Original Research Article

Mathematical Model for the Cyclical Dynamics of Plastic Waste Management: A two-state Closed Model

Abstract

The transition to circular economy has become a sustainable technique to plastic waste management. This makes recycling a key driving machinery for achieving sustainable plastic waste management. However, most of the models that predict the volume of plastic products and its waste generation do not reflect the role of the recycling rate and its corelates. The objective of this study is to develop a simple two-dimensional cyclical dynamic model (CDM) that reflects the plastic life cycle to predict the volume of annual plastic production and plastic waste generation. The CDM was formulated using a time-dependent linear system of ordinary differential equations; and the solution methodology was based on the Laplace transform technique. A programme was written using excel implementation codes to compute the models' parameters and predict the values of global annual plastic production and waste generation; while implementation codes in R were applied to predict and forecast with the models. A global data on plastic waste management was used; it was sourced from the annual reports of the Plastic Europe (the Association of Europe Plastic Manufacturers), the Plastic Europe marketing Research Group, and research publications on plastics. The results revealed that the long-run equilibrium solutions of the models are zeros, which also have a realistic implication under the context of a closed model. The performances of the models were investigated via the criterion of the mean absolute percentage error (MAPE), which measures the predictive power of the models. MAPE values of approximately 13% and 18% were obtained, respectively, for the global annual plastic production and plastic waste generation models. These values indicate that on average, the model for global annual plastic production can predict with an accuracy rate of 87%; while that for the global annual plastic waste generation can predict with an 82% accuracy rate. The outperformance of the model was established by comparing with the best performing solid waste model developed in 2017. The model was used to forecast from 2022 to 2050. The models have significant policy implications for waste managers and all stakeholders.

Keywords: Circular economy, CDM, MAPE, plastic production and waste generation, recycling, predicted and forecast values.

1 Introduction

As a major constituent of conventional solid waste management, plastic waste management presents an unwarranted environmental challenge because of its inherent property of being biologically undegradable as against the case of biodegradability associated with many solid wastes. Thus, the entire process of managing plastic waste can be described as a complex one relative to other solid wastes. The complexity that enclaves plastic waste management can be explained not only by their undegradable nature but also their heterogeneity in polymers, which necessitates sorting at the recycling and recovery stages of plastic wastes leading to exorbitant cost of recycled plastics; and the adverse repercussion on human health and wildlife [1-6].

It is imperative to refer to the global demand for plastics, which increased from about 1.5 million tonnes in 1950 to approximately 322 million tonnes in 2015 with an average approximated growth rate of nine percent per annum. In the global market, the dominant customer sector in the plastic industry is packaging applications with a percentage share of approximately 39.9 in Europe, just as the same situation applies in the UK [3]. It is significant, however, to reflect on the adverse environmental consequences that associate with the continuous increase in the global demand for plastic applications [8]. For instance, since plastic is derived from petroleum, an increase in the demand for plastics and consequently its application may lead to the depletion of petroleum which is a nonrenewable fossil fuel. Moreover, the high consumption of plastics by the end-user consumer may result in the upsurge of solid waste by means of suffusing millions of tonnes of plastic waste into the chain of solid waste generated annually. Citing the UK as a case in point, out of approximately 3.7 million tonnes of plastic waste estimated in 2014, as large volume as 2.2 million tonnes of packaging waste could not be recycled right away as a result of contamination with various residues. Plastic waste therefore constitutes a major component of municipal and industrial wastes that end up perpetually in the landfill in all most every economy around the globe. This has unleashed damage on aquatic food chains, resources and creatures caused by the disposal of substantial volumes (closely eight million tons) of plastic waste comprising more than 5 trillion smithereens into the world's oceans and other water bodies [8, 9]. The global challenges that emanate from managing plastic waste are heightened in urban areas of third world countries not only due to increased industrial and consumer applications, increased demand, but also inadequate data on the volume of plastic waste generated [1, 8-12]. This situation aggravates in the event of a rapidly growing population coupled with the complexities of increased urbanisation and industrialisation which make it tough for authorities to regulate [13,14].

Considering the enormity of environmental and ecological threats, the aligned adverse consequences for both present and future generation, and the fact that plastic waste is a source of precious raw materials [4, 8, 10,15-17], there is a need for waste managers to adopt proactive and sustainable approaches to waste management and integrate global strategies to reduce the stream of generated volume of plastic wastes [18-20].

We emphasize that any sustainable strategic approach to plastic waste management must be backed by effective and efficient optimal decisions and planning based on predictions (or estimates) from mathematical models with high predictive accuracy. Thus, adequate and reliable data are functions of best predicting mathematical models. Unfortunately, just as it applies to solid waste management, the sustainability of plastic waste management has been mitigated by lack of data [13]. This necessitates the need to develop robust models that can predict and forecast, with high predictive power the volume of plastic products and plastic wastes generated over the years. This will surmount the challenges imposed by data unavailability and unreliability.

In the literature, more emphasis was placed on optimal decisions and planning of the volume of municipal solid waste [21-23]; and the volume of recycled plastics [5, 24-26], with a dominant application of optimization models. In addition to the fact that these optimization models do not reflect the periodicity or time heterogeneous properties of plastic products and wastes, none attempted to develop models to reflect the plastic life cycle. It is rational to contend that efficient optimal decision and planning in connection to the volume of plastic waste and recycled plastics, as well as facility location, *inter-alia*, depends on a reliable and timely data which can be derived from accurate predictions and forecasts of the volume of plastic products (primary production or recycled) and wastes generated from models typified by the plastic-life-cycle. It is our candid view that every optimal decision towards achieving a plastic circular economy must depend on a time-dependent model that integrates the roles of waste generation, recycling, incineration, and discarding, which constitutes a significant accomplishment of this present study.

The study therefore proposes a two-state (or decentralized) cyclical dynamic closed model for plastic waste management. The motivation was driven out of the reports of various studies that have uncovered the alarming rate of plastic pollution in the environment and the ocean as a result of the high global volume of mismanaged plastic waste. Another motivation emanated out of the fact that the global data on plastic waste still have incineration as a treatment option as against the EU's data on plastic waste management. Finally, most of the models that predict the volume of plastic products and its waste generation do not reflect the role of the recycling rate and its corelates, albeit the fact that recycling remains the key driving machinery for achieving sustainable plastic waste management.

The main aim is to develop a two-state closed model that reflects the cyclical dynamics of plastic waste management to account for the roles of plastic: recycling, incineration and discard rates in determining the volume of plastic production and waste generation. The specific objectives are of fourfold to: develop closed cyclical linear models, find analytical solutions to the models, fit the models to a global data to validate and predict or forecast the global annual volume of plastic production and waste generation. It is expected that a model with high predictive accuracy will be developed to inform efficient planning and optimal policy formulation in solid waste management.

This study will contribute significantly to science or theory in the following perspectives; it introduces a complete representation of the time dynamics of the plastic-life-cycle which has not been covered in the literature, especially in the mathematics literature. Additionally, the idea of the closed model which places a restriction on the production of virgin plastic has been introduced in this study so that the sustainability of plastic recycling in the event of higher waste incineration and discarding rates can be assessed; this has also received little or no attention in the literature. The plastic: recycling rate, waste incineration, and discarding rates are jointly formulated in this study as corelates of both plastic: production and waste generation, but this has not been dealt with in the literature. Finally, the models were formulated using a system of time-dependent linear differential equations, which has attracted little or no attention in the area of plastic waste management vis-á-vis waste management in general.

