Original Research Article

Femtosecond pulse propagation in optical fibers under higher order effects: a moment method approach

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Abstract

In this paper, we use the moment method approach to investigate the evolution of pulse parameters in nonlinear medium. The pulse propagation is modelled by higher order nonlinear Schrödinger equation (NLSE). The application of moment method leads to variational equations that are be integrated by the fourth order Runge-Kutta method (RK4). The results obtained show the variations of some important parameters of the pulse namely the energy, the pulse position, the frequency shift, the chirp and the width. For this form of the NLSE, the energy and frequency don't vary. The coefficient of quintic self phase modulation governs the dynamics of the pulse propagation. It reveals the effects of the quintic coefficient α. The moment method is able to study the dynamics of the optical pulse modelled by higher order nonlinear Schrödinger equations.

Keywords: optical soliton, moment method, higher order nonlinear Schrödinger equation, nonlinear optical phenomena.

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Introduction

Nonlinear optical phenomena occur typically at high optical intensities and most of wave equations involved are governed by the nonlinear Schrödinger equations (NLSE)[1]. The nonlinear Schrödinger equation plays the role of Newton's laws and conservation of energy in classic mechanics. It predicts the futur behaviour of a dynamic system. The nonlinear Schrödinger equation is an example of a universal nonlinear model that describes many physical nonlinear scenarios [2]. In order to better understand nonlinear phenomena, it's important to seek their exact solution. They can help to analyse the stability of these solution and the mouvement role of the wave by making the graphs of the exacts solution [3]-[13]. The propagation of optical soliton through optical fibers is governed by nonlinear Schrödinger equation and it was first suggested by Hasegawa and Tappert [14] and first experimented by Mollenaner et al. [15]. After, this study has been expanded all accross with various results due to their application in telecommunicatin [16]-[25], severals forms of this nonlinear Schrödinger exist The focus is one of them which take account of the second as well as third order dispersion effects, cubic and quintic self phase modulation, self steepening and nonlinear dispersion effect ; it is known as RKL and was proposed by Radhakrishnan, Kundu and Lakshman. The normalized (RKL) model of a higher order nonlinear Schrödinger equation for the propagation of femtosecond pulse can be written as:

$$i\frac{\partial\psi}{\partial z}+\frac{\partial^2\psi}{\partial t^2}+ic_1\frac{\partial^3\psi}{\partial t^3}+2|\psi|^2\psi+ic_2\frac{\partial}{\partial t}(|\psi|^2\psi)+ic_3\frac{\partial}{\partial t}(|\psi|^4\psi)+\alpha|\psi|^4\psi=0 \tag{0.1}$$

and when

$$c_1=c_2=c_3=0 \tag{0.2}$$

we have the case of cubic-quintic law which has been studied by Shwetanshumoler and Biswas in 2007 with collective variables approach. The mathematical methodology adopted in this paper is know as "moment method" which transform the nonlinear Schrödinger equation (NLSE) to system of ordinary differential equations [26]-[29] solving by Runge-Kutta algorithm. Another method of searching for exact solution to (NLSE) have been presented in the literature : the inverse scattering method, the blacklund transformation, the Hirota bilinear method, the lie group method, the variable separation method, the variation iteration, the Jacobi elliptic function, the expansion method, the split Fourrier method [12]-[14], [23]. The aim of this paper is to apply the moment method to find a solitary wave solution for (NLSE) which is straight forward and concise. The outline of the present paper is as follows. In Section 1, we solve the equation by variational moment method. In Section 2, we use a Gaussian function. We obtain the variational equations of the pulse parameters which are solve by the fourth order Runge Kutta method. In Section 3, we present results in discussions. Finally, we point out the concluding remarks.

