Original Research Article

Evaluation of Generalized Extreme (GEV), Log-Pearson Type 3 (LP3), Pearson Type 3(P3) and Gumbel (EV1) Distributions For Development of IDF Equations for Warri, Nigeria.

Abstract:

The application of Gumbel (EVI) in the development of rainfall intensity – duration – frequency (IDF) curves has often been criticized on theoretical and empirical grounds as it may underestimate the largest extreme rainfall amounts. The consequences of underestimation are economic losses, property damages, and loss of life. Therefore, it is important that water resources engineering infrastructure be accurately design to avoid these consequences. This paper evaluates the performances of four probability distributions; GEV, EV1, LP3 and P3 using the annual maxima precipitation series of 26 years for Warri Metropolis obtained from Nigerian Meteorological Agency (NiMet). The strength and weakness of the four probability distributions were examined with the goodness of fit (GOF) module of Easyfit software which implemented Kolmogorov - Smirnov (KS) and Anderson - Darling (AD) tests at 5% significance level. The Easyfit software fitted the precipitation series data to the four probability distributions and ranked the four probability distributions across the fifteen rainfall durations.

Results show that for both KS and AD tests, GEV distribution was found to be best-fit distribution and it was applied to the development of IDF curves in Warri Metropolis, Nigeria. Furthermore, the IDF values obtained were applied in the development of three-parameter IDF models for return periods of 10 - , 15 -, 20 -, 25 - , 50 -, and 100-years. The mean absolute errors computed for the IDF models increase with increasing return periods. The IDF curves and models depicted the general attributes of IDF curves and models. This study could be of significant academic value and improvement to professional practice in the design of storm water drainage systems.

Key words: IDF curves, Frequency analysis, Goodness – fit – tests, Warri, Probability distributions.

INTRODUCTION:

The IDF Equation is a mathematical relationship connecting rainfall intensity (I), duration (D) and return period (T) or its inverse, the exceedance probability (P). It is a standard water resources engineering tool in most countries for planning, design and operation of hydraulic structures and storm water drainage systems. Beginning with the earlier works of [1] to [4]. The development of IDF equations has reached a level of maturity as IDF equations and curves are presented and discussed in many water resources engineering related texts, e.g. [5] to [10] etc. [11] reported the application of IDF equations/curves as standard tool for design of hydrologic, hydraulic and water resources systems amongst the South East Asia and Pacific countries. [12] compared Gumbel (EV1) and Log-Pearson Type 3 distributions in the development of IDF curves for Koforidua city in Ghana and found Gumbel EV1 more suitable.

[13] derived intensity – duration- frequency curves for the kingdom of Saudi Arabia using Gumbel EV1 as default distribution. In Nigeria, the applications of rainfall intensities at various return periods as input into rainfall-runoff modes (e.g. RFM) for design of a variety of civil Infrastructure, especially in urban environment, has become a common and standard practice. It is important that these civil infrastructure be appropriately sized to avoid economic losses, higher risks and loss of human life [14]. Consequently, the accurate estimation of IDF curves/equations is crucial to proper sizing of water infrastructure such as urban storm – water drainage systems.

There is wealth of literature dealing with the Developing of IDF Curves/Equations in Nigeria. Some of the reviewed literature include [15] – [23], etc,. Analysis of the cited literature show that Gumbel extreme type 1 (EV1) has dominated the development of IDF curves and models in Nigeria. But Gumbel EVI may significantly underestimate the largest extreme rainfall amount (albeit their

predictions for small return periods of 5-10 years are satisfactory) [24]. Consequently, the applicability of Gumbel (EV1) has often been critized both on theoretical and empirical grounds. The use of return periods (≤10 years) are longer in vogue in view of climate change and urbanization causing non-stationarity of observed rainfall series.
The objective of this study, therefore is to evaluate the four selected probability distribution functions; GEV,LP3,P3 and Gumbel (EV1). Using the Easyfit software, the rainfall data is fitted to the four probability distribution functions, perform goodness – of – fit (GOF) tests using Kolmogonov-Smirnov, and Anderson-Darling tests and finally rank the distributions. The best-fit-distribution (the one selected) will be used to develop rainfall Intensity-Duration-Frequency (IDF) relationships for Warri, Nigeria. To the best of the Author's knowledge and literature search, no attempt had been made in the past to evaluate candidate probability distribution functions first, thereafter the best fit distribution is applied in the development of IDF curves for Warri, Nigeria. Meanwhile the Warri metropolis continues to suffer from the devastating impacts of urban flooding, causing loss of property due to absence good drainage systems and poor planning practices [25]).