1.1 Conceptual frame work

Some studies have focused on predicting the volume of generated municipal solid wastes of which plastic waste is a constituent. Least square regression approach by [27] to predict the volume of residential solid waste (RSW) generation was proposed. The goal was to confirm the mathematical model that expresses such variables as education, income per household, and number of residences as correlates of RSW generation. The data applied originates from a study on generation, quantification, and composition of residential wastes in a Mexican city in three stages. Five other variables were further identified for inclusion in the models to define prediction models. A separate mathematical model was developed at each sampling stage to determine a model that exhibited the best linear relation in predicting RSW generation. Normality, multicollinearity, and heteroskedasticity tests were conducted, models exploring the combination of included variables with higher values of R^2 were selected. Finally, a general mathematical model which was proposed to predict RSW generation accounted for 51% of the total. Apart from the fact that this study concentrated on the volume of generated RSW, the model is time independent, which makes it inadequate for predicting the volume of generated RSW which is inherently time dependent. Again, nonlinear regression models have demonstrated superior predictive performance over linear regression models [28]. ARIMA/ARMA and exponential smoothening models were introduced by [29] to predict the volume of solid waste generation using monthly data of solid waste collected by city or municipal authorities that spans a six-year period. Another ARIMA time series model has been applied to study the dynamics of solid waste management [30] based on a monthly solid waste volume generated. In respect of performance assessment metrics such as MAPE, MAD, and RSE, ARIMA (1, 1, 1) was revealed to be the best performing model amidst all parametric-characterised time series models. Notwithstanding, the outperformance of ARIMA (1, 1, 1) is associated with an intrinsic biasedness driven by the underlying axiom of a stationary stochastic process, which expressly implies constant mean and variance. Thus, automatically, the volume of generated solid/plastic waste is subject to flow in a similar fashion albeit its nonparametric nature which confers time-dependency, seasonality, or time-heterogeneity [31]. As it is for solid waste, the same applies to plastic waste and so it will be inappropriate to fit a parametric model to a plastic-based nonparametric data. To explore predictive accuracy, a grey fuzzy dynamic modelling was proposed by [32] in a study to forecast the volume of solid waste generation in general. In this study, extensive parameters were proposed to investigate a fuzzy logic intelligent system, where data was generated from a spatial geodatabase integrated in a GIS environment. However, the application of various fuzzy systems is subject to the choice of the researcher and the interpretation of fuzziness. For a continuous time-dependent data that cannot be discretized as characterised by monthly generation of plastic wastes, continuous modelling and predictive analysis come with some level of complexity. Improving the predictive accuracy of such data is subject to the choice of models with similar behavioural patterns. Such models should be nonparametric, time heterogeneous, and periodic as indicated in [33]. To say the least, [33] also proposed a Fourier series approach to optimize and forecast the volume of solid waste generated. This approach has proven a high degree of predictive accuracy as it outperforms the existing high performing ARIMA models in [29, 30]. The fact is reiterated that data on monthly volume of solid waste vis á vis plastic wastes and products is characterized by periodicity, nonparametric, and time heterogeneity. The Fourier series approach proposed by Asante-Darko *et al.* fits well into these properties and this suggestively explains its outstanding performance relative to the ARIMA and time series models. Two limitations are clear here. First, by nature, the Fourier function is a continuous function and so it will have been appropriate to apply it in the stochastic sense with continuous time integrals. The challenge would have related to finding an appropriate numerical technique to approximate the predicted values. Discretization might have its own challenge in terms of computational complexities, thereby making the approach questionable in terms of time efficiency. The efficient model selected depends on the period k that gives the least value of MAPE and RSE. This makes it unsuitable for large-scale application. Second, the objective of optimizing and forecasting the volume of generated solid waste represented just a single phase, which was a partial representation of the complete coverage of solid waste management.

Aside the few limiting factors, any of the aforementioned best performing models for predicting municipal solid wastes represents the general solid waste, hence, plastic waste seems to be overshadowed in terms of modelling and forecasting. This therefore, makes it imperative to recommend studies, particularly on model development, to reflect the intrinsic characterization of plastic products and plastic wastes for accurate prediction and forecasting. An extensive and a first global analysis of all mass-produced plastics ever manufactured was performed in 2017 [9]. In this study, dispersed data on the production, use, and end-of-life management of polymer resins, synthetic fibers, and additives were identified and synthesized. Although the study modelled and projected the volumes of plastic waste under various characterizations using discretized log-normal distributions, which denote the fractions of plastics in the industrial sector utilized over a specified number of years, the study does not reflect the complete dynamics of the plastic life cycle which is cyclical. Consequently, the transitional connections between plastic products and plastic wastes are mislaid. In addition, as indicated, all estimates made between 2014 and 2050 in respect of growth rates of global recycling, incineration, and discard were based on a simple assumption of forward projections of the historical global growth trends, which therefore, should not be misconstrued for predictions or forecasts. Thus, there is a need to develop a model that will reflect both the forward and reverse transitional dynamics of plastic waste management with specific emphasis on plastic production through recycling and waste generation.

2 Materials and Methods

We define the 2-compartment model using a time-dependent system of homogeneous linear ordinary differential equations. Linearity is considered under the assumption that technology is fixed at a value of 1. The states comprise the source, which is the consumption unit (or compartment) responsible for the consumption of plastic products and the generation of plastic waste; and the production compartment also responsible for the production of plastic products and the recycling of plastic wastes; and supply the same to the consumption unit. Plastic products can be produced from two sources, namely, virgin resource and plastic waste [34, 35]. In the closed model, we ignore the role of virgin resource and introduce just a simple model that mimics the rate of transition of plastic products to plastic waste and the rate of transition of plastic waste to plastic products. To state in simple terms, we attempt to model the rate at which plastic products degenerate or transforms into plastic wastes; and the rate at which plastic wastes are converted into plastic products. The aim is to aid to determine in a simple time discrete way, the volume of plastic that can be recovered from a given volume of plastic waste. The system also neglects the role of a central agent (waste receptor) which serves as a link between the production and consumption units. Therefore, the closed system basically involves only two compartments, the household (consumption unit) and the production unit with emphasis on plastic products (consumable plastics) at the consumption unit supplied from the production unit at a given rate/proportion; and plastic wastes at the production unit supplied from the consumption unit at a given rate.

We assume that a given volume of plastic products that reaches the consumption compartment will become (or degenerate into) waste overtime through disintegration or after usage. In this context, we define plastic waste as plastic products that have outstayed their original usage (plastic bottles, packaging, polytene, and others) or have disintegrated (for example plastic bowls, PVC among others).

Other assumptions underlying the decentralized closed system of plastic waste management are as follows:

Sources of virgin raw materials are ignored to ensure that plastics can only be produced through recycling, thus the models are assumed to operate under a closed system of plastic waste management. Moreover, the volume of plastic products and plastic wastes follow a Poisson process and so do their rates, which per time period, represent the proportion of recycled volume of plastic wastes and that of waste plastic out of plastic products. Additionally, there is no central agent acting as a waste receptor.

This guarantees that plastic waste generated at the household is directly discharged to the production unit.

Let us denote the volume of plastic products at the consumer compartment at any given time by $x(t)$ and its rate of transition into plastic wastes at any time be given by μ. This means that at any point in time, the volume of plastic waste that is expected to be generated out of plastic products is μx. We denote also; the volume of plastic wastes at the production unit per time by $y(t)$ and the rate at which these plastic wastes are converted into plastic products through production by ψ. Thus, at any given time, ψy volume of plastic products will be generated out of the available plastic waste at the production unit to be resupplied to the consumption unit. The volume of waste incineration also counts in the global annual data and becomes a source of decrement to the total volume of the global annual waste generated; denoted by w_i; the rate at which global annual plastic wastes are generated, the total volume of global annual plastic wastes that can be incinerated within any specified period is $w_i y$. We also denote by w_d; the total volume of global annual wastes that are discarded. Now, bringing into force the assumption that plastic wastes generated by the household are directly discharged to the production unit and that there is no central agent to act as a waste receptor who serves as a link between the consumption and the production unit, a simple cyclical dynamic model that defines the rates and volumes of transitions of plastic products into plastic wastes and plastic waste into plastic products is obtained as illustrated in Figure 1.

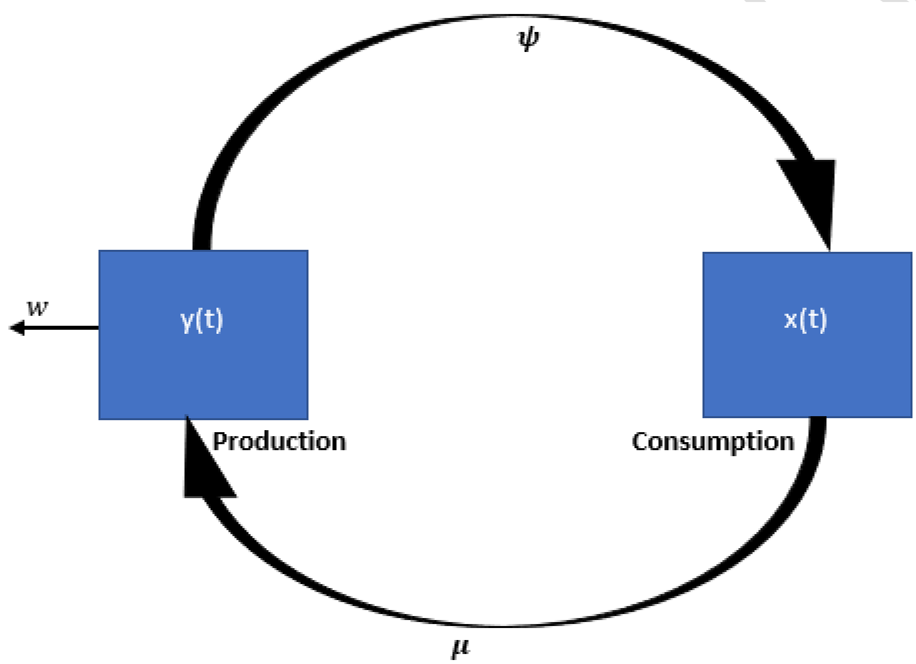

Fig. 1. A decentralized closed system for plastic waste management

The guiding equations for the unregulated version of the decentralized open system of plastic wastes management are given by the system of homogeneous linear ordinary differential equations below.

$$\left.\begin{aligned} \frac{dy(t)}{dt} &= \mu x - (\psi + w)y \\ \frac{dx(t)}{dt} &= \psi y - \mu x \end{aligned}\right\} \quad (1)$$

The above equation, (1) represents the closed model for a 2-multistate cyclical dynamics of plastic waste management. The linearity of (1) was obtained by adopting the popular assumption of a fixed technology (of unit 1) that underpins the Cobb-Douglas production function. This assumption transforms a general nonlinear system of ordinary differential equations (ODEs) to the form presented in [36].

