equation for the black hole entropy. We also briefly described how the equation for the black hole entropy emerges in the framework of LQG and why the degrees of freedom responsible for the entropy remain undetermined in this framework.

In section three we considered the propagation of gravitational waves. We reviewed the process of linearization of the Einstein equations. We described in detail the steps for the elimination of the unphysical degrees of freedom of the spacetime metric to arrive at the equations of motion in the transverse-traceless gauge. The solution for a gravitational wave propagating along the z direction was displayed.

Section four contains our contribution to the subject. We reviewed the energy spectrum for a gravitational wave propagating in the quantized spacetime of LQG which was obtained in [9]. We reviewed the notion of a distant observer.

Then, using the energy spectrum obtained in [9] and de Broglie´s relation, we imposed a quantization process on the classical gravitational wave described in section three. Since the quantum gravitational wave can only have wavelengths which are integer multiples of the Planck length L_P, we interpreted L_P as defining the quantum of length. Using the spherical symmetry of the Schwarzschild black hole and the notion of the quantum of length we quantized the area of the event horizon and found that the microstate responsible for the black hole entropy is the quantum of length L_P. Using the quantized event horizon area, we showed that the mass of the black hole is quantized in terms of the Planck mass M_P, a result that confirms the ideas presented in this paper. Finally, using the relation between the Planck mass M_P and the quantum of length L_P, we displayed equations that show that a black hole can increase or decrease its mass by absorbing or emitting quantum gravitational radiation. In particular, the black hole can convert all its mass into quantum gravitational radiation with well defined polarization, energy and momentum, thus avoiding the information loss paradox.

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