2. MATERIALS AND METHODS

2.1 Description of Study Area and Data Description

The rainfall station at Warri is located at an elevation of 2.44m above mean sea level and the coordinates fall within Latitude $05^{o}31'$, Longitude 05^{o} 44'.Warri Metropolis itself is geographically located between $5^{o}30'$N and $5^{o}35'$N and $5^{o}29'$E to $5^{o}48'$E . The study area is bounded to the north by Okpe and Sapele Local Government Areas; to the southern axis by Warri South West and the Atlantic Ocean; to the east, the metropolis is bounded by Ughelli South Local Government Area while it shares its western boundary with Warri North Local Government Area. Figure 1 shows the map of Warri metropolis [25]
The rainfall data were obtained from Nigerian Meteorological Agency (NiMet) office Abuja, Nigeria. The rainfall intensities were extracted from FROM MET 414 (Tabulation of Autographic Rainguage Records. The length of data is 26years (1962-1990), with two years missing due to the Nigeria civil war. NIMET has the responsibility of measuring, analyzing, hydro meteorological data storage and forecasting the weather in Nigeria.

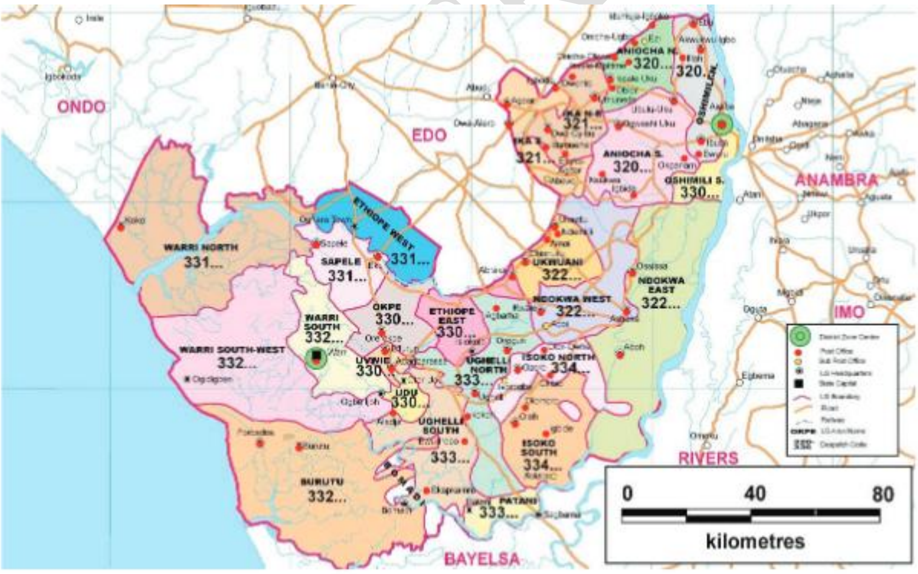

Figure 1 : Map of Warri Metropolis, Nigeria.

2.3 Derivation of IDF Curves

The derivation of IDF relationships involved fitting a theoretical extreme value distribution to the observations and then use the theoretical distribution to estimate the rainfall events with given exceedance probabilities. The IDF were derived using the method of frequency analysis as follows.

(i) Gather time series records of different duration. (eg. 5.10,15,20,30,60,90,120 min, etc.,)
(ii) Extract annual precipitation extremes from the record of each duration
(iii) Fit the annual precipitation extremes of each duration to the selected probability distribution; GEV, Gumbel EV1, LP3 and P3, using the easyfit software.
(iv) The best –fit distribution of each duration was determined using Kolmogorov – Smirnov (KS) and Anderson – Darling tests of goodness – of – fits at 5% significance level.
(v) Rank the four distributions to determine the best – fit – distribution in each duration.

(vi) Following steps (i) to (iv), select the best – fit – distribution across the fifteen (15) durations.

(vii) Finally, the selected best – fit distribution in (vi) will be applied to the development of Rainfall Intensity-Duration – Frequency (IDF) relationship for Warri metropolis, Nigeria.

2.4 Procedure for Fitting GEV distribution to Annual Precipitation Series.

The main objective of frequency analysis is to fit geophysical data to a probability distribution to establish a relationship between the event magnitude and its exceedance probability, and then use the quantile relation as basis for extrapolation to higher return periods. The steps followed in fitting GEV distribution are detailed in subsection 2.4.1.