Laplace transform (LT) is the solution technique applied in solving the system represented by Equation 1. Following the LT technique, we obtain as follows:

$$\ell\left(\frac{dy(t)}{dt}\right) = \ell[\mu x - (\psi + w)y]$$

$$sY(s) - y(0) = \mu X(s) - (\psi + w)Y(s)$$

By applying the initial condition, we simplify to get

$$Y(s)(s + \psi + w) = \mu X(s) + y_0$$

$$\Rightarrow Y(s) = \frac{\mu X(s) + y_0}{s + \psi + w} \tag{2}$$

Following a similar approach, we have

$$\ell\left(\frac{dx(t)}{dt}\right) = \ell[\psi y - \mu x]$$

$$sX(s) - x(0) = \psi Y(s) - \mu X(s)$$

Applying the initial value condition $x(0) = x_0$ and simplifying, we obtain

$$X(s)(s + \mu) = \psi Y(s) + x_0$$

$$\Rightarrow X(s) = \frac{\psi Y(s) + x_0}{s + \mu} \tag{3}$$

If Equation 2 is substituted into Equation 3, we have

$$X(s) = \frac{\psi\left[\frac{\mu X(s) + y_0}{s + \psi + w}\right] + x_0}{s + \mu}$$

$$X(s) = \frac{\psi\mu X(s) + \psi y_0 + x_0(s + \psi + w_i)}{(s + \psi + w)(s + \mu)}$$

Grouping like terms and simplifying, we have

$$X(s) = \frac{\psi y_0 + x_0(s + \psi + w)}{(s + \psi + w_i)(s + \mu) - \mu\psi}$$

$$X(s) = \frac{\psi y_0 + x_0(s + \psi + w)}{(s + A_1)^2 - \lambda^2}.$$

Where we define the following;

$$A_1 = \frac{A}{2},$$

$$A = \mu + \psi + w$$

$$\lambda = \frac{\sqrt{A^2 - 4\mu w}}{2}$$

We decompose $X(s)$ into a partial fraction as follows:

$$X(s) = \frac{\psi y_0 + x_0(s + \psi + w)}{(s + A_1)^2 - \lambda^2} = \frac{Ms + N}{(s + A_1)^2 - \lambda^2}$$

$$\Rightarrow X(s) = \frac{Ms}{(s+A_1)^2-\lambda^2} + \frac{N}{(s+A_1)^2-\lambda^2}$$

$$\Rightarrow M = x_0, N = \psi(x_0+y_0) + wx_0$$

Thus, we have

$$X(s) = \frac{x_0 s}{(s+A_1)^2-\lambda^2} + \frac{\psi(x_0+y_0)+wx_0}{(s+A_1)^2-\lambda^2}$$

Applying ℓ^{-1}, the inverse Laplace transform to $X(s)$, we have

$$\ell^{-1}[X(s)] = \ell^{-1}\left[\frac{x_0 s}{(s+A_1)^2-\lambda^2} + \frac{\psi(x_0+y_0)+wx_0}{(s+A_1)^2-\lambda^2}\right]$$

$$= \ell^{-1}\left[\frac{x_0 s}{(s+A_1)^2-\lambda^2}\right] + \ell^{-1}\left[\frac{\psi(x_0+y_0)+wx_0}{(s+A_1)^2-\lambda^2}\right]$$

$$= x_0\ell^{-1}\left\{\frac{s+A_1}{(s+A_1)^2-\lambda^2} - \frac{\lambda A_1}{\lambda[(s+A_1)^2-\lambda^2]}\right\} + \ell^{-1}\left\{\frac{[\psi(x_0+y_0)+wx_0]\lambda}{\lambda[(s+A_1)^2-\lambda^2]}\right\}$$

We obtain

$$x(t) = x_0\exp(-A_1 t)\left[\cosh(\lambda t) - \frac{A_1}{\lambda}\sinh(\lambda t)\right] - A_2\sinh(\lambda t)$$

$$\Rightarrow x(t) = \exp(-A_1 t)[x_0\cosh(\lambda t) + A_3\sinh(\lambda t)]. \quad (4)$$

Where,

$$A_2 = \frac{\psi(x_0+y_0)+wx_0}{\lambda}$$

$$A_3 = \frac{\lambda A_2 - x_0 A_1}{\lambda}$$

Similarly, we solve for $y(t)$ by substituting Equation 3 into Equation 2 as follows:

$$Y(s) = \frac{\mu\left[\frac{\psi Y(s)+x_0}{s+\mu}\right] + y_0}{s+\psi+w}$$

$$Y(s) = \frac{\mu\psi Y(s) + \mu x_0 + y_0(s+\mu)}{(s+\psi+w)(s+\mu)}$$

Simplifying, we obtain

$$Y(s) = \frac{y_0 s + \mu(x_0+y_0)}{(s+A_1)^2-\lambda^2}.$$

Where,

$$A_1 = \frac{A}{2},$$

$$A = \mu + \psi + w$$

$$\lambda = \frac{\sqrt{A^2 - 4\mu w}}{2}$$

Resolving into a partial fraction and taking the inverse LT, we have

$$\ell^{-1}[Y(s)] = \ell^{-1}\left[\frac{y_0 s}{(s + A_1)^2 - \lambda^2}\right] + \ell^{-1}\left[\frac{\mu(x_0 + y_0)}{(s + A_1)^2 - \lambda^2}\right]$$

$$\Rightarrow y(t) = \exp(-A_1 t)\left[y_0 \cosh(\lambda t) - \frac{y_0 A_1}{\lambda}\sinh(\lambda t)\right] - A_4 \sinh(\lambda t)$$

$$\Rightarrow y(t) = \exp(-A_1 t)[y_0 \cosh(\lambda t) + A_5 \sinh(\lambda t)]. \quad (5)$$

Where,

$$A_4 = \frac{\mu(x_0 + y_0)}{\lambda}$$

$$A_5 = \frac{\lambda A_4 - y_0 A_1}{\lambda}$$

The solution $(x, y,)$ is the region $\Omega \in R^2$ given by

$$\Omega = (\exp(-A_1 t)[x_0 \cosh(\lambda t) + A_3 \sinh(\lambda t)], \exp(-A_1 t)[y_0 \cosh(\lambda t) + A_5 \sinh(\lambda t)]) \quad (6)$$

The steady state equilibrium of the system defined by

$$\lim_{t\to\infty} x(t) = \lim_{t\to\infty} y(t) = 0$$

The steady state solutions of both $x(t)$ and $y(t)$ are zero (0) since both solutions decay to zero as $t \to \infty$.

3 Results and Discussions

3.1 Results

Now that the models have been developed, we will fit them to a real data which comprises a 34-data point of global volume of annual: plastic production, plastic waste generation, incineration, discarding and recycling all measured in metric tonnes (Mt). The main components of the data to be modelled [global annual plastic production and plastic waste generation] were sourced from various editions of the annual report of Plastic Europe, Plastic Europe Market Research Group (PEMRG) in conjunction with Conversio Markets and Strategy GmbH, which was most of the time published, for instance, in the form of [37, 38]; and a publication on plastic wastes statistics of the World Bank Group [39]. Data on waste incineration, discarding, and recycling was generated by applying the percentages of waste distribution by disposal methods in [9]. It is a publication of global data on the production and distribution of plastic wastes by disposal methods from 1950 to 2015. It is eminent to mention that reference was made to [37, 38] in [9] in respect of data sources. For instance, data on plastic resin production (1950 – 2015) was obtained from the publications of PEMRG, while that on global annual fiber production (1970 – 2015) was based on the publication of the Fiber Year and Tecnon OrbiChem. This explains our decision to rely heavily on [37, 38] for the global data on annual plastic production and waste generation. However, the two sources were compared to assist in filling data gaps and updating estimates of the volumes of global annual plastic production, waste generation, and recycling. Based on the list of regions provided by Plastic Europe and its associates, we simplified the data distribution by region as follows: Asia, NAFTA, EU_{28}+2, and the rest of the world (Middle East, Africa, Latin America and CIS). Worth noting is the fact that the EU data includes Thermoplastics, Polyurethanes, Thermosets, Elastomers, Adhesives, Coatings and Sealants and PP-Fibers, however, PET-, PA- and Polyacryl-Fibers are not included.