1 Solving the problem by variational moment method

The basic idea of moment method is to treat the optical pulse like a particule whose energy E, position T, the frequency Ω, the root mean square (RMS) σ and the moment related to the chirp of the pulse are defined as [25, 27, 28, 30, 31] :

$$E = \int_{-\infty}^{+\infty} |\psi|^2 dt \quad (1.1)$$

$$T = \frac{1}{E}\int_{-\infty}^{+\infty} t|\psi|^2 dt \quad (1.2)$$

$$\Omega = \frac{i}{2E}\int_{-\infty}^{+\infty} \left(\psi^*\frac{\partial\psi}{\partial t} - \psi\frac{\partial\psi^*}{\partial t}\right) dt \quad (1.3)$$

$$\sigma^2 = \frac{1}{E}\int_{-\infty}^{+\infty} (t-T)^2|\psi|^2 dt \quad (1.4)$$

$$\tilde{C} = \frac{i}{2E}\int_{-\infty}^{+\infty} (t-T)\left(\psi^*\frac{\partial\psi}{\partial t} - \psi\frac{\partial\psi^*}{\partial t}\right) dt \quad (1.5)$$

where ψ is the envelop of the pulse. Obviously, the evolution of these pulse parameters depends on the evolution on the pulse itself in the fiber which is governed by the NLS equation (0.1). *In order to find the evolution of these pulse parameters, we use the equations* (1.1) *to* (1.5) *along with equation* (0.1).

1.1 Energy evolution

Firstly, consider the evolution of the pulse energy. To find that, we differentiate (1.1) *with respect to z and get :*

$$\frac{dE}{dz} = \int_{-\infty}^{+\infty} \left(\psi^*\frac{\partial\psi}{\partial z} + \psi\frac{\partial\psi^*}{\partial z}\right) dt \quad (1.6)$$

Using (0.1) *we find that :*

$$\frac{\partial\psi}{\partial z} = i\frac{\partial^2\psi}{\partial t^2} - c_1\frac{\partial^3\psi}{\partial t^3} + 2i|\psi|^2\psi - c_2\frac{\partial}{\partial t}(|\psi|^2\psi) - c_3\frac{\partial}{\partial t}(|\psi|^4\psi) + i\alpha|\psi|^4\psi \quad (1.7)$$

After performing calculations, we have :

$$\begin{aligned}\frac{dE}{dz} = & \int_{-\infty}^{+\infty} -i\Big(\psi\frac{\partial^2\psi^*}{\partial z^2} - \psi^*\frac{\partial^2\psi}{\partial z^2}\Big)dt - c_2\int_{-\infty}^{+\infty}\Big[\psi^*\frac{\partial}{\partial t}\Big(|\psi|^2\psi\Big) + \psi\frac{\partial}{\partial t}\Big(|\psi|^2\psi^*\Big)\Big]dt \\ & - \int_{-\infty}^{+\infty} c_1\Big(\psi\frac{\partial^3\psi^*}{\partial z^3} + \psi^*\frac{\partial^3\psi}{\partial z^3}\Big)dt - c_3\int_{-\infty}^{+\infty}\Big[\psi^*\frac{\partial}{\partial t}\Big(|\psi|^4\psi\Big) + \psi\frac{\partial}{\partial t}\Big(|\psi|^4\psi^*\Big)\Big]dt. \quad (1.8)\end{aligned}$$

When we compute each over the integrals, the right handside of (1.8)*, we have*

$$\frac{dE}{dz} = 0. \quad (1.9)$$

1.2 Evolution of pulse position

Differentiating (1.2) *with respect to z we get :*

$$\frac{dT}{dz} = \frac{1}{E}\int_{-\infty}^{+\infty} t\Big(\psi^*\frac{\partial\psi}{\partial z} + \psi\frac{\partial\psi^*}{\partial z}\Big)dt \quad (1.10)$$