2.4.1 Generalized Extreme Value (GEV) Distribution

The generalized extremes value (GEV) is a three-parameter distribution; shape (k), location (μ), and scale (σ). The three – parameters of the (GEV) distribution; shape (k), location (beta), and scale (a) may be estimated from the sample moments; mean (E(Q)), variance (Var [Q]), and skew coefficient (Cs) using Equations 1 – 3 as follows;

$$\alpha = \left(\frac{K^2\ Var[Q]}{\Gamma\ (1+2K) - \Gamma^2(1+K)} \right)^{1/2} \qquad (1)$$

$$\beta = E[Q] - \frac{\alpha}{K}\left[1-\Gamma(1+K)\right] \qquad (2)$$

The shape parameter (k) is calculated from the skew coefficient (Cs) using the equation given by [22] for -2 < Cs < 1.1396 (EV3).

K = 0.277648 – 0.32201Cs +0.060278 C_s^2 + 0.016759 C_s^3 – 0.005873 C_s^4 – 0.00244 C_s^5 – 0.00005 C_s^6 (3)

The following steps may be followed to compute the GEV quantiles:

i) Using the MS Excel built-in-functions compute the three first sample moments, namely; the mean E[Q], variance, Var [Q], and coefficient of skewness (Cs).

ii) Using the calculated coefficient of skewness (Cs), select the appropriate range of inequality, thus Equation 3was selected based on the estimated shape parameter, k according to **[25]**.

iii) Estimate the other two MoM estimators; α and β from Equations 1 and 2 respectively.

iv) Note the MoM estimates for GEV (α, β and k) followed the substitution of Q, σ_Q^2 and Cs by their respective sample estimates.α

v) Compute the T-year quantile estimate as:

$$Q_T = \beta + \frac{\alpha}{k}\left[1-\left\{-\ln\left(\frac{T-1}{T}\right)\right\}^K\right] \qquad (4)$$

2.5 Calibration of IDF Equation Parameters

2.5 Calibration of IDF Equation Parameters

The IDF data is fitted to Equation 5 as:

$$i(D) = \frac{\alpha}{(D+\theta)\delta} \qquad (5)$$

Equation 5 is a three parameter function , the optimum values of equations 1 are estimated by least squares method. Plots of rainfall intensity (I) versus duration (D) for each return period is then produced from the fitted IDF data to Equations 5 [27]. Taking logarithms on both sides of Equation 5, gives:

$$\log(i,D) = \log\alpha - \delta\log(D+\theta)$$

The optimum values of α, δ and θ are those for which the error sum of the square deviations is minimum. That is

$$S = \sum\left[\log(i,D) - \{\log\alpha - \delta\log(D+\theta)\}\right]^2$$

Partial differentiation of S with respect to α and δ yields:

$$\sum\log 1 = n\log\alpha - \eta\sum\log(D+\theta) \qquad (6)$$

$$\sum\left[\log(i,D)\times\log(D+\theta) = \log y \sum\log(D+\theta) - n\sum\log(D+\theta)\right] \qquad (7)$$

where n is the number of observations. Equations 6 and 7 was solved simultaneously to find α and δ for any assumed value of θ and the best value of θ itself will be found by trial and error [28].

2.6 Goodness of fit tests

The performance of the selected distribution fits are ranked using two goodness of fit tests namely; Kolmogorov –Smirnov test and Anderson Darling estimate. The two test were carried out using Easyfitsoftware,at:http://www.mathwave.com/easyfit-distribution-fitting.html.

2.6.1 Kolmogorov-Smirnov Test

The Kolmogorov-Smirnov (KS) Statistics (D) is based on the largest vertical different between the theoretical and empirical cumulative distribution function (CDF):

$$D = \underset{1\le i\le b}{Max}\left(F(Xi) - \frac{i-1}{n}, \frac{i}{n} - F(Xi) \right) \qquad (8)$$

The hypothesis is rejected, if the KS *Statistics* is greater than the critical value at a chosen significance level α = 0.05

2.6.2 Anderson –Darling Estimate (AD)

The Anderson –Darling Estimate compares the fit of an observed cumulative distribution function (CDF) to an expected cumulative distribution function. The method gives greater weight to the tail of the distribution than the KS statistics test. The (AD) statistics (A^2) is expressed as;

$$A^2 = -n - \frac{1}{n}\sum_{i=1}^{n}(2i-1)\times\left(InF(Xi) + In(1 - F(Xn - i + 1))\right) \quad (9)$$

The test hypothesis is rejected if the AD statistics is greater than a critical value of 2.5018 at a given significance level $\alpha = 0.05$. For further reading on the Easy fit software, interested reader may consult [27].

