Original Research Article

$\mathcal{H}_\infty$ PID control for discrete-time network control systems with redundant channels under dynamic event-triggered scheme ☆

Abstract

This paper is concerned with the $\mathcal{H}_\infty$ proportional-integral-derivative (PID) control problem for a class of discrete-time network control systems (NCSs). First, a dynamic event-triggered control (DETC) scheme has been introduced to save the constrained network bandwidth. Moreover, in order to improve the reliability of network communication, a redundant channels transmission mechanism has been constructed during the transmission process. Then, with the aid of an appropriate Lyapunov function, some sufficient conditions are established to guarantee the exponentially stability and the prescribed $\mathcal{H}_\infty$ performance for the controlled system. Meanwhile, the gains of the PID controller can be derived by solving linear matrix inequalities (LMIs). Finally, a simulation example is presented to demonstrate the va-

☆This work was supported in part by the National Natural Science Foundation of China under Grants 11661028; the Natural Science Foundation of Guangxi, PR China under Grants 2020GXNSFAA159141.

*

lidity of the proposed method.

Keywords: Proportional-integral-derivative (PID) control; Redundant channels; Dynamic event-triggered control (DETC); $\mathcal{H}_\infty$ performance.

---

1. Introduction

In the development and application of automatic control, with the complexity of the practical systems, various control methods have been proposed. Among these control methods, the proportional-integral-derivative (PID) control is considered to be one of the most effective control methods. The success of the PID control method is mainly due to its simple structure, strong flexibility and easy adjustment [1]. Hence, the PID control problem has aroused widespread attention, and certain research results have been obtained, see [2]-[6]. Specifically, in [2], a new type PID-like neural network controller has been constructed by using a mix locally recurrent neural network for multivariable single-input/multi-ouppput system. A novel fuzzy PID control method combining the PID cntrol and the optimal fuzzy resoning model has been proposed in [5] to enhance the robustnesss of the control system. Most of the existing study on PID control issues is based on the assumption that the states of the system is fully available. However, in the actual industrial process, the system states is often not directly available. Therefore, it has become one of the most popular methods to obtain the measurement system state through the observer. In recent decades, a lot of results about complex dynamic network state measurement have been proposed [7]-[10]. Therefore, it is very meaningful to study the observer-based PID control in most network systems where the system states is unavailable.

For several decades, network control systems (NCSs) have been widely used in many fields for their advantages in reducing costs, saving energy, and improving flexibility and reliability. The signals between the various components of the NCSs are exchanged through some communication network medium [11]. However, the communication resources of these communication networks are usually limited. Specifically, the limited network bandwidth will inevitably cause network-induced phenomenon(e.g. data packet dropouts, data congestion and communication delays) and these behaviors have attracted considerable research attention, see [12]-[14]. Therefore, how to effectively save limited bandwidth resources and improve the utilization efficiency of network bandwidth has significant research value and attracted widespread attention in the field of control and signal processing [15]-[18]. In the past few years, event-triggered schemes have been extensively studied to reduce the communication burden [19]-[21]. Under the event-triggered schemes, data will be released only when the trigger conditions are met. It is worth noting that the above studies are based on static event-triggered schemes. However, in actual research, due to the existence of network-induced phenomenon and network delays, the data transmission rate may be time-varying, so the real-time states of bandwidth utilization should be considered. Therefore, a dynamic event-triggered control (DETC) scheme is proposed, which can dynamically adjust the threshold parameters according to the external environment. Recently, control issues based on the dynamic event-triggered schemes have begun to receive attention, see [22]-[26]. For example, in [22], under the framework of observer-based PID control system, a DETC scheme has been presented to improve resource utilization efficiency. The DETC scheme and the complex dynamic network synchronization controller has been integrated in [23], and the usefulness of the dynamic event-triggered synchronization control law has been confirmed by a simulation example. However, to the best of the authors knowledge, PID control based on dynamic event-triggered schemes has not been fully studied, which is one of our research motivations.

In addition to the aforementioned event-triggered schemes, introducing the transmission protocol in the data transmission process is another way to improve the reliability of network transmission. And the frequently-used communication protocols include stochastic communication (SC) protocol [27],[28], round-robin (RR) protocol [29],[30], try-once-discard (TOD) protocol [31]-[32] and redundant channel transmission (RCT) protocol [33]-[34]. In most existing networks, data is transmitted via a single channel. When a severe communication environment occurs on the network, the data transmitted in the channel may occur packet dropouts. In practical, however, two or more channels can be used at the same time to improve the reliability of communication services. Therefore, inserting redundant channels in network transmission can reduce the probability of packet dropouts. The key idea of the redundant channels transmission mechanism is that if the main channel suffers certain communication failure, other channels will be introduced for signal transmission to protect data transmission, thereby greatly improving the reliability of network communication. As an effective way to deal with data packet dropouts, redundant channels transmission has been widely adopted in networked evaluation/control systems. The redundant channels transmission mechanism has received special attention, and has achieved fruitful results in [35]-[39]. Specifically, in order to improve the reliability of data transmission, the redundant channels transmission mechanism has been employed for the singularly perturbed systems in [35]. By taking time-varying random delays into account, a novel state estimator has been designed in [36] for neural networks via redundant channels. However, a thorough literature search showed that the related research work has not been extended to the observer-based PID control problem under the DETC schemes, which constitutes another research motivation of ours.

To summarize the discussions above, this paper focuses on the $\mathcal{H}_\infty$ PID control problem for discrete-time network control systems with redundant channels under DETC scheme. The main contributions of this article are stressed as follows: (1)A new $\mathcal{H}_\infty$ PID control problem is addressed for the discrete-time NCSs where both the redundant channels transmission mechanism and the dynamical event-triggered scheme are considered; (2)Sufficient conditions are proposed to guarantee the exponetial stability as well as the prescribed $\mathcal{H}_\infty$ performance of the controlled systems; (3)Based on the Lyapunov stability theory and the matrix inequality approach, an easy-to-implement PID controller parameter design method is derived.

The organization of the rest of this paper is given as follows. Section 2 presents the main problem considered in this paper. The observer-based $\mathcal{H}_\infty$ PID control issue for discrete-time NCSs subject to redundant channels and the DETC scheme has been addressed in Section 3. Section 4 provides an example to examine the presented method. Finally, conclusions are drawn in Section 5.

Notations: $\mathbb{R}^{n\times m}$ denotes the set of all $n\times m$ real matrices. $E\{\cdot\}$ stands for the expectation and $diag\{\cdot\}$ refers to a block diagonal matrix. $\lambda_{min}(\cdot)$ and $\lambda_{max}(\cdot)$ denote, respectively, the minimal and maximal eignvalues of a matrix. I and 0 represent identity matrix and zero matrix respectively. $sym(Z) = Z + Z^T$. If there are no special instructions, the matrices are considered to have appropriate dimensions.

2. Preliminaries

Consider the following discrete-time network control systems (NCSs)

$$\begin{cases} x(k+1) = Ax(k) + Bu(k) + D\varpi(k) \\ z(k) = Fx(k) \end{cases} \tag{1}$$

where $x(k) \in \mathbb{R}^{n_x}$ is the state vector and $u(k) \in \mathbb{R}^{n_u}$ denotes the control input; $\varpi(k) \in (l_2[0,\infty), \mathbb{R}^{n_\omega})$ and $z(k) \in \mathbb{R}^{n_z}$ represents, respectively, the system noise and the control output. A, B, D, F are known constant matrices with suitable dimensions and assume the matrix B is of full column rank.

In networked systems, the single channel of data transmisson is often unreliable due to the existence of packet loss. Therefore, in order to reduce the probability of data packet loss, this article considers introducing the following redundant channels transmission mechanism as shown in Fig.1, and its mathematical model is as follows:

$$y(k) = \delta_1(k)C_1x(k) + \sum_{i=2}^{N}\left\{\prod_{j=1}^{i-1}(1-\delta_j(k))\delta_i(k)C_ix(k)\right\} \tag{2}$$

where $y(k) \in \mathbb{R}^{n_y}$ is the measurement output and C_i with $i \in [1, N]$ are know real matrices. The random variablies $\delta_i(k)(i = 1, \cdots, N)$ which denotes the randomly occurring packet dropout phenomenon for the ith channel, are mutally independent Bernoulli distributions with the following probailities:

$$Prob\{\delta_i(k) = 1\} = \bar{\delta}_i, \quad Prob\{\delta_i(k) = 0\} = 1 - \bar{\delta}_i.$$

where $\bar{\delta}_i \in [0, 1]$ are known constants.