The 34-data point for this study starts from 1988 through to 2021. This starting point was chosen based on the fact that the values for recycling right from 1950 through to 1987 are zeroes (no plastic recycling), which has the possibility to affect the predictive power of the model when considered in computing the values of the parameters. The first 33 data points (1988 – 2020) for the global plastic production and waste generation were based on historical estimates, while the last (2021) was generated based on an extrapolated estimate of 8.5% growth [38]. However, for recycling, incineration, and discarding, the first 27 data points (1988 – 2015) were generated based on historical estimates, while the remaining 6 (2016 – 2021) are based on extrapolated estimates as in [9]. It is also important to mention that a simplified version of the data in [9] was published in [39].

3.1.1 Data and computation of values of parameters

In this section, the values of the parameters which are the global: recycling rate ψ, waste generation rate μ and the separation rate β, are computed for the simple closed model for plastic waste management. By applying the formulae in Equations 7 – 15 to the global annual volume of plastic: production, waste generation, and recycling (Table 1), we compute the values of the parameters as follows:

$$\psi = \frac{\sum_{n=0}^{33} x_n^{rec}}{\sum_{n=0}^{33} y_n}. \tag{7}$$

Table 1. Global annual volume of production, waste generation, and recycled plastics in metric tonnes (Mt)

Year	Annual Plastic Production (Mt)	Annual Plastic Wastes Generation (Mt)	Annual Plastic Wastes Recycled (Mt)	Annual Plastic Wastes Discarded (Mt)	Annual Plastic Wastes Incinerated (Mt)
1988	95000000	82900000	497400	76931200	5471400
1989	100000000	86800000	1128400	79335200	6336400
1990	105000000	89500000	1790000	80550000	7160000
1991	109000000	93400000	2521800	82752400	8125800
1992	115000000	97400000	3311600	84932800	9155600
1993	120000000	102600000	4206600	88030800	10362600
1994	130000000	107900000	5179200	91067600	11653200
1995	134000000	113200000	6226000	93956000	13018000
1996	145000000	118400000	7340800	96614400	14444800
1997	157000000	126300000	8714700	101292600	16292700
1998	165000000	134200000	10199200	105749600	18251200
1999	175000000	142100000	11794300	109985400	20320300
2000	185000000	150000000	13500000	114000000	22500000
2001	195000000	160500000	15568500	119733000	25198500
2002	204000000	165800000	17243200	121365600	27191200
2003	210000000	171100000	18992100	122849800	29258100
2004	225000000	181600000	21428800	127846400	32324800
2005	227000000	192100000	24012500	132549000	35538500
2006	240000000	200000000	26400000	135200000	38400000
2007	257000000	207900000	28898100	137629800	41372100
2008	245000000	221100000	32280600	143272800	45546600
2009	250000000	223700000	34226100	141825800	47648100
2010	270000000	218400000	34944000	135408000	48048000
2011	279000000	227600000	38009200	137925600	51665200
2012	288000000	244700000	42577800	144862400	57259800
2013	299000000	252600000	45720600	146002800	60876600
2014	311000000	265800000	49970400	149911200	65918400
2015	322000000	300000000	58500000	165000000	76500000
2016	335000000	242000000	48884000	129712000	63404000
2017	348000000	261000000	54549000	136242000	70209000

2018	359000000	269250000	58158000	136779000	74313000
2019	368000000	276000000	61548000	136344000	78108000
2020	367000000	275250000	63307500	132120000	79822500
2021	398195000	298646250	70779161	139169153	88697936.25
Total	**7732195000**	**6299746250**	**922407561.3**	**4076946353**	**1300392336**

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$$\mu = \frac{\sum_{n=0}^{33} y_n}{\sum_{n=0}^{33} x_n}. \tag{8}$$

$$w_i = \frac{\sum_{n=0}^{33} I_n}{\sum_{n=0}^{33} y_n}. \tag{9}$$

$$w_d = \frac{\sum_{n=0}^{33} y_d}{\sum_{n=0}^{33} y_n}. \tag{10}$$

Where, we denote in addition; the number of years (n), the volume of incinerated plastic wastes (I), the volume of recycled wastes (x^{rec}), and the volume of discarded wastes (y_d).

The accuracy of the predicted values will depend on the values of A_1 and λ; since both the plastics production and waste generation models in Equations 5 and 6 are interactively decay exponential and hyperbolic functions, higher values of A_1 will produce consistently lower values over time. However, higher values of λ will produce consistently higher values overtime. Therefore, in order to obtain values with minimal amount of error, adjustment is required in the values of either A_1 or λ, or both. Considering the faster rate of decadence in the models as time grows indefinite, we chose to adjust the value of λ after all parameter values have been computed. In general, the value of λ was adjusted using ${\lambda^{(\bullet)}}_{\mathrm{adjust}}$, which was defined separately for both models such that $A_1 < \lambda < {\lambda^{(\bullet)}}_{\mathrm{adjust}}$. That is, for both models, it is required that $A_1 < \lambda$, so, based on the values of the computed parameters for the global annual data on plastic wastes management presented in Table 2, we select ${\lambda^{(\bullet)}}_{\mathrm{adjust}}$ to satisfy

$$A_1 < {\lambda^{(\bullet)}}_{\mathrm{adjust}} < \frac{\lambda}{g + \psi}.$$

The values of the global annual volume of plastic products produced $x(t)$, and plastic wastes generated $y(t)$ were computed using Equations 11 and 12 specified below.

.

$${\lambda^{x}}_{\mathrm{adjust}} = \frac{\lambda}{g + \psi} - \Delta_x. \tag{11}$$

$${\lambda^{y}}_{\mathrm{adjust}} = \frac{\lambda}{g + \psi} - \Delta_y. \tag{12}$$

Where, g is defined as the rate of plastic products in stock (that is, the proportion of total global annual plastic products that did not degenerate into waste over the period, given by

$$\frac{\sum_{n=0}^{33} (x_n - y_n)}{\sum_{n=0}^{33} x_n}. \tag{13}$$

We select Δ_x and Δ_x, which are defined as follows:

$$\frac{g}{2\mu} \leq \Delta_x \leq \frac{g}{2\mu} + k_x. \tag{14}$$

$$\frac{\mu}{6w} - k_y \leq \Delta_y \leq \frac{\mu}{6w}. \tag{15}$$

Where, $w = w_i + w_d$.

The domain Δ_x is selected to ensure that the cumulative volume of the predicted production does not fall below 5 percent or above 1.2 percent of the cumulative volume of the observed plastic production. Similarly, the domain for Δ_y is chosen to ensure that the cumulative value of the predicted plastic waste generation does not exceed the cumulative observed value by 7% or fall below it by 2%. Consequently, we carefully choose k_x and k_y such that

$$\left.\begin{array}{r} k_x = 0.0013 \\ k_y = 0.009 \end{array}\right\}$$

Thus, given the data, it is expected that

$$\left.\begin{array}{l} \Delta_x \in \left[\frac{g}{2\mu}, \quad 0.0013 + \frac{g}{2\mu}\right] \\ \Delta_y \in \left[\frac{\mu}{6w} - 0.009, \quad \frac{\mu}{6w}\right] \end{array}\right\}$$

By applying excel implementation codes, we compute the following parameters or rates:

$$\psi = 0.146419796$$

$$\mu = 0.814742289$$

$$w_i = 0.206419796$$

$$w_d = 0.647160408$$

$$w = 0.853580204$$

$$g = 0.185257711$$

The next step is to compute the values of the other parameters $A, A_1, \lambda, A_2, A_3, A_4$ and A_5. These play significant roles in the validation process of the models. By setting our initial values at $x_0 = 95,000,000$ and $y_0 = 82,900,000$ for the initial year 1988, we have

$$A = 1.814742289$$

$$A_1 = 0.907371144$$

$$\lambda = 0.357595448$$

$$A_2 = 299607284.1$$

$$A_3 = 58552038.3$$

$$A_4 = 405325777.5$$

$$A_5 = 194973357.8$$

For the global annual volume of plastics produced and waste generated, we compute, respectively,

$$\lambda^x{}_{\text{adjust}} = 1.078141991 - \Delta_x,$$

and

$$\lambda^{y}{}_{\text{adjust}} = 1.078141991 - \Delta_y.$$

Where,

$$\Delta_x \in [0.113691, \quad 0.115000] \text{ and } \Delta_y \in [0.1500, \quad 0.159083]$$