The quantile equation (Equations 4), was used to calculate maximum precipitation for return periods 10,15,20,25, 50 and 100 years in Warri metropolise.

2.6.3 Mean Absolute Error (MAE)

The MAE statistics is computed and reported in the same units as the model output and work well for continuous simulation and commonly used in model performance evaluation.

$$MAE = \frac{1}{n}\sum_{i=1}^{n}\left|O_i - P_i\right| \quad (10)$$

Where *n* is data length, O_i i is observed annual maximum precipitation series and P_i is the predicted series.

(3) Results and Discussion

Tables 1 – 4 and Figure 2 show the results of this study. The analysis was executed in Microsoft Excel 2010 while the fitting of the probability distributions, goodness of fit tests and ranking of the probability distributions across the fifteen durations were performed using Easy fit Software. Table 1 shows the descriptive statistic and GEV parameters computed from the annual precipitation series across the all the rainfall durations considered in this study. The data exhibit asymmetry with positive and negative skewness coefficient, which implies a non-normal distribution. Using the kurtosis coefficient in conjunction with the excess coefficient (E), a platykurtic- type distribution was obtained, confirming the non-nomality of the data. The shape parameter (k) is generally greater than zero (0) leading to a EV-III distribution.

Table 2 shows the outcome of the goodness of fit tests conducted using KS and AD tests at 5% significance level. The chi-squared test was not considered because it is weaker test compared to KS and AD tests and also not distribution free. The results of fitting of KS and AD tests to GEV, Gumbel (EVI), LP3 and P3 distributions and ranking their performances across the fifteen (15) rainfall durations considered in this study, are shown in Table 2.

Table 2 revealed that in terms of KS GOF test, GEV is best fit distribution in eleven (11) durations out of fifteen (15), Gumbel (EVI), scored (0) zero, LP3 scored one (1) and P3 scored 3. Similarly, in terms of AD GOF test, GEV scored ten (10) out of fifteen (15), Gumble (EV1) scored one (1), LP3 scored zero (0) and P3 scored four (4) out of fifteen (15). Consequently, GEV distribution is the best-fit probability distribution in this study. It is selected for frequency analysis and development of IDF curves for Warri Metropolis, Nigeria. Rainfall intensities for different durations and return periods using the GEV quantile relation (Equation 4) are presented in Table 3. Finally, using the computed intensities data in Table 3, the empirical equations in Table 4 and mean absolute error (MAE) of each equation and return period were computed. Figure 2 shows the IDF curves. Generally, Tables 3 and 4 and Figure 2 follow the characteristics of IDF relations. They reveal that rainfall intensity is a decreasing function of rainfall duration for a given return period [28]. The mean absolute error is a measure of the accuracy of a probability distribution to the fitted time series. Table 4 shows that the accuracy of IDF models decreases with increasing return periods. Figure 2 shows that rainfall intensity and duration are inversely related, meaning that as the duration increases, the intensity reduces. The findings in this work cannot be corroborated with [29] who developed rainfall intensity-duration-frequency models for Lokoja Metropolis, Nigeria using Normal, Log-Normal Gumbel, Pearson Type III and Log-Pearson Type III distributions and found Log-Pearson Type 3 the best fit distribution. [29] did not include GEV distribution in their study. This study agrees with [30] and [31], who applied GEV distribution in the construction of IDF curves using the annual maxima rainfall of Netherland, England and Wales respectively. They used GEV distribution due to the superiority of the distribution in describing the upper tail characteristics.