We get :

$$\begin{aligned}\frac{dT}{dz} &= \frac{-i}{E}\int_{-\infty}^{+\infty} t\Big(\psi\frac{\partial^2\psi^*}{\partial t^2}-\psi^*\frac{\partial^2\psi}{\partial t^2}\Big)dt-\frac{c_2}{E}\int_{-\infty}^{+\infty} t\Big[\psi^*\frac{\partial}{\partial t}\Big(|\psi|^2\psi\Big)+\psi\frac{\partial}{\partial t}\Big(|\psi|^2\psi^*\Big)\Big]dt \\ &\quad -\frac{c_1}{E}\int_{-\infty}^{+\infty} t\Big(\psi\frac{\partial^3\psi^*}{\partial t^3}+\psi^*\frac{\partial^3\psi}{\partial t^3}\Big)dt-\frac{c_3}{E}\int_{-\infty}^{+\infty} t\Big[\psi^*\frac{\partial}{\partial t}\Big(|\psi|^4\psi\Big)+\psi\frac{\partial}{\partial t}\Big(|\psi|^4\psi^*\Big)\Big]dt \end{aligned} \tag{1.11}$$

After integrating by parts and the definition of frequency in (1.3)*, we obtain:*

$$\frac{dT}{dz}=2\Omega-\frac{3c_1}{E}\int_{-\infty}^{+\infty}\Big|\frac{\partial\psi}{\partial t}\Big|^2dt+\frac{3c_2}{2E}\int_{-\infty}^{+\infty}|\psi|^4dt+\frac{c_3}{3E}\int_{-\infty}^{+\infty}|\psi|^6dt \tag{1.12}$$

1.3 Evolution of frequency schift

Differentiating (1.3) *with respect to* z*, we get :*

$$\frac{d\Omega}{dz}=\frac{i}{2E}\int_{-\infty}^{+\infty}\Big[\frac{\partial}{\partial z}\Big(\psi^*\frac{\partial\psi}{\partial t}\Big)-\frac{\partial}{\partial z}\Big(\psi\frac{\partial\psi^*}{\partial t}\Big)\Big]dt \tag{1.13}$$

$$\frac{\partial}{\partial z}\Big(\psi^*\frac{\partial\psi}{\partial t}\Big)=\psi^*\frac{\partial^2\psi}{\partial z\partial t}+\frac{\partial\psi^*}{\partial z}\frac{\partial\psi}{\partial t} \tag{1.14}$$

From (1.7)*, we can write :*

$$\psi^*\frac{\partial^2\psi}{\partial z\partial t}=i\psi^*\frac{\partial^3\psi}{\partial t^3}-c_1\psi^*\frac{\partial^4\psi}{\partial t^4}+2i\psi^*\frac{\partial}{\partial t}(|\psi|^2\psi)-c_2\psi^*\frac{\partial^2}{\partial t^2}(|\psi|^2\psi)-c_3\psi^*\frac{\partial^2}{\partial t^2}(|\psi|^4\psi)+i\alpha\psi^*\frac{\partial}{\partial t}(|\psi|^4\psi) \tag{1.15}$$

and

$$\frac{\partial\psi^*}{\partial z}\frac{\partial\psi}{\partial t}=-i\frac{\partial^2\psi^*}{\partial t^2}\frac{\partial\psi}{\partial t}-c_1\frac{\partial^3\psi^*}{\partial t^3}\frac{\partial\psi}{\partial t}-2i|\psi|^2\psi^*\frac{\partial\psi}{\partial t}-c_2\frac{\partial}{\partial t}\Big(|\psi|^2\psi^*\Big)\frac{\partial\psi}{\partial t}-c_3\frac{\partial}{\partial t}\Big(|\psi|^4\psi^*\Big)\frac{\partial\psi}{\partial t}-i\alpha|\psi|^4\psi^*\frac{\partial\psi}{\partial t} \tag{1.16}$$