Remark 2.1. *In order to reduce the probability of packeet loss, the redundant channels transmission protocol is introduced in this paper. Under this protocol, when* $\delta_1(k) = 1$ *implies that there is no packet loss occurred in the first channel, and the other channels will not be activated. Moreover, when* $\delta_i(k) = 0(i = 1, 2, \cdots, q-1)$ *and* $\delta_q(k) = 1$*, which implies that the packet loss happen from channel* 1 *to channel* $q-1$ *at time* k*, then the information will be transmitted through the* q*th channel. In particular, when* $\delta_i(k) = 0(i = 1, 2, \cdots, N)$*, which means that all channels are nont availiable. As such, compared with the traditional one-channel transmisson system, the probability of data packet loss is reduced from* $1 - \bar{\delta}_1$ *to* $\prod_{i=1}^{N}(1 - \bar{\delta}_i)$ *after imploying the redundant channels transmission protocol. Based on the above discussions, although the introduction of redundant channels in the data transmission process will increase the cost of equipment, the reliability of the data transmission is guaranteed. Therefore, in engineering practice, the number of redundant channels should be determined after weighing data reliability and cost.*

Now, considering the effect of redundant channels transmission mechanism, we introuce the following mathematical notations:

$$\delta(k) \triangleq \delta_1(k)C_1 + \sum_{i=2}^{N} \left\{ \prod_{j=1}^{i-1} (1 - \delta_j(k))\, \delta_i(k) C_i \right\} \tag{3}$$

$$\mathbb{E}\{\delta(k)\} = \bar{\delta}_1 C_1 + \sum_{i=2}^{N} \left\{ \prod_{j=1}^{i-1} \left(1 - \bar{\delta}_j\right) \bar{\delta}_i C_i \right\} \triangleq \bar{\delta} \tag{4}$$

Then, (2) can be represented by the following form:

$$y(k) = \delta(k)x(k) \tag{5}$$

In this paper, the DETC scheme is introduced to reduce the burden of the communication network. We employ the event-generator function $f(.,.,.)$ as follows:

$$f(\psi(k), \phi(k), \theta) = \psi^T(k)\psi(k) - \frac{1}{\sigma}\phi(k) - \theta y^T(k)y(k) \tag{6}$$

where $\psi(k) = y(k_t) - y(k)(k \in [k_t, k_{t+1}))$. k and k_t denote, respectively, the sampling instant and the latest triggered time; $\sigma > 0$ and $\theta > 0$ are given scalars. $\phi(k)$ is an internal dynamical variable satisfying

$$\begin{cases} \phi(k+1) = \lambda\phi(k) + \theta y^T(k)y(k) - \psi^T(k)\psi(k) \\ \phi(0) = 0 \end{cases} \tag{7}$$

where $\lambda \in (0, 1)$ is a given scalar.

It is obvious that once the triggering condition $f(\psi(k), \phi(k), \theta) > 0$ is satisfied, the measurement output $y(k)$ is sent to the observer. Let us define the triggering time as $0 < k_0 < k_1 < \cdots < k_t < \cdots$. Then the next transmitted instant k_{t+1} can be described as

$$k_{t+1} = min\left\{k \in N | k > k_t, f(\psi(k), \phi(k), \theta) > 0\right\} \tag{8}$$

Remark 2.2. *It can be observed from (6) that the triggering threshold is time-varying, which depends on the time-varying $\phi(k)$. Compared with the traditional static event-triggered scheme proposed in [19], the dynamic event-triggered scheme whose threshold parameters can be dynamically adjusted can* *better meet the engineering needs. In particularly, when σ approaching to infinity, the dynamic event-triggered scheme can be regarded as the traditional static event-triggered scheme proposed in [19]. Therefore, the proposed dynamic event-triggered method includes the static one as a special case.*

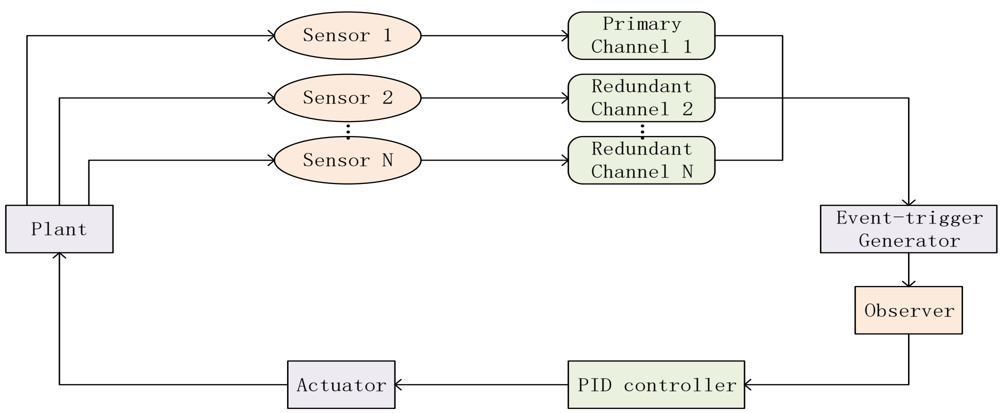

Figure 1: Discrete-time PID control system under redundant channels and DETC scheme.

In practice, it is difficult to directly obtain the states information of the system. Thus, the observer is designed to estimate the system states. The specific design is that

$$\hat{x}(k+1) = A\hat{x}(k) + Bu(k) + L(y(k_t) - \hat{y}(k)) \tag{9}$$

where $\hat{x}(k) \in \mathbb{R}^{n_x}$ is the estimate of $x(k)$ and $\hat{y}(k) \in \mathbb{R}^{n_y}$ is the estimate of $y(k)$, L is the observer gain.

Then, the considered observer-based PID controller is described as follows:

$$u(k) = K_P\hat{x}(k) + K_I \sum_{s=k-d}^{k-1} \hat{x}(s) + K_D(\hat{x}(k) - \hat{x}(k-1)) \tag{10}$$

where K_P, K_I, K_D are the controller gains and $d > 1$ is a give scalar representing the length of time window.

Remark 2.3. *Considering the fact that the state of system is often obtained directly unavailiable, this paper introduces an observer to measure the state of system. Moreover, due to the strong practicability of the PID controller in engineering practice, the observer-based PID controller has been designed. The PID controller consists of three parts: the proportional part (reflecting the present), the integral part (reflecting the past), and the derivative part (reflecting the future). In particular, a time window of finite length is applied in the integral part, which greatly reduces the computational burden.*

Defining $e(k) = x(k) - \hat{x}(k)$ as the estimation error. Then, according to (1) and (9), the estimation error system can be obtained as follows:

$$e(k+1) = (A - L\delta(k))e(k) + D\varpi(k) - L\psi(k) \tag{11}$$

Now, based on the system (11) and the observer-based PID control law (10), we derive the following closed-loop system:

$$\begin{cases} \mathcal{X}(k+1) = (\bar{A} + \tilde{A})\mathcal{X}(k) + \bar{B}\eta(k) + \bar{D}\varpi(k) + \bar{L}\psi(k) \\ z(k) = \bar{F}\mathcal{X}(k) \end{cases} \tag{12}$$

where

$$\mathcal{X}(k) = [x^T(k) \quad e^T(k)]^T, \quad \eta(k) = [\mathcal{X}^T(k-1) \quad \mathcal{X}^T(k-2) \cdots \mathcal{X}^T(k-d)]^T,$$

$$\bar{A} = \begin{bmatrix} A + B(K_P + K_D) & -B(K_P + K_D) \\ 0 & A - L\bar{\delta} \end{bmatrix}, \quad \tilde{A} = \begin{bmatrix} 0 & 0 \\ 0 & -L\tilde{\delta}(k) \end{bmatrix},$$

$$\bar{B} = \begin{bmatrix} B\bar{K} \\ 0 \end{bmatrix}, \quad \bar{D} = \begin{bmatrix} D \\ D \end{bmatrix}, \quad \bar{L} = \begin{bmatrix} 0 \\ -L \end{bmatrix}, \quad \bar{F} = \begin{bmatrix} F & 0 \end{bmatrix},$$

$$\bar{K} = \begin{bmatrix} \bar{K}_1 - \bar{K}_2 & \underbrace{\bar{K}_1 \quad \bar{K}_1 \cdots \bar{K}_1}_{d-1} \end{bmatrix}, \quad \bar{K}_1 = [K_I \quad -K_I],$$

$$\bar{K}_2 = [K_D \quad -K_D], \quad \tilde{\delta}(k) = \delta(k) - \bar{\delta}.$$

Definition 2.1. *The closed-loop system (12) with $\varpi(k) = 0$ is exponentially stable if there exist two scalars $\alpha(\alpha > 0)$ and $\beta(0 < \beta < 1)$, satisfying*

$$\mathbb{E}\left\{\|\mathcal{X}(k)\|^2\right\} \leq \alpha\beta^k \max_{-d\leq q\leq 0} \mathbb{E}\left\{\|\Psi(q)\|^2\right\} \tag{13}$$

Definition 2.2. *The considered system (1) under the DETC scheme and the redundant channels transmission mechanism is exponentially stable and satisfy a prescribed $\mathcal{H}_\infty$ performance index γ, if the following requirements are satisfied simultaneously*

(1)The considered system is exponentially stable in the sense of Definition (2.1).