Restricting all computations to the boundaries, we have

$$0.9631 \le \lambda^{x}{}_{\text{adjust}} \le 0.9645$$

$$0.9191 < \lambda^{y}{}_{\text{adjust}} \le 0.9281$$

The values for $x(t)$ and $y(t)$ were computed using Equations 14 and 15 given by

$$x(t) = exp(-A_1 t)\left[x_0 \cosh\left(\lambda^{x}{}_{\text{adjust}} t\right) + A_3 \sinh\left(\lambda^{x}{}_{\text{adjust}} t\right)\right]. \tag{16}$$

$$y(t) = exp(-A_1 t)\left[y_0 \cosh\left(\lambda^{y}{}_{\text{adjust}} t\right) + A_5 \sinh\left(\lambda^{y}{}_{\text{adjust}} t\right)\right]. \tag{17}$$

Where, $A_1, A_2, \ldots, A_5$ retain their original values. Table 2 summarizes the computations for the predicted values for $\lambda^{x}{}_{\text{adjust}} = 0.9645$ and $\lambda^{y}{}_{\text{adjust}} = 0.9281$.

Table 2. Predicted against historical values of global annual recycled and generated plastics

Index	Year	Annual Plastic Production (Mt)	Annual Plastic Wastes Generation (Mt)	Predicted Annual Plastic Production (Mt)	Predicted Annual Plastic Wastes Generation (Mt)
0	1988	95000000	82900000	95000000	82900000
1	1989	100000000	86800000	84093348.77	132906736.6
2	1990	105000000	89500000	86500342.06	143391468.8
3	1991	109000000	93400000	91195603.02	147623431.1
4	1992	115000000	97400000	96497156.11	150911440.9
5	1993	120000000	102600000	102161203.1	154103580.7
6	1994	130000000	107900000	108166067.5	157336298.5
7	1995	134000000	113200000	114525173.6	160632533.2
8	1996	145000000	118400000	121258330.6	163997139.2
9	1997	157000000	126300000	128387373.5	167432110.8
10	1998	165000000	134200000	135935553.1	170939011.6
11	1999	175000000	142100000	143927507.7	174519362.3
12	2000	185000000	150000000	152389327.2	178174703.7
13	2001	195000000	160500000	161348636	181906606.8
14	2002	204000000	165800000	170834682.7	185716675.4
15	2003	210000000	171100000	180878435.1	189606546.5
16	2004	225000000	181600000	191512682.2	193577891.8
17	2005	227000000	192100000	202772140.5	197632417.7
18	2006	240000000	200000000	214693567.4	201771866.3
19	2007	257000000	207900000	227315881.7	205998016.5
20	2008	245000000	221100000	240680289.9	210312684.3
21	2009	250000000	223700000	254830421.6	214717723.5
22	2010	270000000	218400000	269812471.2	219215027.2
23	2011	279000000	227600000	285675348.8	223806527.7
24	2012	288000000	244700000	302470840.5	228494198.1
25	2013	299000000	252600000	320253776.7	233280052.6
26	2014	311000000	265800000	339082211.3	238166147.8
27	2015	322000000	300000000	359017611.7	243154583.2
28	2016	335000000	242000000	380125058.8	248247502.3

39	2017	348000000	261000000	402473459.9	253447093.6
30	2018	359000000	269250000	426135773.3	258755591.3
31	2019	368000000	276000000	451189247	264175276.5
32	2020	367000000	275250000	477715670.3	269708478
33	2021	398195000	298646250	505801641.4	275357573.5
Total		**7732195000**	**6299746250**	**7824656834**	**6721916298**

3.1.2 The mean absolute percentage error (MAPE)

One way of determining the predictive power or accuracy of a model is by using the mean absolute percentage error (MAPE) as applied in [29, 30, 33, 40]. A MAPE value of $x\%$ indicates that on average, the predicted values of the model will deviate from the actual or observed values by $x\%$. Thus, the smaller the value of the MAPE, the more accurate are the predicted values. The MAPE is given by

$$MAPE = \frac{1}{t}\sum_{t=1}^{33}\left|\frac{h_t - \overline{h_t}}{h_t} \times 100\right|. \qquad (18)$$

Here, h_t is the observed values and $\overline{h_t}$ is the predicted values all at time t. Since in plastic waste management, cumulative value from the start to the end of the period is critical in decision and planning towards attaining a circular economy, the first step in this study is to select the values of λ within the respective neighbourhood such that the percentage errors in terms of the cumulative global annual plastic products and wastes generated are minimal. This is only a necessary but not a sufficient condition for determining the predictive accuracy of a model. When this value is smaller, it gives an idea of how the model can correctly predict the total values from the beginning to the end of the period, however, this is not a guarantee that the value of MAPE will be smaller. Table 3 summarizes the computation of MAPE for both the global annual volume of plastic products produced, and the wastes generated.

Table 3. Computation of MAPE

Annual Plastic Production (Mt) (x_n)	Predicted Annual Plastic Production (Mt) ($\overline{x_n}$)	Absolute Percentage Error $\left\lvert\frac{x_n - \overline{x_n}}{x_n} \times 100\right\rvert$	Annual Plastic Wastes Generation (Mt) (y_n)	Predicted Annual Wastes Generation (Mt) ($\overline{y_n}$)	Absolute Percentage Error $\left\lvert\frac{y_n - \overline{y_n}}{y_n} \times 100\right\rvert$
95000000	95000000	0	82900000	82900000	0
100000000	84093348.77	15.90665123	86800000	132906736.6	53.11836012
105000000	86500342.06	17.61872185	89500000	143391468.8	60.21393164
109000000	91195603.02	16.33430915	93400000	147623431.1	58.05506544
115000000	96497156.11	16.08942947	97400000	150911440.9	54.93987774
120000000	102161203.1	14.86566408	102600000	154103580.7	50.19842178
130000000	108166067.5	16.7953327	107900000	157336298.5	45.81677343
134000000	114525173.6	14.53345257	113200000	160632533.2	41.90153112
145000000	121258330.6	16.37356509	118400000	163997139.2	38.51109733
157000000	128387373.5	18.22460288	126300000	167432110.8	32.56699196
165000000	135935553.1	17.61481632	134200000	170939011.6	27.37631263
175000000	143927507.7	17.75570991	142100000	174519362.3	22.81447027
185000000	152389327.2	17.6273907	150000000	178174703.7	18.78313577
195000000	161348636	17.25710972	160500000	181906606.8	13.33744972
204000000	170834682.7	16.2575085	165800000	185716675.4	12.01247006
210000000	180878435.1	13.86741185	171100000	189606546.5	10.81621656
225000000	191512682.2	14.88325234	181600000	193577891.8	6.595755399
227000000	202772140.5	10.67306587	192100000	197632417.7	2.879967545
240000000	214693567.4	10.54434691	200000000	201771866.3	0.88593316

257000000	227315881.7	11.5502406	207900000	205998016.5	0.914854965
245000000	240680289.9	1.763146962	221100000	210312684.3	4.878930684
250000000	254830421.6	1.932168654	223700000	214717723.5	4.015322519
270000000	269812471.2	0.069455123	218400000	219215027.2	0.373180943
279000000	285675348.8	2.392598154	227600000	223806527.7	1.666727717
288000000	302470840.5	5.024597406	244700000	228494198.1	6.622722473
299000000	320253776.7	7.108286513	252600000	233280052.6	7.648435214
311000000	339082211.3	9.029649949	265800000	238166147.8	10.39648313
322000000	359017611.7	11.49615271	300000000	243154583.2	18.94847227
335000000	380125058.8	13.47016681	242000000	248247502.3	2.581612521
348000000	402473459.9	15.65329306	261000000	253447093.6	2.893833883
359000000	426135773.3	18.70077251	269250000	258755591.3	3.897644843
368000000	451189247	22.60577364	276000000	264175276.5	4.284320127
367000000	477715670.3	30.16775759	275250000	269708478	2.013268672
398195000	505801641.4	27.02360435	298646250	275357573.5	7.798081017
7732195000	**7824656834**	**461.2100**	**6299746250**	**6721916298**	**629.7576527**
MAPE (Plastic Production)		**13.5650**	**MAPE (P. Waste Generation)**		**17.9931**

The MAPE for the global annual plastic production model is approximately 14%, whilst the MAPE for the global annual plastic waste generation model is approximately 18%. This means that on average, the predicted values of global annual plastic products deviate from the respective observed values by 14%, indicating that the model for predicting the volume of global annual plastic production can predict to an accuracy degree of 86% approximately. Similarly, the predicted values of the global annual plastic waste generation deviate from the observed global annual plastic waste generation values by 18% on average. This implies that the said model can predict correctly with an accuracy degree of 82% approximately.