Adding (1.15) *and* (1.16) *and substituting into* (1.14)*, we find :*

$$\begin{aligned}\frac{\partial}{\partial z}\Big(\psi^*\frac{\partial\psi}{\partial t}\Big)&=i\left[\psi^*\frac{\partial^3\psi}{\partial t^3}-\frac{\partial^2\psi^*}{\partial t^2}\frac{\partial\psi}{\partial t}\right]-c_1\left[\psi^*\frac{\partial^4\psi}{\partial t^4}+\frac{\partial^3\psi^*}{\partial t^3}\frac{\partial\psi}{\partial t}\right]+i2\psi^*\frac{\partial}{\partial t}(|\psi|^2\psi)-i2\frac{\partial\psi}{\partial t}\psi^*|\psi|^2\\ &\quad -c_2\psi^*\frac{\partial^2}{\partial t^2}(|\psi|^2\psi)-c_2\frac{\partial\psi}{\partial t}\frac{\partial}{\partial t}(\psi^*|\psi|^2)-c_3\psi^*\frac{\partial^2}{\partial t^2}(|\psi|^4\psi)-c_3\frac{\partial\psi}{\partial t}\frac{\partial}{\partial t}(\psi^*|\psi|^4)\\ &\quad +i\alpha\psi^*\frac{\partial}{\partial t}(|\psi|^4\psi)-i\alpha\frac{\partial\psi}{\partial t}\psi^*|\psi|^4\end{aligned} \tag{1.17}$$

Also, we can write

$$\begin{aligned}\frac{\partial}{\partial z}\Big(\psi\frac{\partial\psi^*}{\partial t}\Big)&=-i\left[\psi\frac{\partial^3\psi^*}{\partial t^3}-\frac{\partial^2\psi}{\partial t^2}\frac{\partial\psi^*}{\partial t}\right]-c_1\left[\psi\frac{\partial^4\psi^*}{\partial t^4}+\frac{\partial^3\psi}{\partial t^3}\frac{\partial\psi^*}{\partial t}\right]-i2\psi\frac{\partial}{\partial t}(|\psi|^2\psi^*)+i2\frac{\partial\psi^*}{\partial t}\psi|\psi|^2\\ &\quad -c_2\psi\frac{\partial^2}{\partial t^2}(|\psi|^2\psi^*)-c_2\frac{\partial\psi^*}{\partial t}\frac{\partial}{\partial t}(\psi|\psi|^2)-c_3\psi\frac{\partial^2}{\partial t^2}(|\psi|^4\psi^*)-c_3\frac{\partial\psi^*}{\partial t}\frac{\partial}{\partial t}(\psi|\psi|^4)\\ &\quad -i\alpha\psi\frac{\partial}{\partial t}(|\psi|^4\psi^*)+i\alpha\frac{\partial\psi^*}{\partial t}\psi|\psi|^4\end{aligned} \tag{1.18}$$

Using (1.17) *and* (1.18) *into* (1.13)*, we can find the evolution of frequency along the fiber to be*

$$\begin{aligned}\frac{d\Omega}{dz} &= \frac{1}{2E}\int_{-\infty}^{+\infty}\Big[\Big(\frac{\partial^2\psi^*}{\partial t^2}\frac{\partial\psi}{\partial t}+\frac{\partial\psi^2}{\partial t^2}\frac{\partial\psi^*}{\partial t}\Big)-\Big(\psi\frac{\partial^3\psi^*}{\partial t^3}+\psi^*\frac{\partial^3\psi}{\partial t^3}\Big)\Big]dt\\ &-\frac{ic_1}{2E}\int_{-\infty}^{+\infty}\Big[\Big(\psi^*\frac{\partial^4\psi}{\partial t^4}-\psi\frac{\partial^4\psi^*}{\partial t^4}\Big)+\Big(\frac{\partial^3\psi^*}{\partial t^3}\frac{\partial\psi}{\partial t}-\frac{\partial\psi^3}{\partial t^3}\frac{\partial\psi^*}{\partial t}\Big)\Big]dt\\ &-\frac{ic_2}{2E}\int_{-\infty}^{+\infty}|\psi|^2\Big(\psi^*\frac{\partial^2\psi}{\partial t^2}-\psi\frac{\partial^2\psi}{\partial t^2}\Big)dt-\frac{ic_2}{2E}\int_{-\infty}^{+\infty}\frac{\partial}{\partial t}|\psi|^2\Big(\psi^*\frac{\partial\psi}{\partial t}-\psi\frac{\partial\psi}{\partial t}\Big)dt\\ &-\frac{2}{E}\int_{-\infty}^{+\infty}|\psi|^2\frac{\partial}{\partial t}|\psi|^2dt-\frac{ic_3}{2E}\int_{-\infty}^{+\infty}|\psi|^4\Big(\psi^*\frac{\partial^2\psi}{\partial t^2}-\psi\frac{\partial^2\psi}{\partial t^2}\Big)dt+\\ &-\frac{ic_3}{2E}\int_{-\infty}^{+\infty}\frac{\partial}{\partial t}|\psi|^4\Big(\psi^*\frac{\partial\psi}{\partial t}-\psi\frac{\partial\psi}{\partial t}\Big)dt-\frac{3\alpha}{E}\int_{-\infty}^{+\infty}|\psi|^4\frac{\partial}{\partial t}|\psi|^2dt\end{aligned}\tag{1.19}$$