(2)Under zero initial condition, for all nonzero $\varpi(k) \in l_2[0, \infty)$, there exist a scalar $\gamma > 0$, such that the controlled output $z(k)$ satisfies

$$\mathbb{E}\left\{\sum_{k=0}^{\infty} z^T(k)z(k)\right\} \leq \gamma^2\mathbb{E}\left\{\sum_{k=0}^{\infty} \varpi^T(k)\varpi(k)\right\} \tag{14}$$

Lemma 2.1. *[35] For stochastic varying matrix $\tilde{\delta}(k) = \delta(k) - \bar{\delta}$, a positive-definite matrix P and a real matrix M, it has:*

$$\mathbb{E}\{M\tilde{\delta}(k)\} = 0; \tag{15}$$

$$\mathbb{E}\{(M\tilde{\delta}(k))^T P(M\tilde{\delta}(k))\}$$

$$
\begin{aligned}
&= -\bar{\delta}^T M^T P M \bar{\delta} + \bar{\delta}_1 C_1^T M^T P M C_1 \\
&\quad + \sum_{i=2}^{N} \left\{ \prod_{j=1}^{i-1} (1-\bar{\delta}_j)\bar{\delta}_i C_i^T M^T P M C_i \right\}.
\end{aligned} \tag{16}
$$

Lemma 2.2. *[40] For given positive matrix Y, any matrix X and any scalar θ, the following inequality holds:*

$$
-X^T Y X \leq \theta^2 Y^{-1} - \theta X - \theta X^T \tag{17}
$$

Lemma 2.3. *[41] For positive definite matrix $R \in \mathbb{R}^{n_x \times n_x}$, and vectors $x(x > 0)$, $y(y > 0)$, it has:*

$$
2x^T y \leq x^T R x + y^T R^{-1} y \tag{18}
$$

Lemma 2.4. *[42] Suppose that the matrix Q with appropriate dimension, the following two items are equivalent:*

1. There exist two symmetric and positive-definitive matrices X, Y such that

$$
\begin{bmatrix} -X & * \\ Q & -Y^{-1} \end{bmatrix} < 0
$$

2. There exist two symmetric and positive-definitive matrices X, Y, and constant matrix Z satisfying

$$
\begin{bmatrix} -X & * \\ ZQ & sym(-Z) + Y \end{bmatrix} < 0
$$

Lemma 2.5. *[43] For the DETC scheme consists of (6) and (7) with the initial value $\phi(0) \geq 0$, the internal dynamic variable satisfies $\phi(k) \geq 0$ for all $k \geq 0$ if the parameters $\sigma(\sigma > 0)$ and $\lambda(0 < \lambda < 1)$ satisfy $\lambda\sigma \geq 1$.*

3. Main Results

In this section, sufficient conditions are given to guarantee the $\mathcal{H}_\infty$ performance requirement for designed systems (1) under DETC scheme and redundant channels transmission mechanism.

Theorem 3.1. *Consider the matrices K_P, K_I, K_D and L are given. Asumme that the parameters $\sigma(\sigma > 0)$ and $\lambda(0 < \lambda < 1)$ satisfy $\lambda\sigma \geq 1$. If there exist positive scalar θ, τ, positive definite matrices P, $Q_i(i = 1, 2, \cdots, d)$ satisfying the following matrix inequalities:*

$$\bar{\Pi} = \begin{bmatrix} \bar{\Pi}_{11} & * \\ \bar{\Pi}_{21} & \bar{\Pi}_{22} \end{bmatrix} < 0 \tag{19}$$

where

$$\bar{\Pi}_{11} = \begin{bmatrix} \bar{Q} + \bar{\tau}\hat{C} - P + \Sigma & * & * & * \\ 0 & -Q & * & * \\ 0 & 0 & -(\frac{1}{\sigma} + \tau)I_{n_y} & * \\ 0 & 0 & 0 & \frac{\lambda+\tau-1}{\sigma}I_1 \end{bmatrix},$$

$$\bar{\Pi}_{21} = \begin{bmatrix} P\bar{A} & P\bar{B} & P\bar{L} & 0 \\ P\Lambda_1 & 0 & 0 & 0 \\ P\Lambda_2 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots \\ P\Lambda_N & 0 & 0 & 0 \end{bmatrix},$$

$$\bar{\Pi}_{22} = diag\left\{\underbrace{-P, -P, \cdots, -P}_{N+1}\right\}, \ \bar{Q} = \sum_{i=1}^{d} Q_i, \ \Sigma = \Sigma_1 - \Sigma_2 - \Sigma_3,$$

$$\Sigma_1 = \begin{bmatrix} 0 & * \\ 0 & \hat{P} \end{bmatrix}, \ \Sigma_2 = \begin{bmatrix} 0 & * \\ 0 & \bar{\delta}^T L^T \hat{P} \end{bmatrix}, \ \Sigma_3 = \begin{bmatrix} 0 & * \\ 0 & \hat{P}L\bar{\delta} \end{bmatrix},$$

$$P = diag\{\hat{P}, \quad \hat{P}\}, \quad Q = diag\left\{Q_1, \quad Q_2, \cdots, Q_d\right\},$$

$$\hat{C} = \begin{bmatrix} -\bar{\delta}^T\bar{\delta} + \bar{\delta}_1 C_1^T C_1 + \sum_{i=2}^{N}\left\{\prod_{j=1}^{i-1}(1-\bar{\delta}_j)\bar{\delta}_i C_i^T C_i\right\} & * \\ 0 & 0 \end{bmatrix},$$

$$\bar{\tau} = \theta(\frac{1}{\sigma} + \tau), \quad \Lambda_1 = \begin{bmatrix} 0 & * \\ 0 & \sqrt{\bar{\delta}_1} L C_1 \end{bmatrix},$$

$$\Lambda_n = \begin{bmatrix} 0 & * \\ 0 & \sqrt{\prod_{j=1}^{n-1}(1-\bar{\delta}_j)\bar{\delta}_n} L C_n \end{bmatrix}, \; n = 2, 3, \cdots, N.$$

then the closed-loop system (12) is exponentially stable and satisfies the prescribed $\mathcal{H}_\infty$ performance index.

Proof. First, we choose the following Lyapunov functional:

$$V(k) = \sum_{i=1}^{3} V_i(k) \tag{20}$$

where

$$\begin{aligned} V_1(k) &= \mathcal{X}^T(k) P \mathcal{X}(k) \\ V_2(k) &= \sum_{i=1}^{d}\sum_{q=k-i}^{k-1} \mathcal{X}^T(q) Q_i \mathcal{X}(q) \\ V_3(k) &= \frac{1}{\sigma}\phi(k). \end{aligned}$$

Then, according to the state evolution of the system (12), calculating the difference of $V(k)$, and taking the mathematical expectation, we have

$$\begin{aligned} \mathbb{E}\left\{\triangle V_1(k)\right\} =& \mathbb{E}\left\{\mathcal{X}^T(k+1) P \mathcal{X}(k+1) - \mathcal{X}^T(k) P \mathcal{X}(k)\right\} \\ =& \mathbb{E}\{((\bar{A}+\tilde{A})\mathcal{X}(k) + \bar{B}\eta(k) + \bar{D}\varpi(k) + \bar{L}\psi(k))^T P((\bar{A}+\tilde{A})\mathcal{X}(k) \\ &+ \bar{B}\eta(k) + \bar{D}\varpi(k) + \bar{L}\psi(k)) - \mathcal{X}^T(k) P \mathcal{X}(k)\} \end{aligned}$$

$$
\begin{aligned}
=&\mathbb{E}\{\mathcal{X}^T(k)(\bar{A}^TP\bar{A}+2\bar{A}^TP\tilde{A}+\tilde{A}^TP\tilde{A}-P)\mathcal{X}(k)\\
&+\eta^T(k)\bar{B}^TP\bar{B}\eta(k)+\varpi^T(k)\bar{D}^TP\bar{D}\varpi(k)+\psi^T(k)\bar{L}^TP\bar{L}\psi(k)\\
&+2\mathcal{X}(\bar{A}+\tilde{A})^TP\bar{B}\eta(k)+2\mathcal{X}^T(k)(\bar{A}+\tilde{A})^TP\bar{D}\varpi(k)\\
&+2\mathcal{X}^T(k)(\bar{A}+\tilde{A})^TP\bar{L}\psi(k)+2\eta^T(k)\bar{B}^TP\bar{D}\varpi(k)\\
&+2\eta^T(k)\bar{B}^TP\bar{L}\psi(k)+2\varpi^T(k)\bar{D}^TP\bar{L}\psi(k)\}
\end{aligned}
\tag{21}
$$

$$
\begin{aligned}
\mathbb{E}\left\{V_2(k)\right\}=&\mathbb{E}\left\{\sum_{i=1}^{d}\sum_{q=k+1-i}^{k}\mathcal{X}^T(q)Q_i\mathcal{X}(q)-\sum_{i=1}^{d}\sum_{q=k-i}^{k-1}\mathcal{X}^T(q)Q_i\mathcal{X}(q)\right\}\\
=&\mathbb{E}\left\{\sum_{i=1}^{d}\mathcal{X}^T(k)Q_i\mathcal{X}(k)-\sum_{i=1}^{d}\mathcal{X}^T(k-i)Q_i\mathcal{X}(k-i)\right\}
\end{aligned}
\tag{22}
$$