It is important to note that these MAPE values may not necessarily be the best, so we consider other choices of ${\lambda^x}_{\text{adjust}}$. If we consider a reduction of approximately 0.18% in the value of ${\lambda^x}_{\text{adjust}} = 0.9645$, we obtain ${\lambda^x}_{\text{adjust}} = 0.9628$ in addition to ${\lambda^x}_{\text{adjust}} = 0.9631$, which occurs at the end of the interval for Δ_x . Table 4 summarizes the predicted values and the resulting MAPE values.

Table 4. Computation of MAPE for ${\lambda^x}_{\text{adjust}} = 0.9628$ and 0.9631

Index	Year	Annual Plastic Production (Mt) (x_n)	Predicted Annual Plastic Production (Mt) at ${\lambda^x}_{\text{adjust}} = 0.9628$ $(\overline{x_{1n}})$	Predicted Annual Plastic Production (Mt) at ${\lambda^x}_{\text{adjust}} = 0.9631$ $(\overline{x_{2n}})$	Absolute Percentage error $\left\lvert\frac{x_n - \overline{x_{1n}}}{x_n} \times 100\right\rvert$	Absolute Percentage Error $\left\lvert\frac{x_n - \overline{x_{2n}}}{x_n} \times 100\right\rvert$
0	1988	95000000	95000000	95000000	0	0
1	1989	100000000	83960043.41	83983550.24	16.03995659	16.01644976
2	1990	105000000	86209673	86260895.02	17.89554953	17.84676664
3	1991	109000000	90732366.16	90813942.02	16.7592971	16.68445687
4	1992	115000000	95843340.22	95958396.6	16.65796503	16.557916
5	1993	120000000	101296539.7	101448593.8	15.58621691	15.45950518
6	1994	130000000	107068386.2	107261282	17.63970289	17.49132153
7	1995	134000000	113170401.8	113408309.2	15.5444763	15.36693346
8	1996	145000000	119620380.8	119907814.5	17.50318566	17.30495555
9	1997	157000000	126437997.4	126779841.3	19.46624366	19.24850872
10	1998	165000000	133644180.5	134045715.1	19.00352696	18.76017269
11	1999	175000000	141261072.1	141728003.7	19.27938735	19.01256932
12	2000	185000000	149312079.6	149850571.8	19.2907678	18.99969094
13	2001	195000000	157821944.7	158438652	19.0656694	18.74940921

14	2002	204000000	166816819.4	167518923.5	18.2270493	17.88288066
15	2003	210000000	176324346.4	177119594	16.03602551	15.65733621
16	2004	225000000	186373743.7	187270488.1	17.16722504	16.76867195
17	2005	227000000	196995894.4	198003139.8	13.21766767	12.77394725
18	2006	240000000	208223442	209350890	13.24023249	12.77046251
19	2007	257000000	220090890.4	221348990.7	14.36152124	13.87198806
20	2008	245000000	232634710	234034714.1	5.047057149	4.475626915
21	2009	250000000	245893449.7	247447468.4	1.642620126	1.021012625
22	2010	270000000	259907855.5	261628920.7	3.737831279	3.100399755
23	2011	279000000	274720995.9	276623125.5	1.533693237	0.85192633
24	2012	288000000	290378393.6	292476662.7	0.825831101	1.554396774
25	2013	299000000	306928166.1	309238781.3	2.651560572	3.424341583
26	2014	311000000	324421173.3	326961553.1	4.315489816	5.13233218
27	2015	322000000	342911173.8	345700033.9	6.494153344	7.360258979
28	2016	335000000	362454989.9	365512435.1	8.195519367	9.108189578
29	2017	348000000	383112682.6	386460304	10.08985133	11.0518115
30	2018	359000000	404947736.1	408608715.4	12.79881229	13.81858369
31	2019	368000000	428027252.6	432026473.6	16.31175343	17.39849826
32	2020	367000000	452422158.8	456786326	23.27579259	24.46493897
33	2021	398195000	478207423.8	482965189.3	20.09377912	21.2886122
		7732195000	**7543171704**	**7591968296**	**438.9954**	**441.2749**
				MAPE	**12.912%**	**12.979**

As evidenced in Table 4, the MAPE that corresponds to $\lambda^x{}_{\text{adjust}} = 0.9628$ and $\lambda^x{}_{\text{adjust}} = 0.9631$ are, approximately, 12.91% and 12.98% respectively; indicating that the model for global annual plastic production can predict with approximately; 87.1% degree of accuracy for $\lambda^x{}_{\text{adjust}} = 0.9628$ and 87% degree of accuracy for $\lambda^x{}_{\text{adjust}} = 0.9631$. Further computations revealed the following MAPE values; 12.956% (for $\lambda^x{}_{\text{adjust}} = 0.9630$) and 13.261(for $\lambda^x{}_{\text{adjust}} = 0.9639$). Following the MAPE criterion, the model for global annual plastic production can predict with the highest degree of accuracy at $\lambda^x{}_{\text{adjust}} = 0.9628$. This corresponds to an absolute percentage error of approximately 2.4% (below the observed) in the cumulative volume of plastics produced from 1988 to 2021. The absolute percentage errors in the cumulative value of plastics produced from 1988 to 2021 for $x(t)$ at $\lambda^x{}_{\text{adjust}} = 0.9645, 0.9639, 0.9331, 0.9630$ are, respectively, 1.2 percent, 0.11 (below the cumulative observed values), 1.8 percent (below the cumulative observed values), and 2.0 percent (above the cumulative observed values).

It is also necessary to consider other values of $\lambda^y{}_{\text{adjust}}$ within the defined neighbourhood in order to determine its value that produces the best predicting model for global annual plastic wastes generation $y(t)$. By considering a 0.48% leftward location of $\lambda^y{}_{\text{adjust}}$ within the neighbourhood, we have $\lambda^y{}_{\text{adjust}} = 0.9236$; and another value $\lambda^y{}_{\text{adjust}} = 0.9245$ (which is between $\lambda^y{}_{\text{adjust}} = 0.9236$ and $\lambda^y{}_{\text{adjust}} = 0.9281$), we summarize the predicted values and the corresponding MAPE values in Table 5.

Table 5. Computation of MAPE for $\lambda^y{}_{\text{adjust}} = 0.9236$ and 0.9245

Index	Year	Annual Plastic Wastes Generation (Mt) (y_n)	Predicted Annual Plastic Wastes Generation (Mt) at $\lambda^y{}_{\text{adjust}} = 0.9236$ ($\overline{y_{1n}}$)	Predicted Annual Plastic Wastes Generation (Mt) at $\lambda^y{}_{\text{adjust}} = 0.9245$ ($\overline{y_{2n}}$)	Absolute Percentage Error $\left\lvert\frac{y_n - \overline{y_{1n}}}{y_n} \times 100\right\rvert$	Absolute Percentage Error $\left\lvert\frac{y_n - \overline{y_{2n}}}{y_n} \times 100\right\rvert$
0	1988	82900000	82900000	82900000	0	0