In order to calculate $\frac{d\Omega}{dz}$*, we evaluate one by one the integrals on right hand side of the* (1.19)*. After computation, we get :*

$$\frac{d\Omega}{dz}=0\tag{1.20}$$

1.4 Evolution of chirp parameter

Next we find the evolution of the chirp parameter. Differentiating (1.5) *with respect to* z*, we can write*

$$\frac{d\tilde{C}}{dz}=\frac{i}{2E}\int_{-\infty}^{+\infty}(t-T)\Big[\frac{\partial}{\partial z}\Big(\psi^*\frac{\partial\psi}{\partial t}\Big)-\frac{\partial}{\partial z}\Big(\psi\frac{\partial\psi^*}{\partial t}\Big)\Big]dt\tag{1.21}$$

From (1.17) *and* (1.18)*, we have :*

$$\begin{aligned}\frac{d\tilde{C}}{dz} &= \frac{1}{2E}\int_{-\infty}^{+\infty}(t-T)\Big[\Big(\frac{\partial^2\psi^*}{\partial t^2}\frac{\partial\psi}{\partial t}+\frac{\partial\psi^2}{\partial t^2}\frac{\partial\psi^*}{\partial t}\Big)-\Big(\psi\frac{\partial^3\psi^*}{\partial t^3}+\psi^*\frac{\partial^3\psi}{\partial t^3}\Big)\Big]dt\\ &-\frac{i}{2E}c_1\int_{-\infty}^{+\infty}(t-T)\Big[\Big(\psi^*\frac{\partial^4\psi}{\partial t^4}-\psi\frac{\partial^4\psi^*}{\partial t^4}\Big)+\Big(\frac{\partial^3\psi^*}{\partial t^3}\frac{\partial\psi}{\partial t}-\frac{\partial\psi^3}{\partial t^3}\frac{\partial\psi^*}{\partial t}\Big)\Big]dt\\ &-\frac{ic_2}{2E}\int_{-\infty}^{+\infty}(t-T)|\psi|^2\Big(\psi^*\frac{\partial^2\psi}{\partial t^2}-\psi\frac{\partial^2\psi}{\partial t^2}\Big)dt-\frac{ic_2}{2E}\int_{-\infty}^{+\infty}(t-T)\frac{\partial}{\partial t}|\psi|^2\Big(\psi^*\frac{\partial\psi}{\partial t}-\psi\frac{\partial\psi}{\partial t}\Big)dt+\\ &-\frac{2}{E}\int_{-\infty}^{+\infty}(t-T)|\psi|^2\frac{\partial}{\partial t}|\psi|^2dt-\frac{ic_3}{2E}\int_{-\infty}^{+\infty}(t-T)|\psi|^4\Big(\psi^*\frac{\partial^2\psi}{\partial t^2}-\psi\frac{\partial^2\psi}{\partial t^2}\Big)dt+\\ &-\frac{ic_2}{2E}\int_{-\infty}^{+\infty}(t-T)\frac{\partial}{\partial t}|\psi|^4\Big(\psi^*\frac{\partial\psi}{\partial t}-\psi\frac{\partial\psi}{\partial t}\Big)dt-\frac{3\alpha}{E}\int_{-\infty}^{+\infty}(t-T)|\psi|^4\frac{\partial}{\partial t}|\psi|^2dt\end{aligned}\tag{1.22}$$