3.1 Descriptive Statistics and GEV Parameters

Table 1: Descriptive Statistics and GEV Parameters

Duration	Mean	Stdev	CV	Skewness	Kurtosis	A	β	K
10	11.213	8.744	0.78	2.062	4.547	4.962	7.257	-0.184
20	21.968	9.079	0.413	0.255	-0.05	8.632	18.434	0.1997
30	31.47	9.691	0.308	0.869	0.723	8.029	27.213	0.0498
45	40	16.338	0.408	0.864	1.114	13.55	32.83	0.0507
60	55.613	14.198	0.255	-0.053	-0.678	14.315	50.673	0.0295
90	42.083	24.912	0.592	0.53	-0.501	22.306	31.704	0.126
120	54.419	20.576	0.378	0.077	-0.926	20.269	46.874	0.253
180	57.995	24.7	0.426	0.602	-0.138	21.757	47.561	0.108
240	56.402	26.377	0.468	1.41	2.695	19.253	44.35	-0.005
300	60.678	18.249	0.301	-0.958	1.252	19.881	57.375	0.624
360	59.135	21.87	0.37	0.578	-0.404	19.369	49.938	0.1142
420	54.616	23.738	0.435	0.97	0.687	19.217	44.078	0.023
480	63.666	27.528	0.443	0.832	1.082	23	51.628	0.0573
540	65.377	24	0.367	1.52	4.088	17.014	54.37	-0.066
600	67.6496	27.18	0.402	0.595	1.56	23.983	56.184	0.111

3.2 Results and Probability Distribution for all Durations.

Table 2: Ranking of Probability Distributions Across The Fifteen Durations.

	Kolmogorov - Smirnov(KS)				Anderson - Darling(AD)			
Dur.(mins)	GEV	GUM	LP3	P3	GEV	GUM	LP3	P3
10	0.08752^{1}	0.1358^{4}	0.09895^{2}	0.09964^{3}	0.25512^{1}	0.89484^{4}	0.46006^{3}	0.26529^{2}
20	0.08791^{1}	0.13763^{4}	0.09737^{2}	0.10346^{3}	0.24394^{1}	0.59184^{4}	0.26542^{3}	0.2599^{2}
30	0.07423^{1}	0.0843^{4}	0.08168^{3}	0.07571^{2}	0.18484^{1}	0.18551^{2}	0.18838^{4}	0.18715^{3}
45	0.10355^{3}	0.09428^{2}	0.10482^{4}	0.09415^{1}	0.21821^{2}	0.2288^{3}	0.24147^{4}	0.2054^{1}
60	0.0876^{1}	0.16809^{5}	0.11166^{2}	0.12237^{3}	0.22653^{1}	1.0666^{5}	0.30954^{2}	0.36024^{3}
90	0.09602^{1}	0.12315^{4}	0.10274^{2}	0.11449^{3}	0.27729^{1}	0.42116^{4}	0.40676^{3}	0.36726^{2}
120	0.12907^{2}	0.15762^{4}	0.12751^{1}	0.14618^{3}	0.35001^{1}	0.70849^{4}	0.35357^{2}	0.40963^{3}
180	0.09601^{1}	0.11455^{4}	0.09687^{2}	0.10867^{3}	0.21238^{1}	0.26353^{4}	0.21256^{2}	0.23104^{3}
240	0.10236^{1}	0.10309^{2}	0.10873^{3}	0.12944^{5}	0.29282^{1}	0.35573^{4}	0.29465^{2}	0.36369^{5}
300	0.09177^{1}	0.22162^{4}	0.14825^{2}	0.17176^{3}	0.30639^{1}	1.8834^{3}	14.254^{4}	0.60116^{2}
360	0.16893^{3}	0.16926^{4}	0.16099^{2}	0.15736^{1}	0.71275^{3}	0.68603^{1}	0.68966^{2}	0.78919^{4}
420	0.09297^{1}	0.10892^{3}	0.11537^{4}	0.1085^{2}	0.25412^{1}	0.26853^{3}	0.30473^{4}	0.26851^{2}
480	0.13375^{3}	0.12873^{2}	0.13802^{4}	0.12563^{1}	0.38244^{2}	0.41488^{3}	0.4279^{4}	0.35798^{1}
540	0.09972^{1}	0.11335^{3}	0.11485^{4}	0.10725^{2}	0.36556^{2}	0.38539^{3}	0.42516^{4}	0.34709^{1}
600	0.11468^{1}	0.12305^{3}	0.13392^{4}	0.11927^{2}	0.38556^{2}	0.57704^{3}	4.3661^{4}	0.33344^{1}
Score	**11/15**	**0/15**	**1/15**	**3/15**	**10/15**	**1/15**	**0/15**	**4/15**