1	1989	86800000	132229539.7	132364764.4	52.33817936	52.49396822
2	1990	89500000	142081062.3	142342219.6	58.74979031	59.04158609
3	1991	93400000	145637762.8	146032761.5	55.92908218	56.35199303
4	1992	97400000	148218029.7	148752842.3	52.17456843	52.72365738
5	1993	102600000	150674706.1	151354323.4	46.85643868	47.51883377
6	1994	107900000	153145005	153974235.1	41.93234937	42.70086667
7	1995	113200000	155651462.7	156635164.2	37.50129215	38.37028644
8	1996	118400000	158198246.8	159341384.9	33.61338415	34.57887239
9	1997	126300000	160786590.2	162094250.5	27.30529705	28.34065755
10	1998	134200000	163417264.5	164894658.2	21.77143408	22.87232357
11	1999	142100000	166090977.2	167743444.1	16.88316485	18.04605499
12	2000	150000000	168808434.8	170641446.3	12.53895654	13.76096423
13	2001	160500000	171570353.3	173589515.5	6.8974164	8.155461384
14	2002	165800000	174377460.3	176588516.7	5.173377739	6.506946136
15	2003	171100000	177230495.1	179639329.8	3.582989515	4.990841493
16	2004	181600000	180130209.1	182742849.9	0.809356239	0.629322659
17	2005	192100000	183077366.1	185899987.7	4.696842243	3.227492064
18	2006	200000000	186072742.2	189111669.5	6.963628884	5.444165247
19	2007	207900000	189117126.5	192378837.5	9.034571171	7.465686604
20	2008	221100000	192211320.8	195702450.5	13.06588838	11.48690616
21	2009	223700000	195356140	199083483.5	12.67047833	11.00425415
22	2010	218400000	198552412.4	202522928.5	9.08772328	7.269721376
23	2011	227600000	201800979.8	206021794.8	11.33524614	9.480758006
24	2012	244700000	205102697.8	209581108.8	16.18197881	14.35181494
25	2013	252600000	208458436.2	213201915	17.47488671	15.59702493
26	2014	265800000	211869078.6	216885275.7	20.29003814	18.40283082
27	2015	300000000	215335523.5	220632271.5	28.22149218	26.45590949
28	2016	242000000	218858683.7	224444002	9.562527394	7.254544639
29	2017	261000000	222439487.3	228321585.4	14.7741428	12.52046537
30	2018	269250000	226078877.3	232266159.4	16.03384315	13.73587393
31	2019	276000000	229777812.4	236278881.5	16.74716943	14.39170959
32	2020	275250000	233537266.6	240360929	15.15448987	12.67541181
33	2021	298646250	237358230.3	244513499.5	20.52194518	18.12604393
		6299746250	**6186151781**	**6288838487**	**715.8739691**	**695.9732491**
				MAPE	**20.454%**	**19.88%**

The MAPE values corresponding to $\lambda^{y}{}_{\text{adjust}}$ values 0.9236 and 0.9245 are approximately 20.45% and 19.88% in the given order. Thus, the model for the global annual plastic wastes generation $y(t)$ can predict correctly with approximately 79.55 % and 80.12 % degrees of accuracy at 0.9236 and 0.9245 values of $\lambda^{y}{}_{\text{adjust}}$, respectively (Table 5). The absolute percentage error in the cumulative value of global annual plastic waste generation from 1988 to 2021 for both values of $\lambda^{y}{}_{\text{adjust}}$ are, respectively, 1.8% and 1.7% (all below the observed). This MAPE values establishes that $y(t)$ can predict the volume of global annual plastic waste generation with a higher degree of accuracy at $\lambda^{y}{}_{\text{adjust}} = 0.9245$ than it can at $\lambda^{y}{}_{\text{adjust}} = 0.9236$. Among all three values of $\lambda^{y}{}_{\text{adjust}}$, $y(t)$ predicts with the highest degree of accuracy when $\lambda^{y}{}_{\text{adjust}} = 0.9281$. The absolute percentage error in the cumulative volume of global annual plastic waste generation from 1988 to 2021 is 6.7% (above the observed). Any attempt to increase $\lambda^{y}{}_{\text{adjust}}$ exclusively between 0.9281 and 0.9291 will decrease the value of MAPE, but the absolute percentage error in the cumulative value will increase (above the observed) beyond the given threshold. We attempted, as much as possible, not to tolerate persistent increases above the cumulative value. This explains why we selected $\lambda^{y}{}_{\text{adjust}} = 0.9281$ with an equally competitive MAPE value and a small percentage error (6.7%) above the cumulative value of the observed.

3.1.3 Forecasting with the model

In this section, we perform forecasting using the best predicting models. The objective here is to forecast the volume of global annual plastic production and waste generation over a period of 29 years and to compare the cumulative values of global annual plastic production and waste generation with existing findings. Figures 2 and 3 present the forecasting.

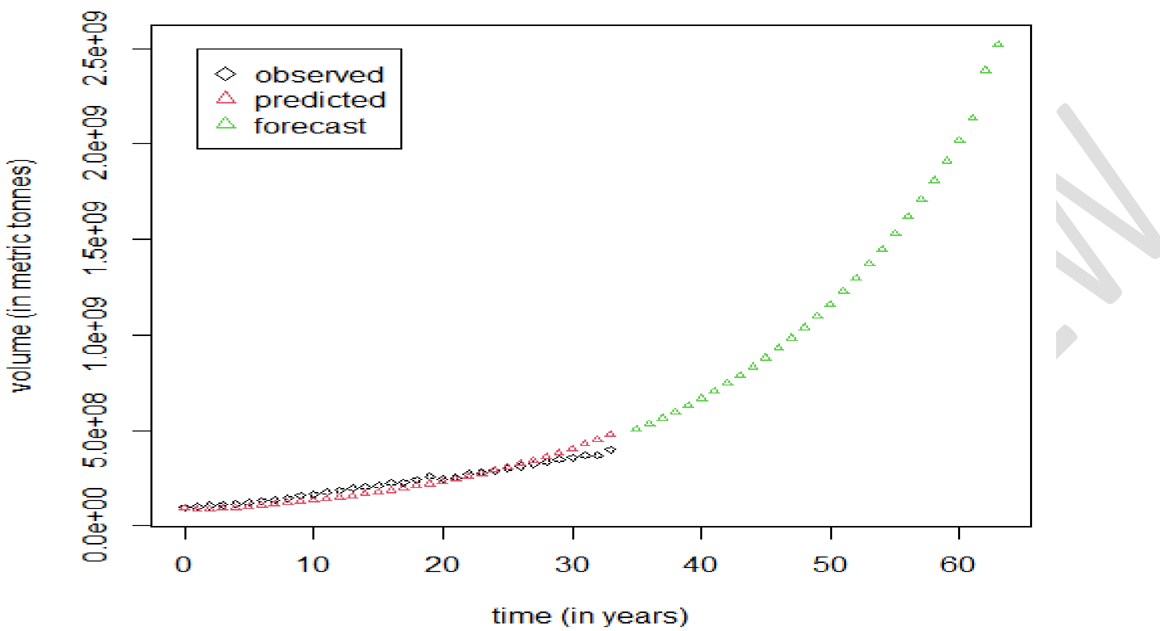

Fig. 2. A time series plot of forecast volume of global annual plastic production from 2022 to 2050. The time $t = 0, 1, 2, \ldots, 63$ correponds to the years 1988, 1989, 1990, …, 2050. The predicted values end at $t = 33$, while the forecast (represented by the green line) starts from time $t = 34$ (2021) and ends at $t = 63$ (2050).

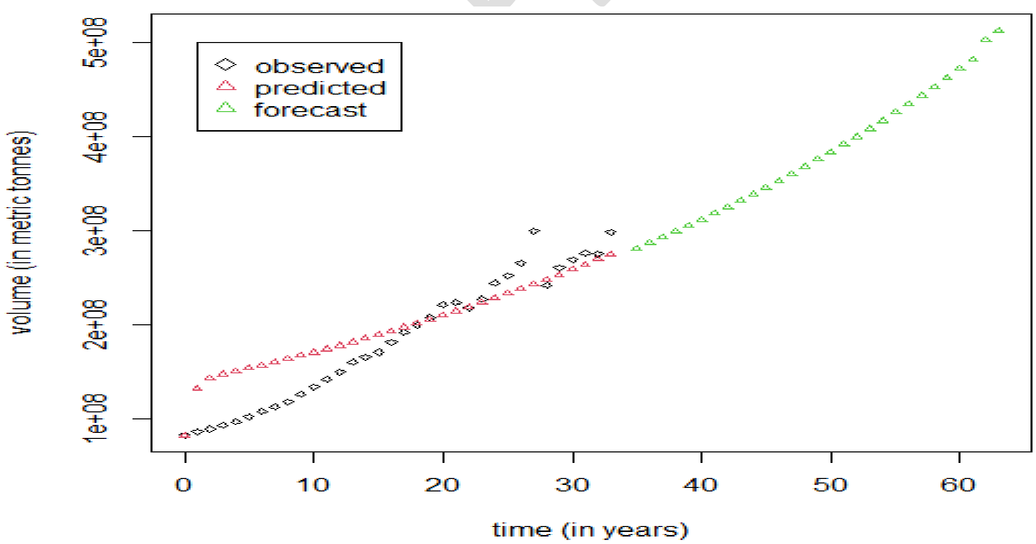

Fig. 3. A time series representation of the volume of global annual plastic waste generation from 1988 to 1950.

3.2 Discussions

Now that the model development process is complete, and a comparison has been made with the observed values of global annual plastic production and plastic waste generation, this section is devoted to a brief discussion of the results.