$$
\begin{aligned}
\mathbb{E}\left\{V_3(k)\right\}=&\mathbb{E}\left\{\frac{1}{\sigma}\phi(k+1)-\frac{1}{\sigma}\phi(k)\right\}\\
=&\mathbb{E}\left\{\frac{1}{\sigma}(\lambda\phi(k)+\theta y^T(k)y(k)-\psi^T(k)\psi(k)-\phi(k))\right\}\\
=&\mathbb{E}\left\{\frac{\lambda-1}{\sigma}\phi(k)+\frac{\theta}{\sigma}x^T(k)\delta^T(k)\delta(k)x(k)-\frac{1}{\sigma}\psi^T(k)\psi(k)\right\}\\
=&\mathbb{E}\left\{\frac{\lambda-1}{\sigma}\bar{\phi}^T(k)\bar{\phi}(k)+\frac{\theta}{\sigma}\mathcal{X}(k)^T\bar{C}^T\bar{C}\mathcal{X}(k)-\frac{1}{\sigma}\psi^T(k)\psi(k)\right\}
\end{aligned}
\tag{23}
$$

where

$$
\bar{\phi}(k)=\phi^{\frac{1}{2}}(k),\quad \bar{C}=[\delta(k)\quad 0].
$$

According to the dynamical event-triggered condition (6), we can obtain

$$
\tau(-\psi^T(k)\psi(k)+\frac{1}{\sigma}\phi(k)+\theta y^T(k)y(k))\geq 0 \tag{24}
$$

where $\tau>0$ is a given scalar.

Letting $\varpi = 0$, by combining (20)-(24), it follows from Lemma(2.1) that

$$\begin{aligned}
\mathbb{E}\{\triangle V(k)\} =& \mathbb{E}\left\{\sum_{i=1}^{3} \triangle V_i(k)\right\} \\
\leq& \mathbb{E}\{\mathcal{X}^T(k)(\bar{A}^T P\bar{A} + \tilde{A}^T P\tilde{A} - P)\mathcal{X}(k) + \eta^T(k)\bar{B}^T P\bar{B}\eta(k) \\
&+ \psi^T(k)\bar{L}^T P\bar{L}\psi(k) + 2\mathcal{X}^T(k)\bar{A}^T P\bar{B}\eta(k) \\
&+ 2\mathcal{X}^T(k)\bar{A}^T P\bar{L}\psi(k) + 2\eta^T(k)\bar{B}^T P\bar{L}\psi(k) \\
&+ \sum_{i=1}^{d} \mathcal{X}^T(k)Q_i\mathcal{X}(k) - \sum_{i=1}^{d} \mathcal{X}^T(k-i)Q_i\mathcal{X}(k-i) \\
&+ \frac{\lambda - 1}{\sigma}\bar{\phi}^T(k)\bar{\phi}(k) + \frac{\theta}{\sigma}\mathcal{X}^T(k)\bar{C}^T\bar{C}\mathcal{X}(k) - \frac{1}{\sigma}\psi^T(k)\psi(k) \\
&+ \tau(-\psi^T(k)\psi(k) + \frac{1}{\sigma}\bar{\phi}^T(k)\bar{\phi}(k) + \theta y^T(k)y(k))\} \\
=& \Phi_1^T(k)(\bar{\Pi}_{11}^* + \Theta_1^T P^{-1}\Theta_1)\Phi_1(k) \qquad (25)
\end{aligned}$$

where

$$\Phi_1(k) = \begin{bmatrix} \mathcal{X}^T(k) & \eta^T(k) & \psi^T(k) & \bar{\phi}^T(k) \end{bmatrix}^T,$$

$$\bar{\Pi}_{11}^* = \begin{bmatrix} \bar{Q} + \bar{\tau}\hat{C} - P + \hat{\Sigma} & * & * & * \\ 0 & -Q & * & * \\ 0 & 0 & -(\frac{1}{\sigma} + \tau)I_{n_y} & * \\ 0 & 0 & 0 & \frac{\lambda+\tau-1}{\sigma}I_1 \end{bmatrix},$$

$$\Theta_1 = \begin{bmatrix} P\bar{A} & P\bar{B} & P\bar{L} & 0 \end{bmatrix}.$$

in which

$$\begin{aligned}
\hat{C} =& \mathbb{E}\{\bar{C}^T\bar{C}\} \\
=& \begin{bmatrix} -\bar{\delta}^T\bar{\delta} + \delta_1 C_1^T C_1 + \sum_{i=2}^{N}\left\{\prod_{j=1}^{i-1}(1-\bar{\delta}_j)\bar{\delta}_i C_i^T C_i\right\} & * \\ 0 & 0 \end{bmatrix},
\end{aligned}$$

$$\hat{\Sigma} = \mathbb{E}\{\tilde{A}^T P \tilde{A}\} = \begin{bmatrix} 0 & * \\ 0 & -\bar{\delta}^T L^T \hat{P} L \bar{\delta} \end{bmatrix} + \Lambda,$$

$$\Lambda = \begin{bmatrix} 0 & * \\ 0 & \delta_1 C_1^T L^T \hat{P} L C_1 + \sum_{i=2}^{N} \left\{ \prod_{j=1}^{i-1} (1 - \bar{\delta}_j) \bar{\delta}_i C_i^T L^T \hat{P} L C_i \right\} \end{bmatrix}.$$

For matrix $\hat{\Sigma}$, it follows from Lemma(2.2) that

$$- \bar{\delta}^T L^T \hat{P} L \bar{\delta} \leq \hat{P} - \bar{\delta}^T L^T \hat{P} - \hat{P} L \bar{\delta} \tag{26}$$

which implies that

$$\hat{\Sigma} \leq \Sigma + \Lambda. \tag{27}$$

Then, by further utilizing the Schur complement lemma, it is clear that $\mathbb{E}\{\triangle V(k)\} < 0$ can be ensured by (19).

From (25), we know that there exists a sufficient small scalar $\ell > 0$ such that the following inequality holds:

$$\bar{\Pi}_{11}^* + \Theta_1^T P^{-1} \Theta_1 + \ell diag\{I, 0, 0, I\} < 0 \tag{28}$$

which implies that

$$\mathbb{E}\{\triangle V(k)\} < -\ell \mathbb{E} \left\{ \left\| \Psi^T(k) \right\|^2 \right\} \tag{29}$$

where

$$\Psi(k) = \begin{bmatrix} \mathcal{X}^T(k) & \bar{\phi}^T(k) \end{bmatrix}^T.$$

In the following, we shall proceed to deal with the exponential stability analysis of the closed-loop system (12). According to the definition of $V(k)$, we have

$$\mathbb{E}\{V(k)\} \leq \mathbb{E} \left\{ a \left\| \Psi(k) \right\|^2 + \bar{d} \sum_{q=k-d}^{k-1} \left\| \Psi(q) \right\|^2 \right\} \tag{30}$$

where

$$a = max\{\lambda_{max}(P), \quad \frac{1}{\sigma}\}, \quad \bar{d} = d\lambda_{max}(Q).$$

Then, for any $r > 0$, it follows from (29) and (30) that

$$\begin{aligned}
&\mathbb{E}\left\{r^{k+1}V(k+1)\right\} - \mathbb{E}\left\{r^{k}V(k)\right\} \\
&= r^{k+1}\mathbb{E}\left\{\triangle V(k)\right\} + r^{k}(r-1)\mathbb{E}\left\{V(k)\right\} \\
&\leq b_1(r)r^{k}\mathbb{E}\left\{\|\Psi(k)\|^2\right\} + b_2(r)\sum_{q=k-d}^{k-1} r^{k}\mathbb{E}\left\{\|\Psi(q)\|^2\right\} \qquad (31)
\end{aligned}$$

where

$$b_1(r) = -\ell r + (r-1)a, \quad b_2(r) = (r-1)\bar{d}.$$

Futhermore, for any $m \geq d+1$, summing up both sides of (31) from 0 to $m-1$ with respect to k, we obtain

$$\begin{aligned}
\mathbb{E}\{r^{m}V(m)\} - \mathbb{E}\{V(0)\} \leq & b_1(r)\sum_{k=0}^{m-1} r^{k}\mathbb{E}\left\{\|\Psi(k)\|^2\right\} \\
& + b_2(r)\sum_{k=0}^{m-1}\sum_{q=k-d}^{k-1} r^{k}\mathbb{E}\left\{\|\Psi(q)\|^2\right\} \qquad (32)
\end{aligned}$$

The last term in (32) can be computed as

$$\begin{aligned}
&\sum_{k=0}^{m-1}\sum_{q=k-d}^{k-1} r^{k}\mathbb{E}\left\{\|\Phi(q)\|^2\right\} \\
&\leq (\sum_{q=-d}^{-1}\sum_{k=0}^{q+d} + \sum_{q=0}^{m-d-1}\sum_{k=q+1}^{q+d} + \sum_{q=m-d}^{m-1}\sum_{k=q+1}^{m-1}) r^{k}\mathbb{E}\left\{\|\Phi(q)\|^2\right\} \\
&\leq d\sum_{q=-d}^{-1} r^{q+d}\mathbb{E}\left\{\|\Phi(q)\|^2\right\} + d\sum_{q=0}^{m-d-1} r^{q+d}\mathbb{E}\left\{\|\Phi(q)\|^2\right\} \\
&\quad + d\sum_{q=m-d}^{m-1} r^{q+d}\mathbb{E}\left\{\|\Phi(q)\|^2\right\}
\end{aligned}$$

$$\leq dr^d \max_{-d\leq q\leq 0} \mathbb{E}\left\{\|\Phi(q)\|^2\right\} + dr^d \sum_{k=0}^{m-1} r^k \mathbb{E}\left\{\|\Phi(k)\|^2\right\} \tag{33}$$