3.2 Results of Rainfall Intensity – Duration Frequency

Table 3: Estimates of Intensity – Duration – Frequency values

Dur.(hrs)	10-Year	15-Year	20- Year	25- Year	50-Year	100- Year
10	88.76323347	108.2066	140.3973	152.1532	179.8677	236.2118
20	76.00033748	91.86636	117.4926	138.9007	163.1145	198.2079
30	72.61377158	86.62909	98.76611	121.9114	131.379	170.4549
45	67.53025924	80.94862	86.92379	94.45196	109.0674	127.2366
60	62.22088251	70.15637	79.00427	80.32155	90.64552	100.7251
90	50.94293457	55.56105	59.9844	60.28712	66.97847	73.05936
120	43.32217305	48.62355	49.5942	52.65406	59.05912	61.4738
180	33.66950829	35.6878	36.93796	38.45678	43.26301	45.45814
240	28.76465581	31.80339	34.93331	37.9544	40.95715	43.31381
300	25.08387453	27.44751	29.85062	32.38339	34.69145	37.48895
360	22.56280785	24.76146	26.95491	29.13994	31.3211	33.62876
420	20.84249518	22.77926	24.95564	26.95571	28.97754	30.95072
480	19.39870891	21.33088	23.18097	25.73115	27.50647	29.45516
540	18.40615889	20.01483	22.35695	23.99744	25.56828	27.53926
600	17.59823958	19.27253	20.99357	22.58348	23.9248	26.07356

Table 4: Derived IDF Models for Various Return Periods.

Return Period	IDF Model	Mean Absolute Error (MAE) – (mm/hr)
10	$I_{10} = \frac{2321.89}{(D+65)^{0.749}}\ mm/hr$	1.24
15	$I_{15} = \frac{2249.62}{(D+50)^{0.745}}\ mm/hr$	1.56
20	$I_{20} = \frac{1225.00}{(D+20)^{0.640}}\ mm/hr$	1.50
25	$I_{25} = \frac{1429.20}{(D+20)^{0.654}}\ mm/hr$	3.17
50	$I_{50} = \frac{1709.58}{(D+18)^{0.672}}\ mm/hr$	3.10
100	$I_{100} = \frac{2552.37}{(D+18)^{0.729}}\ mm/hr$	6.30

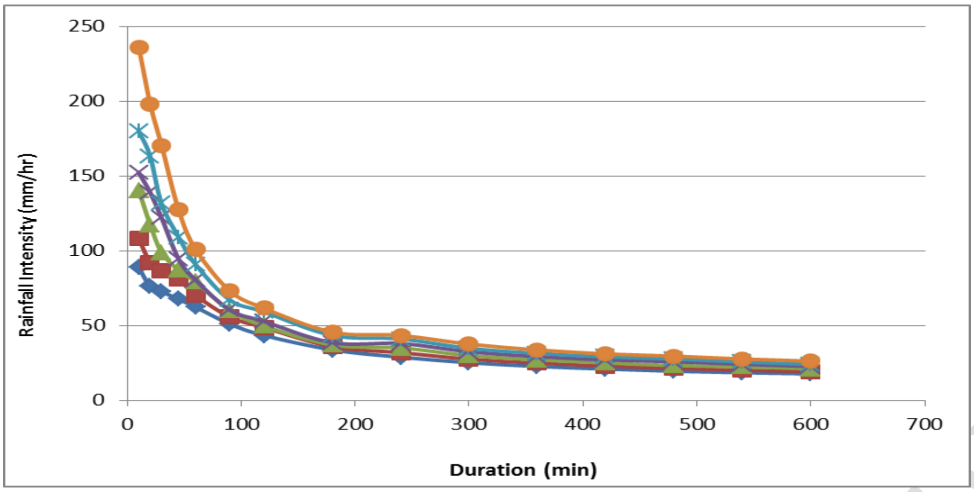

Figure 3. IDF Curves for 10-, 15-, 20-, 25-, 50-, and 100-Year Return Periods

CONCLUSION

The study was conducted to evaluate four probability distributions functions; GEV, Gumbel(EVI), Log-Pearson type III and Person type III and then use the best-fit distribution to derive curves/models for Warri Metropolis.
The study that GEV is the best-fit distribution, seconded by Pearson Type distributions. Consequently, GEV distribution revealed used for the development of IDF curves and Models for Warri Metropolis, Nigeria. The IDF curves and models displayed the general characteristics of IDF curves and models. The mean absolute errors computed for the IDF models increase with increasing return period. Intensity-duration-frequency curves and models are standard tools widely applied for the design of structures such as municipal storm-water drainage systems.

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