The models in general are integration of two important functions; a decay exponential function interacting with (multiplied by) a hyperbolic function. This has a real-life implication for solid waste management in the sense that the decay aspect could represent the diminishing rate of plastic waste as a result of recycling and other integrated government and individual efforts that aid in reducing plastic waste in the environment. In the case of plastic production, several demand and supply shocks which are both internal and external could lead to a fall in production, a case in point is the decline in production in periods of recession. The hyperbolic part can reflect the fact that both plastic production and its waste generation faces a hyperbolic growth rate, the rapid increase in the production of plastics and its corresponding waste generation over the past two decades as analysed in [9, 12] has been confirmed by the nature of the plastic production and waste generation models that have been developed in this study.

The circular plastic economy order; produce-use-recovery, was applied in the development of the two models in the context of a closed economy, where the assumption of no virgin plastic production was operational. Hence, given higher rates of incineration and discarding relative to the volume of recycled plastics, the total decadence of the models in the long run is envisaged. It is obvious that with higher rates of incineration and discarding of waste growing above the rate of recycling, if the cycle repeats without any effort to increase recycling, recyclable waste will eventually diminish until there is virtually nothing to recycle. No recycling under the assumed close system means no production. However, in the event of total riddance of waste incineration and discarding as waste treatment methods, the model can assume a steady state equilibrium in the long run.

Additionally, worth discussing is the predictive power of the models. The assessment of the models' predictive power was accomplished by applying the criterion of the MAPE. This was done alongside comparison of the percentage absolute errors in the cumulative volume from 1988 to 2021, the parameters that yielded the least absolute percentage errors in the cumulative volume were first selected, this constituted the necessary condition in this study, and the sufficient condition was to pass the selected parameters through the MAPE test.

As far as the global data used in this study is concerned, the production model can predict approximately 87% of the global annual plastic production data; while the plastic waste generation data can predict approximately 82% of its corresponding observed global annual data. The models' performance was compared to the performance of the solid waste generation model developed via Fourier series technique in [33], which was reported to be the best solid waste predicting model so far as of 2017. In [33], the best solid waste predicting model had a MAPE of approximately 29% which appeared to be the best in comparison with the existing best ARIMA solid waste models [29, 30]; for example, the MAPE in [29] was approximately 35% as computed in [33]. However, in this study, the values of the MAPE based on the data were approximately 13% for the global annual plastic production model and 18% for the global annual plastic waste generation model. The predicted values showed the cumulative values of plastics produced and plastic waste generated from 1988 to 2021 to be approximately 7.5 and 6.7 billion metric tonnes, respectively. The forecast values show that by 2050, a cumulative volume (from 1988) of approximately 43.2 billion metric tonnes of plastic will be produced; and approximately 17.8 billion metric tonnes of plastic waste will be generated. Our predicted cumulative values show that over the past 34 years, the growth of global production of plastics and its corresponding waste generation has increased than the growth over 65 years from 1950 to 2015 according to the estimates in [9]. In [39], the cumulative global plastic production was projected to reach approximately 34 billion metric tonnes, while plastic waste generation was projected to attain a cumulative volume of about 12 billion metric tonnes by the year 2050. In general, we can ascribe the difference to the difference in base years applied in this study and [9]; and the fact that the forecast made in [9] to 2050 was based on a projection of an assumed constant growth rate of 0.07% in plastic waste generation, incineration and discarding [9]. This assumption is a point of contention in the sense that the increase in plastic production and its waste generation have been tied to population [13, 14] growth, which increases at an exponential growth rate.

The effectiveness of the model depends on its parameters which were computed using a global data on annual plastic production, plastic waste generation, recycling, incineration, and discarding. Thus, the plastic production as well as its waste generation model is a function of time and such wastes treatment rates or parameters as the plastic: recycling rate, waste generation rate, incineration rate, and discarding rate. These parameters were first computed as the basic parameters of the model upon which every other parameter that arose consequent to the development of the model was obtained. The parameters reflect the realities of global plastic waste management as presented and predicted in [9, 12, 15]. In general, over the entire coverage of the data (1988-2021), about 80% of all plastics produced degenerated into waste; about 14% of the total waste generated over the period was recycled, 21% and 64%, approximately were incinerated and discarded respectively. The total rate of waste that could not be recovered stood as high as 85% approximately, which confirms major research findings on the high rate of mismanaged plastic waste [8, 15]. A major feature of the global annual data on plastic waste is the incineration component, which distinguishes it from the plastic waste management data of the EU_{28}+2, a discovery which was made by comparing the annual reports of the Plastic Europe and its associates [37, 38] to the global data used in this study.

Albeit the fact that the development of the parameters of a model is subject to geographical location, culture, and other socio-economic characteristics, the feature of the parameters applied in our model is unique by its heterogeneous nature as they permeate all geographical boundaries, culture, socio-economic, and other socio-demographic characteristics across the global economy. Thus, in this study, the parameters are not limited to any unique socio-demographic characteristics or geographic boundaries. The reflection of the plastic life cycle, the predictive power together with the universality of the models' parameters, constitute the major strengths of the models developed in this study. However, the limitations rest in the reliability of the global data applied, a global data which exhibits different characteristics could reflect different results when applied to the model. Another limitation can be cited in the behaviour of the global annual waste generation model, which exhibited weak performance in the early stages of the model, however, this was stabilized, and it performed better as time increases. To surmount these weaknesses, we suggest that more parameters be included in further researches in this regard to increase the models' representation of the real-world cutting-edge practices. Irrespective of the revealed weaknesses, the model predicts the volume of global annual plastic production and waste generation with a higher degree of accuracy. It is imperative to mention that a good performing model predicts correctly overtime.

This study therefore has significant policy implications for waste managers and all stakeholders. It can therefore be relied upon for optimal and efficient decision-making in the area of complete waste management.

4. Conclusions

In this section, we present the conclusion of the study following the results and its discussions. The main objective was to develop a basic two-dimensional cyclical dynamic model that reflects the plastic life cycle to predict the production of plastics and the wastes that are generated out of them. The model operated under the assumption of a closed system, which assumes no production of virgin plastics; this was to aid in determining the sustainability of plastic recycling and its activities in the long run. Due process was followed right from the development of the model, computation of parameters, fitting the model to a global data on annual plastic production and waste generation. The performance of the model was evaluated using the MAPE, after which they were used to predict and forecast over a period of 29 years.

The nature of the model conforms to the real-world representation of plastic production and its waste generation. The model is an integration of a decay exponential and a hyperbolic growth functions, whose long run equilibrium is zero, and thus, represents a real feature of the assumed closed system in the event of rapid incineration and discarding (mismanaged) rates, resulting in a lower rate of plastic recovery. The parameters of the model also reflected the reality and results of other researches on plastic waste management. The evaluation process of the model revealed its predictive accuracy, which represents the power of every predicting model. MAPE values of 13% and 18%, respectively, were obtained in respect of the annual plastic production and plastic waste generation; thereby indicating that the two models can predict with accuracy rates of 87% and 82% respectively. A comparison of this performance with the best performing Fourier series-based solid waste model developed in 2017 revealed the outperformance of our model. The predicted cumulative values of plastics produced, and plastic wastes generated from 1988 to 2015 were approximately 7.5 and 6.7 billion metric tonnes, respectively. The forecast values show that by 2050, a total (from 1988) of approximately 43.2 billion metric tonnes of plastic will be produced; and approximately 17.8 billion metric tonnes of plastic waste will be generated.

The models developed in this study derive their strength from their reflection of the plastic life cycle, higher predictive power, and the universality of the parameters. That albeit, the models are dogged with weaknesses which can be tied to the data and the highly unstable behaviour of the plastic waste generation model at the initial stages. However, this was stabilized overtime. To reduce these weaknesses, it is our candid suggestion that more parameters be included in further studies to reflect cutting-edge waste management practices across the globe.

The predictive ability of the model suggests that it can be relied upon by all stakeholders involved in waste management in making optimal decisions, planning, and policy formulation. Thus, the model has major policy implications for the transition towards achieving a higher degree of a plastic circular economy. We suggest that further researches in this direction of developing models for solid waste management should reflect the cyclical dynamics of the plastic life cycle and extend the models by including other applicable parameters that were omitted.

COMPETING INTERESTS DISCLAIMER:

Authors have declared that no competing interests exist. The products used for this research are commonly and predominantly use products in our area of research and country. There is absolutely no conflict of interest between the authors and producers of the products because we do not intend to use these products as an avenue for any litigation but for the advancement of knowledge. Also, the research was not funded by the producing company rather it was funded by personal efforts of the authors.

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