Substituting the (33) into (32) results in

$$\begin{aligned}\mathbb{E}\{r^m V(m)\} - \mathbb{E}\{V(0)\} \leq & \pi_1(r) \sum_{k=0}^{m-1} r^k \mathbb{E}\left\{\|\Psi(k)\|^2\right\} \\ & + \pi_2(r) \max_{-d\leq q\leq 0} \mathbb{E}\left\{\|\Phi(q)\|^2\right\}\end{aligned} \tag{34}$$

where

$$\pi_1(r) = b_1(r) + dr^d b_2(r), \quad \pi_2(r) = dr^d b_2(r).$$

It is clear that

$$\mathbb{E}\left\{V(m)\right\} \geq g_1 \mathbb{E}\left\{\|\Psi(m)\|^2\right\}, \tag{35}$$

$$\mathbb{E}\left\{V(0)\right\} \leq g_2 \max_{-d\leq q\leq 0} \mathbb{E}\left\{\|\Phi(q)\|^2\right\}. \tag{36}$$

where

$$g_1 = min\{\lambda_{min}(P),\ \frac{1}{\sigma}\}, \quad g_2 = max\{\lambda_{max}(P),\ d\lambda_{max}(Q)\}.$$

Duing to the fact that $\pi_1(1) = -\ell < 0$ and $\lim_{r\to\infty} = +\infty$, there exists a scalar $r_0 > 1$ such that $\pi_1(r_0) = 0$. Then, considering (34)-(36), we have

$$\begin{aligned}\mathbb{E}\left\{\|\mathcal{X}(m)\|^2\right\} \leq & \mathbb{E}\left\{\|\Psi(m)\|^2\right\} \\ & \leq \frac{1}{r_0^m} \frac{g_2 + dr_0^d b_2(r_0)}{g_1} \max_{-d\leq q\leq 0} \mathbb{E}\left\{\|\Phi(q)\|^2\right\}.\end{aligned} \tag{37}$$

Denoting $k = m$, $\alpha = (g_2 + dr_0^d b_2(r_0))/g_1$, $\beta = 1/r_0$ and considering $\bar{\phi}(q) = 0(-d \leq q \leq 0)$, from Definition 2.1, we can know that the system (1) is exponentially stable. The proof is complete.

Now, we are ready to deal with the $\mathcal{H}_\infty$ performance analysis issue for the closed-loop system (12).

Theorem 3.2. *Assume that the parameters $\sigma(\sigma > 0)$ and $\lambda(0 < \lambda < 1)$ satisfy $\lambda\sigma \geq 1$. And the $\mathcal{H}_\infty$ performance index $\gamma > 0$, the matrices K_P, K_I, K_D and L are given. Then, the closed-loop system (12) is exponentially stable with the prescribed $\mathcal{H}_\infty$ performance if there exist positive scalar θ, τ, and positive definite matrices P, $Q_i (i = 1, 2, \cdots, d)$ satisfying*

$$\hat{\Pi} = \begin{bmatrix} \hat{\Pi}_{11} & * \\ \hat{\Pi}_{21} & \hat{\Pi}_{22} \end{bmatrix} < 0 \tag{38}$$

where

$$\hat{\Pi}_{11} = \begin{bmatrix} \bar{Q} + \bar{\tau}\hat{C} - P + \mathcal{F}^T\mathcal{F} + \Sigma & * & * & * & * \\ 0 & -Q & * & * & * \\ 0 & 0 & -(\frac{1}{\sigma} + \tau)I_{n_y} & * & * \\ 0 & 0 & 0 & \frac{\lambda+\tau+1}{\sigma}I_1 & * \\ 0 & 0 & 0 & 0 & -\gamma^2 I_\omega \end{bmatrix},$$

$$\hat{\Pi}_{21} = \begin{bmatrix} P\bar{A} & P\bar{B} & P\bar{L} & 0 & P\bar{D} \\ P\Lambda_1 & 0 & 0 & 0 & 0 \\ P\Lambda_2 & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ P\Lambda_N & 0 & 0 & 0 & 0 \end{bmatrix}, \quad \hat{\Pi}_{22} = \bar{\Pi}_{22}.$$

and other parameters are defined in Theorem 3.1.

Proof. For any $\varpi(k) \neq 0$, considering the (20)-(24), we have

$$\begin{aligned} &\mathbb{E}\{\triangle V(k)\} + \mathbb{E}\{Z^T(k)Z(k)\} - \gamma^2\mathbb{E}\{\varpi^T(k)\varpi(k)\} \\ \leq &\mathbb{E}\{\mathcal{X}^T(k)(\bar{A}^T P\bar{A} + \tilde{A}^T P\tilde{A} - P)\mathcal{X}(k) + \eta^T(k)\bar{B}^T P\bar{B}\eta(k) \\ &+ \varpi^T(k)\bar{D}^T P\bar{D}\varpi(k) + \psi^T(k)\bar{L}^T P\bar{L}\psi(k) + 2\mathcal{X}^T(k)\bar{A}^T P\bar{B}\eta(k) \end{aligned}$$

$$
\begin{aligned}
&+2\mathcal{X}^T(k)\bar{A}^T P\bar{L}\psi(k) + 2\mathcal{X}^T(k)\bar{A}^T P\bar{D}\varpi(k) + 2\eta^T(k)\bar{B}^T P\bar{L}\psi(k)\\
&+2\eta^T(k)\bar{B}^T P\bar{D}\varpi(k) + 2\varpi^T(k)\bar{D}^T P\bar{L}\psi(k) + \sum_{i=1}^{d}\mathcal{X}^T(k)Q_i\mathcal{X}(k)\\
&-\sum_{i=1}^{d}\mathcal{X}^T(k-i)Q_i\mathcal{X}(k-i) + \frac{\lambda-1}{\sigma}\bar{\phi}^T(k)\bar{\phi}(k) + \frac{\theta}{\sigma}\mathcal{X}^T(k)\bar{C}^T\bar{C}\mathcal{X}(k)\\
&-\frac{1}{\sigma}\psi^T(k)\psi(k) + \tau(-\psi^T(k)\psi(k) + \frac{1}{\sigma}\bar{\phi}^T(k)\bar{\phi}(k) + \theta y^T(k)y(k))\\
&+\mathcal{X}^T(k)(\mathcal{F}^T\mathcal{F})\mathcal{X}(k) - \gamma^2\varpi^T(k)\varpi(k)\}\\
=&\Phi_2^T(k)(\hat{\Pi}_{11}^* + \Theta_2^T P^{-1}\Theta_2)\Phi_2(k) \qquad (39)
\end{aligned}
$$

where

$$
\Phi_2(k) = \begin{bmatrix}\mathcal{X}^T(k) & \eta^T(k) & \psi^T(k) & \bar{\phi}^T(k) & \varpi^T(k)\end{bmatrix}^T,
$$

$$
\hat{\Pi}_{11}^* = \begin{bmatrix}
\bar{Q} + \bar{\tau}\hat{C} - P + \mathcal{F}^T\mathcal{F} + \hat{\Sigma} & * & * & * & * \\
0 & -Q & * & * & * \\
0 & 0 & -(\frac{1}{\sigma}+\tau)I_{n_y} & * & * \\
0 & 0 & 0 & \frac{\lambda+\tau-1}{\sigma}I_1 & * \\
0 & 0 & 0 & 0 & -\gamma^2 I_{n_\omega}
\end{bmatrix}
$$

$$
\Theta_2 = \begin{bmatrix}P\bar{A} & P\bar{B} & P\bar{L} & 0 & P\bar{D}\end{bmatrix}.
$$

Then, by using the Schur complement lemma, it follows from (38) that $\hat{\Pi}_{11}^* + \Theta_2^T P^{-1}\Theta_2 < 0$, which means

$$
\mathbb{E}\left\{\triangle V(k)\right\} + \mathbb{E}\left\{Z^T(k)Z(k)\right\} - \gamma^2\mathbb{E}\left\{\varpi^T(k)\varpi(k)\right\} < 0 \qquad (40)
$$

Moreover, summing up both sides of (40) frrom 0 to ∞ with respect to k, we can obtain

$$
\mathbb{E}\left\{\sum_{k=0}^{\infty} Z^T(k)Z(k)\right\} - \gamma^2\mathbb{E}\left\{\sum_{k=0}^{\infty}\varpi^T(k)\varpi(k)\right\} < \mathbb{E}\left\{V(0)\right\} - \mathbb{E}\left\{V(\infty)\right\} \quad (41)
$$

Noting that $V(0) = 0$ and $V(\infty) \geq 0$, we have

$$\mathbb{E}\left\{\sum_{k=0}^{\infty} Z^T(k)Z(k)\right\} \leq \gamma^2 \mathbb{E}\left\{\sum_{k=0}^{\infty} \varpi^T(k)\varpi(k)\right\} \tag{42}$$

The proof is accomplished.