After many integrations by parts, finally, we get :

$$\begin{aligned}
\frac{d\tilde{C}}{dz} \quad = \quad & \frac{-2}{E}\int_{-\infty}^{+\infty}\Big|\frac{\partial\psi}{\partial t}\Big|^2 dt + \frac{3ic_1}{2E}\int_{-\infty}^{+\infty}\Big[\Big(\frac{\partial^2\psi^*}{\partial t^2}\frac{\partial\psi}{\partial t} - \frac{\partial\psi^2}{\partial t^2}\frac{\partial\psi^*}{\partial t}\Big)dt \\
& -\frac{ic_2}{2E}\int_{-\infty}^{+\infty}|\psi|^2\Big(\psi\frac{\partial\psi^*}{\partial t} - \psi^*\frac{\partial\psi}{\partial t}\Big)dt + \frac{ic_2}{E}\int_{-\infty}^{+\infty}(t-T)\frac{\partial}{\partial t}|\psi|^2\Big(\psi^*\frac{\partial\psi}{\partial t} - \psi\frac{\partial\psi^*}{\partial t}\Big)dt \\
& +\frac{1}{E}\int_{-\infty}^{+\infty}\Big|\psi\Big|^4 dt - \frac{ic_3}{E}\int_{-\infty}^{+\infty}|\psi|^4\Big(\psi\frac{\partial\psi^*}{\partial t} - \psi^*\frac{\partial\psi}{\partial t}\Big)dt \\
& +\frac{ic_3}{2E}\int_{-\infty}^{+\infty}(t-T)\frac{\partial}{\partial t}|\psi|^4\Big(\psi^*\frac{\partial\psi}{\partial t} - \psi\frac{\partial\psi^*}{\partial t}\Big)dt \\
& +\frac{ic_3}{2E}\int_{-\infty}^{+\infty}|\psi|^4\Big(\psi\frac{\partial\psi^*}{\partial t} - \psi^*\frac{\partial\psi}{\partial t}\Big)dt + \frac{\alpha}{E}\int_{-\infty}^{+\infty}\Big|\psi\Big|^6 dt
\end{aligned} \tag{1.23}$$

1.5 Evolution of the RMS width

We differentiate (1.4)*, with respect to* z *to obtain*

$$2\sigma E\frac{d\sigma}{dz} = \int_{-\infty}^{+\infty}(t-T)^2(\psi^*\frac{\partial\psi}{\partial z} + \psi\frac{\partial\psi^*}{\partial z})dt \tag{1.24}$$

After calculating, we can write

$$\begin{aligned}
2\sigma E\frac{d\sigma}{dz} \quad = \quad & \int_{-\infty}^{+\infty} -i(t-T)^2\Big(\psi\frac{\partial^2\psi^*}{\partial z^2} - \psi^*\frac{\partial^2\psi}{\partial z^2}\Big)dt - \int_{-\infty}^{+\infty} c_1(t-T)^2\Big(\psi\frac{\partial^3\psi^*}{\partial z^3} + \psi^*\frac{\partial^3\psi}{\partial z^3}\Big)dt \\
& -c_2(t-T)^2\int_{-\infty}^{+\infty}\Big[\psi^*\frac{\partial}{\partial t}\Big(|\psi|^2\psi\Big) + \psi\frac{\partial}{\partial t}\Big(|\psi|^2\psi^*\Big)\Big]dt \\
& -c_3(t-T)^2\int_{-\infty}^{+\infty}\Big[\psi^*\frac{\partial}{\partial t}\Big(|\psi|^4\psi\Big) + \psi\frac{\partial}{\partial t}\Big(|\psi|^4\psi^*\Big)\Big]dt
\end{aligned} \tag{1.25}$$

After many integrations by parts we get:

$$\frac{d\sigma}{dz} = \frac{2\tilde{c}}{\sigma} - \frac{3c_1}{\sigma E}\int_{-\infty}^{+\infty}(t-T)\Big|\frac{\partial\psi}{\partial t}\Big|^2 dt \tag{1.26}$$