Finally, the designed PID controller and observer are proposed in the following theorem.

Theorem 3.3. *Let the parameters $\sigma(\sigma > 0)$ and $\lambda(0 < \lambda < 1)$ satisfy $\lambda\sigma \geq 1$ and the $\mathcal{H}_\infty$ performance index $\gamma > 0$ be given. Assume that there exist positive scalar θ, τ and positive definite matrices P, $Q_i(i = 1, 2, \cdots, d)$, X satisfying*

$$\begin{bmatrix} \tilde{\Pi} + E_1 Y E_1^T & * & * \\ B^T B \tilde{K}_P E_2 & sym{-}B^T B X & * \\ 0 & \hat{P}B - BX & -Y \end{bmatrix} < 0 \tag{43}$$

where

$$\tilde{\Pi} = \begin{bmatrix} \tilde{\Pi}_{11} & * \\ \tilde{\Pi}_{21} & \tilde{\Pi}_{22} \end{bmatrix}, \quad \tilde{\Pi}_{21} = \begin{bmatrix} \hat{A} & \hat{B} & \hat{L} & 0 & \hat{D} \\ \hat{\Lambda}_1 & 0 & 0 & 0 & 0 \\ \hat{\Lambda}_2 & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \hat{\Lambda}_N & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\tilde{\Pi}_{11} = \begin{bmatrix} \bar{Q} + \bar{\tau}\hat{C} - P + \mathcal{F}^T\mathcal{F} + \tilde{\Sigma} & * & * & * & * \\ 0 & -Q & * & * & * \\ 0 & 0 & -(\frac{1}{\sigma} + \tau)I_{n_y} & * & * \\ 0 & 0 & 0 & \frac{\lambda+\tau+1}{\sigma} I_1 & * \\ 0 & 0 & 0 & 0 & -\gamma^2 I_{n_\omega} \end{bmatrix}$$

$$\tilde{\Pi}_{22} = \bar{\Pi}_{22}, \quad \tilde{\Sigma} = \Sigma_1 - \tilde{\Sigma}_2 - \tilde{\Sigma}_3, \quad \tilde{\Sigma}_2 = \begin{bmatrix} 0 & 0 \\ 0 & \bar{\delta}^T \tilde{L}^T \end{bmatrix}, \quad \tilde{\Sigma}_3 = \begin{bmatrix} 0 & 0 \\ 0 & \tilde{L}\bar{\delta} \end{bmatrix},$$

$$\hat{A} = \begin{bmatrix} \hat{P}A + B(\tilde{K}_P + \tilde{K}_D) & -B(\tilde{K}_P + \tilde{K}_D) \\ 0 & \hat{P}A - \tilde{L}\bar{\delta} \end{bmatrix}, \quad \hat{B} = \begin{bmatrix} B\hat{K} \\ 0 \end{bmatrix},$$

$$\hat{D} = \begin{bmatrix} \hat{P}D \\ \hat{P}D \end{bmatrix}, \quad \hat{L} = \begin{bmatrix} 0 \\ -\tilde{L} \end{bmatrix}, \quad \hat{K} = \begin{bmatrix} \hat{K}_1 - \hat{K}_2 & \underbrace{\hat{K}_1 \quad \hat{K}_1 \cdots \hat{K}_1}_{d-1} \end{bmatrix},$$

$$\hat{K}_1 = \begin{bmatrix} \tilde{k}_I, & -\tilde{K}_I \end{bmatrix}, \quad \hat{K}_2 = \begin{bmatrix} \tilde{K}_D & -\tilde{K}_D \end{bmatrix}, \quad \hat{\Lambda}_1 = \begin{bmatrix} 0 & * \\ 0 & \sqrt{\bar{\delta}_1}\tilde{L}C_1 \end{bmatrix},$$

$$\hat{\Lambda}_n = \begin{bmatrix} 0 & * \\ 0 & \sqrt{\prod_{j=1}^{n-1}(1-\bar{\delta}_j)\bar{\delta}_n}\tilde{L}C_n \end{bmatrix}, \ n = 2, 3, \cdots, N,$$

$$E_1^T = \begin{bmatrix} I_{n_x} & 0_{n_x \times (2(N+d+1)n_x + n_x + 2n_y + n_\omega + 1)} \end{bmatrix},$$

$$E_2 = \begin{bmatrix} 0_{n_x \times 2n_x} & I_{n_x} & 0_{n_x \times (2(N+d)n_x + n_x + 2n_y + n_\omega + 1)} \end{bmatrix}.$$

and other parameters are defined in Theorem 3.1 and Theorem 3.2. Moreover, if the inequality (43) is available, the desired PID control gains and observer gain are given by:

$$K_P = X^{-1}\tilde{K}_P, \quad K_I = X^{-1}\tilde{K}_I, \quad K_D = X^{-1}\tilde{K}_D, \quad L = \hat{P}^{-1}\tilde{L}. \tag{44}$$

Proof. The inequality (38) is equivalent to

$$\tilde{\Pi} + sym\{E_1(\hat{P}B - BX)(X^{-1}\tilde{K}_P)E_2\} < 0 \tag{45}$$

From Lemma 2.3, it is clear that there exist a positive definite matrix $Y \in \mathbb{R}^{n_x \times n_x}$ satisfying

$$\tilde{\Pi} + E_1 Y E_1^T + E_2^T (X^{-1}\tilde{K}_P)^T (\hat{P}B - BX)^T Y^{-1} (\hat{P}B - BX)(X^{-1}\tilde{K}_P) E_2 < 0 \tag{46}$$

According to the Schur complement, the above inequality is equivalent to

$$\begin{bmatrix} \tilde{\Pi} + E_1 Y E_1^T & * \\ (X^{-1}\tilde{K}_P)E_2 & -\Omega \end{bmatrix} < 0 \tag{47}$$

where $\Omega = (\hat{P}B - BX)^T Y^{-1}(\hat{P}B - BX)$.

Then, based on the Lemma 2.4, we have

$$\begin{bmatrix} \tilde{\Pi} + E_1 Y E_1^T & * \\ B^T B\tilde{K}_P E_2 & sym\{-B^T BX\} - \Omega \end{bmatrix} < 0 \tag{48}$$

Finally, by further using the Schur complement lemma, it is obvisous that (43) can be ensured by (48). Thus, the proof is now complete.

Remark 3.1. *It is worth noting that Theorem 3.3 derives a LMI-based solution to co-design the observer and the PID controller for the discrete-time NCSs under the redundant channel transmission protocol and the DETC scheme. As such, the conservative effect of the results has been reduced. Moreover, compared with other linearization methods, the linearization method used in Theorem 3.3 can directly calculate the variable X matrix in the L-MI, so as to quickly obtain the PID controller gain and greatly reduce the computational complexity.*

4. Examples

In this section, a numerical simulation example is presented to illustrate the effectiveness of proposed observer-based PID controller design scheme.

Consider system (1) with the following parameters:

$$A = \begin{bmatrix} -0.02 & 0.1 \\ 0.01 & -0.02 \end{bmatrix}, \quad B = \begin{bmatrix} -0.2 & 0.03 \\ 0.2 & 0.03 \end{bmatrix},$$

$$D = \begin{bmatrix} -0.03 \\ 0.1 \end{bmatrix}, \quad F = \begin{bmatrix} 0.2 & -0.3 \end{bmatrix}.$$

In this example, the redundant channels' number is $N = 3$. The probabilities of successfully transmitting data packets on three different channels are taken as $\delta_1 = 0.8$, $\delta_2 = 0.7$ and $\delta_3 = 0.6$. The measurement matrices are

$$C_1 = \begin{bmatrix} 0.1 & 0.02 \\ 0.03 & -0.1 \\ 0.1 & 0.01 \\ 0.04 & -0.1 \end{bmatrix}, \quad C_2 = \begin{bmatrix} 0.2 & 0.01 \\ 0.02 & -0.2 \\ 0.2 & 0.03 \\ 0.01 & -0.2 \end{bmatrix}, \quad C_3 = \begin{bmatrix} 0.3 & 0.03 \\ 0.01 & -0.3 \\ 0.3 & 0.02 \\ 0.01 & -0.3 \end{bmatrix}.$$

The target for us is to design a state observer like (9) and a PID controller like (10) such that the controller closed-loop systems exponetially stable and achieve the desired performace index $\gamma = 1$. The other relevant parameters are given as $\theta = 0.2$, $\sigma = 5$, $\tau = 0.3$, $\lambda = 0.6$, $d = 2$.

By solving the linear matrix inequality in Theorem(3.3), we have:

$$P = \begin{bmatrix} 4.1659 & 0.0905 & 0 & 0 \\ 0.0905 & 4.5781 & 0 & 0 \\ 0 & 0 & 4.1659 & 0.0905 \\ 0 & 0 & 0.0905 & 4.5781 \end{bmatrix},$$

$$Q_1 = Q_2 = \begin{bmatrix} 1.2424 & 0.0387 & 0.0001 & -0.0002 \\ 0.0387 & 1.3410 & -0.0005 & 0.0007 \\ 0.0001 & -0.0005 & 0.1236 & 0.0286 \\ -0.0002 & 0.0007 & 0.0286 & 0.1260 \end{bmatrix},$$

$$X = \begin{bmatrix} 4.9775 & 0.0363 \\ 0.0363 & 5.5443 \end{bmatrix}, \quad Y = \begin{bmatrix} 1.0221 & 0.0421 \\ 0.0421 & 1.1006 \end{bmatrix},$$

$$\tilde{L} = \begin{bmatrix} 0.7238 & -0.0651 & 0.7131 & -0.0257 \\ 0.1928 & -0.8480 & 0.1510 & -0.8443 \end{bmatrix}, \quad \tilde{K}_P = \begin{bmatrix} 0.0283 & -0.0644 \\ 0.0344 & -0.6450 \end{bmatrix},$$

$$\tilde{K}_I = \begin{bmatrix} 0.0010 & -0.0027 \\ 0.0005 & -0.0135 \end{bmatrix}, \quad \tilde{K}_D = \begin{bmatrix} 0.0020 & -0.0053 \\ 0.0010 & -0.0269 \end{bmatrix}.$$

Then, by taking (44) into consideration, the observer gain and the controller gains can be expressed as follows:

$$L = \begin{bmatrix} 0.1729 & -0.0116 & 0.1705 & -0.0020 \\ 0.0387 & -0.1850 & 0.0296 & -0.1844 \end{bmatrix}, \quad K_P = \begin{bmatrix} 0.0047 & -0.0121 \\ 0.0062 & -0.1163 \end{bmatrix},$$

$$K_I = \begin{bmatrix} 0.0002 & -0.0005 \\ 0.0001 & -0.0024 \end{bmatrix}, \quad K_D = \begin{bmatrix} 0.0004 & -0.0010 \\ 0.0002 & -0.0049 \end{bmatrix}.$$

In this simulation, the initial value of the state is set to be $x(0) = [-0.3\ 0.4]^T$, and $x(-2) = x(-1) = [0\ 0]^T$. The system noise is assumed to be $\varpi(k) = 0.2sin(k)/k$. The simulation results are shown in Fig.2-3. Specifically, Fig.2 reveals the trajector of the system state $x(k)$. Fig.3 plots the trajectories of the system noise $\varpi(k)$ and the control output $z(k)$, respectively. It is easy to see that the fluctuation of $z(k)$ is smaller than that of $\varpi(k)$. Furthermore, by computation, the desired $\mathcal{H}_\infty$ performance is satisfied.

In order to examine the efffectiveness of the redundant channel transmission mechanism in reducing data packet loss, the random data packet dropouts of three transmission channels are shown in Fig.4. Therefore, we can know that compared with a single channel, the possibility of data packet loss in the transmission channel is greatly reduced by adopting the redundant channels.

Fig.5 and Fig.6, respectively, describe the dynamic triggering instants and the static triggering instants. Comparing the above two figures, we can conclude that compared to the static event-triggered control (SETC) scheme, the DETC scheme can better save network bandwidth resources and increase the reliability of data transmission.

Remark 4.1. *Based on the above considerations, we can summarize as follows: the network system that combines the redundant channel transmission mechanism and DETC scheme can effectively improve the reliability of network transmission. At the same time, in our future work, we can consider applying the above framework to the more complex network system.*

5. Conclusions

This paper has addressed the $\mathcal{H}_\infty$ PID control issue for a class of discrete-time NCSs network control under dynamic event-triggered control scheme. The redundant channel transmission mechanism has been introduced to improve the reliability of network communication during the transmission process. Based on the Lyapunov theory, the sufficient conditions have been proposed to guarantee the exponentially stability and the $\mathcal{H}_\infty$ performance index for the designed system. Futhermore, by using linear matrix inequality technology, the PID controller gains have been obtained. Finally, an illustrative example has been provided to show the validity of the proposed method. In near futher, we will consider extending the proposed method to the PID security control for T-S fuzzy systems under cyber attacks.

[1] K.H. Ang, G. Chong, Y. Li, PID control system analysis, design, and technology. IEEE Trans. Control Syst. Technol, 3(4)(2005) 559C576.

[2] S. Cong, Y. Liang, PID-like neural network nonlinear adaptive control for uncertain multivariable motion control systems, IEEE Trans. Ind. Electron., 56(10)(2009) 3872C3879.

[3] P.H. Chang, J.H. Jung, A systematic method for gain selection of robust PID control for nonlinear plants of second-order controller canonical form, IEEE Trans. Control Syst. Technol., 17(2)(2009) 473C483.

[4] M. Van, An enhanced robust fault tolerant control based on an adaptive fuzzy PID-nonsingular fast terminal sliding mode control for uncertain nonlinear systems, IEEE Trans. Mechatron. 23(3)(2018) 1362C1371.

[5] H.-X. Li, L. Zhang, K.-Y. Cai, and G. Chen, An improved robust fuzzy-PID controller with optimal fuzzy reasoning, IEEE Trans. Syst., Man,Cybern. B, Cybern., 35(6)(2005) 1283C1294.

[6] Y. Wang, L. Zou, Z. Zhao, and X. Bai, $\mathcal{H}_\infty$ fuzzy PID control for discrete time-delayed T-S fuzzy systems, Neurocomputing, 332(2019) 91C99.

[7] J. Hu, Z. Wang, S. Liu, and H. Gao, A variance-constrained approach to recursive state estimation for time-varying complex networks with missing measurements, Automatica, 64(2016) 155C162.

[8] B. Shen, Z. Wang, D. Ding, and H. Shu, $\mathcal{H}_\infty$ state estimation for complex networks with uncertain inner coupling and incomplete measurements, IEEE Trans. Neural Netw. Learn. Syst., 24(12)(2013) 2027C2037.

[9] D. Zhang, Q.-G. Wang, D. Srinivasan,et al., Asynchronous state estimation for discrete-time switched complex networks with communica-

tion constraints, IEEE Trans. Neural Netw. Learn. Syst., 29(5)(2018) 1732C1746.

[10] Y. Liu, B. Shen, H. Shu, Finite-time resilient $\mathcal{H}_\infty$ state estimation for discrete-time delayed neural networks under dynamic event-triggered mechanism, Neural Networks, 121(2020) 356-365.

[11] X.-M. Zhang, Q.-L. Han, X.-H. Ge, et al., Networked control systems: a survey of trends and techniques, IEEE/CAA J.Automatica Sinica 7(1)(2020) 1C17.

[12] D. Zhang, W. Cai, L. Xie, et al., Nonfragile distributed filtering for T-S fuzzy systems in sensor networks, IEEE Trans. Fuzzy Syst., 23(5)(2015) 1883C1890.

[13] L. Zou, Z. Wang, Q.-L. Han, et al., Ultimate boundedness control for networked systems with try-once-discard protocol and uniform quantization effects, IEEE Trans. Autom. Control, 62(12)(2017) 6582C6588.

[14] D. Ding, Z. Wang, B. Shen, et al., Event-triggered consensus control for discrete-time stochastic multi-agent systems: the input-to-state stability in probability. Automatica, 62(2015) 284C291.

[15] L. Zou, Z. Wang, H. Gao, Set-membership filtering for time-varying systems with mixed time-delays under Round-Robin and Weighted Try-Once-Discard protocols, Automatica, 74(2016) 341-348.

[16] J. Liu, M. Yang, L. Zha, et al., Multi-sensors-based security control for T-S fuzzy systems over resource-constrained networks, J. Frankl. Inst., 357(2020) 4286C4315.

[17] L. Zha, E. Tian, X. Xie, et al., Decentralized event-triggered $\mathcal{H}_\infty$ control for neural networks subject to cyber-attacks, Inf.Sci., 457-458(2018) 141-155.

[18] X. Yin, D. Yue, S. Hu, et al., Model-based event-triggered predictive control for networked systems with data dropout, SIAM J. Control Optim., 54(2)(2016) 567C586.

[19] H. Dong, Z. Wang, B. Shen, et al., Variance-constrained $\mathcal{H}_\infty$ control for a class of nonlinear stochastic discrete time-varying systems: The event-triggered design, Automatica, 72(2016) 28-36.

[20] S. Hu, D. Yue, X. Xie, et al., Resilient event-triggered controller synthesis of networked control systems under periodic DoS jamming attacks, IEEE Trans. Cybern., 49(12)(2019) 4271-4281.

[21] S. Hu, D. Yue, Q. Han, et al., Observer-based event-triggered control for networked linear systems subject to denial-of-service attacks, IEEE Trans. Cybern., 50(5)(2020) 1952-1964.

[22] D. Zhao, Z. Wang, G. Wei, et al., A dynamic event-triggered approach to observer-based PID security control subject to deception attacks, Automatica, 120(4):109128(2020).

[23] Q. Li, B. Shen, Z. Wang, et al., Synchronization control for a class of discrete time-delay complex dynamical networks: a dynamic event-triggered approach, IEEE Trans. Cybern., 49(5)(2019)1979-1986.

[24] Q. Li, Z. Wang, N. Li, et al., A dynamic event-triggered approach to

recursive filtering for complex networks with switching topologies subject to random sensor failures, IEEE Trans. Neural Netw. Learn. Syst., 31(10)(2020) 4381-4388.

[25] K.-Z. Liu, A.R. Teel, X.-M. Sun, et al., Model-based dynamic event-triggered control for systems With uncertainty: a hybrid system approach, IEEE Trans. Autom. Control, 66(1)(2021)444-451.

[26] F. Shu, J. Zhai, Dynamic event-triggered output feedback control for a class of nonlinear systems with time-varying delays, Inf.Sci., 569(2021) 205-216.

[27] L. Zou, Z. Wang, H. Gao, et al., Finite-horizon $\mathcal{H}_\infty$ consensus control of time-varying multiagent systems with stochastic communication protocol, IEEE Trans. Cybern., 47(8)(2017) 1830C1840.

[28] L. Sheng, Y. Niu, M. Gao, Distributed resilient filtering for time-varying systems over sensor networks subject to Round-Robin/stochastic protocol, ISA Trans., 87(2019) 55C67.

[29] Y. Xu, H. Su, Y.J. Pan, et al., Stability analysis of networked control systems with Round-Robin scheduling and packet dropouts, J. Franklin Inst., 350(8)(2013) 2013C2027.

[30] F. Wang, Z. Wang, J. Liang, et al., Resilient filtering for linear time-varying repetitive processes under uniform quantizations and Round-Robin protocols, IEEE Trans. Circ. Syst. I, 65(9)(2018) 2992C3004.

[31] K. Zhu, Y. Song, D. Ding, Resilient RMPC for polytopic uncertain systems under TOD protocol: a switched system approach, Int. J. Robust Nonlinear Control, 28(16)(2018) 5103C5117.

[32] M.H. Mamduhi, S. Hirche, Try-once-discard scheduling for stochastic networked control systems, Int. J. Control, 92(11)(2019) 2532C2546.

[33] A.R. Mesquita, J.P. Hespanha, G.N. Nair, Redundant data transmission in control/estimation over lossy networks, Automatica, 48(8)(2012) 1612C1620.

[34] L. Zhang, Z. Ning, Z. Wang, Distributed filtering for fuzzy time delay systems with packet dropouts and redundant channels, IEEE Trans. Syst., Man, Cybern., Syst., 46(4)(2016) 559C572.

[35] J. Song, Y. Niu and H. Lam, Reliable sliding mode control of fast sampling singularly perturbed systems: a redundant channel transmission protocol approach, IEEE trans. circuits syst., 66(11)(2019) 4490-4501.

[36] Y. Chen, J. Ren, X. Zhao, et al., State estimation of Markov jump neural networks with random delays by redundant channels, Neurocomputing, 453(2021) 493-501.

[37] Q. Li, X. Liu, Q. Zhua, et al., Distributed state estimation for stochastic discrete-time sensor networks with redundant channels, Applied Mathematics and Computation, 343(2019) 230C246.

[38] S. Liu, Y. Song, G. Wei, et al., Event-triggered dynamic output feedback RMPC for polytopic systems with redundant channels: Input-to-state stability, J. Frankl. Inst., 54(7)(2017) 2871-2892.

[39] D. Li, J. Liang, F. Wang, Dissipative networked filtering for two-dimensional systems with randomly occurring uncertainties and redundant channels, Neurocomputing, 369(2019) 1-10.

[40] F. Li, W.X. Zheng and S. Xu, Finite-time fuzzy control for nonlinear singularly perturbed systems with input constraints, IEEE Trans. Fuzzy Syst., doi: 10.1109/TFUZZ.2021.3072737.

[41] Y. Wang, L. Xie, C.E.D. Souza, Robust control of a class of uncertain nonlinear systems, Syst. Control Lett., 19(2)(1992) 139C149 .

[42] M. Shen, J.H. Park, D. Ye, A separated approach to control of markov jump nonlinear systems with general transition probabilities, IEEE Trans. on Cybern., 46(9)(2015) 2010C2018.

[43] W. Li, Z. Wang, Q. Liu, et al., An information aware event-triggered scheme for particle filter based remote state estimation, Automatica, 103(2019) 151C158.

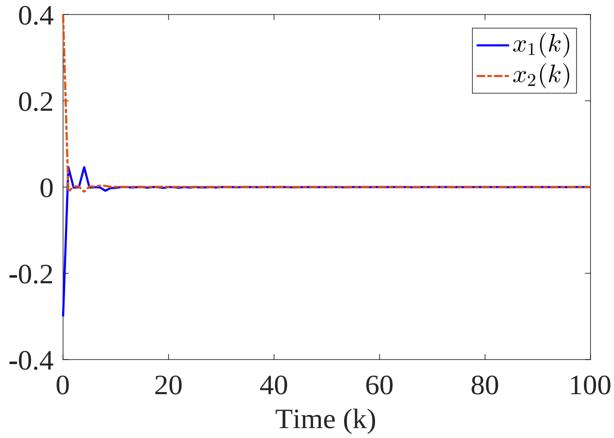

Figure 2: State trajectories $x(k)$ of the system with PID control.

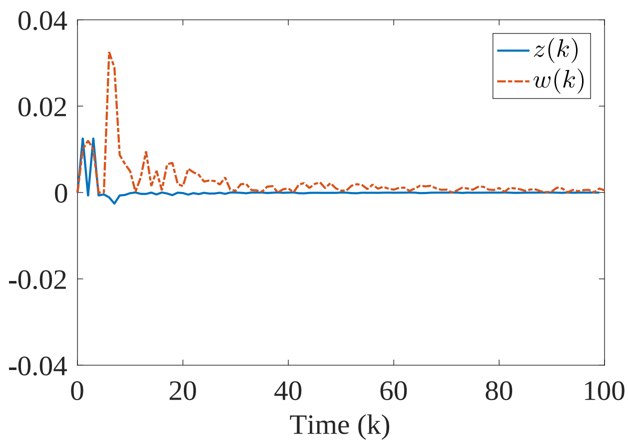

Figure 3: The trajectory of controller output $z(k)$ and system noise $\varpi(k)$.

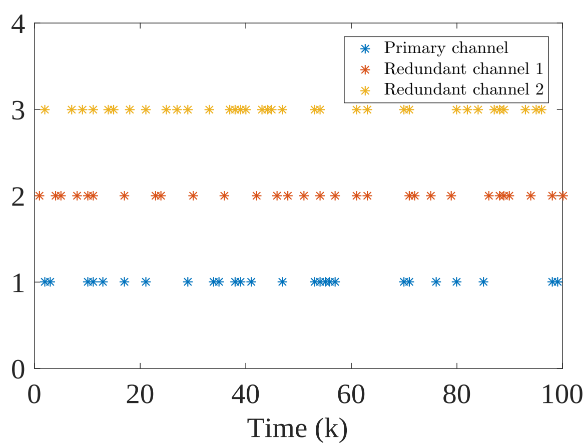

Figure 4: Random packet dropouts in three channels.

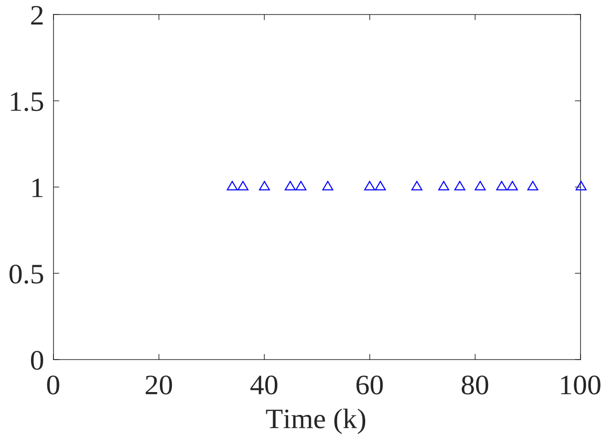

Figure 5: Dynamic triggering instants.

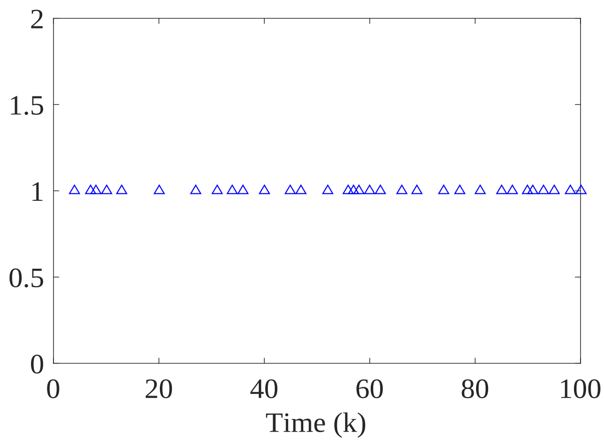

Figure 6: Static triggering instants.