2 Numerical simulation with Runge-Kutta 4

Let's choose the pulse shape on the Gaussian form [30, 32]:

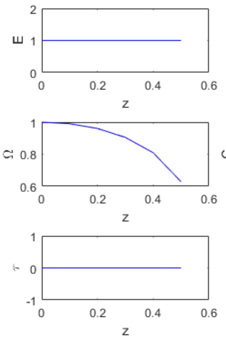
$$\psi(z,t) = A\exp\Big[(i\varphi - i\Omega(t-T) - (1+iC)\frac{(t-T)^2}{2\tau^2}\Big], \tag{2.1}$$

with $\tau^2 = K\sigma^2$, $C = 2\tilde{C}$, $K = cte$

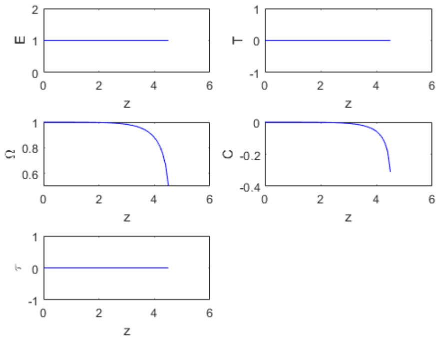
We obtain a variational equations for each parameter as follows:

$$
\begin{cases}
\dfrac{dE}{dz} &= 0; \\
\dfrac{dT}{dz} &= 2\Omega - 3c_1\left(\Omega^2 + \dfrac{1+C^2}{2\tau^2}\right) + \dfrac{3c_2E}{\sqrt{2\pi}2\tau} + \dfrac{c_3E^2}{3\pi\sqrt{3}\tau^2}; \\
\dfrac{d\Omega}{dz} &= 0; \\
\dfrac{dC}{dz} &= -4\Omega^2 - 4\dfrac{1+C^2}{2\tau^2} - 3c_1\left(2\Omega^3 + 3\Omega\dfrac{1+C^2}{\tau^2}\right) + \dfrac{2E + 2Ec_1\Omega - 2Ec_2\Omega + 2Ec_3\Omega}{\sqrt{2\pi}\tau} \\
&\quad + \dfrac{4c_3E^2\Omega + 4c_3E^2C}{3\pi\sqrt{3}\tau^2} + \dfrac{2\alpha E^2}{\pi\sqrt{3}\tau^2}; \\
\dfrac{d\tau}{dz} &= \dfrac{4C}{\tau} - \dfrac{12\Omega c_1 C}{\tau}.
\end{cases}
\tag{2.2}
$$

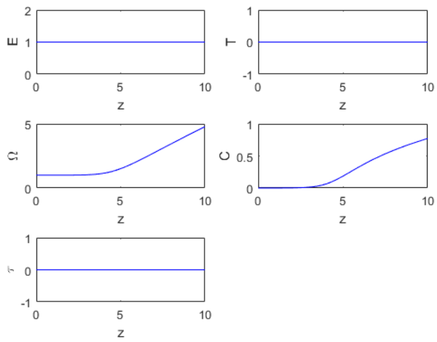
We solve the variational equations by the fourth order Runge-Kutta method [24].
The results obtained for the case $c_i = 0$ $(i \in \{1,2,3\})$ *and* $\alpha \neq 0$ *are showed in Figure 1.*



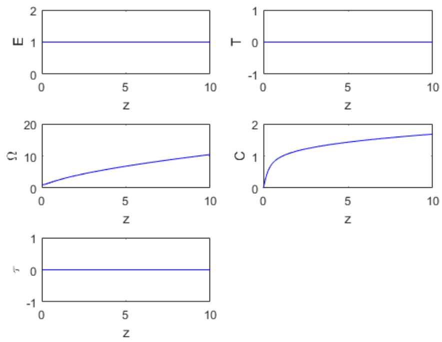
(a) c_1 = 0; c_2 = 0 ; c_3 = 0 ; α = 2



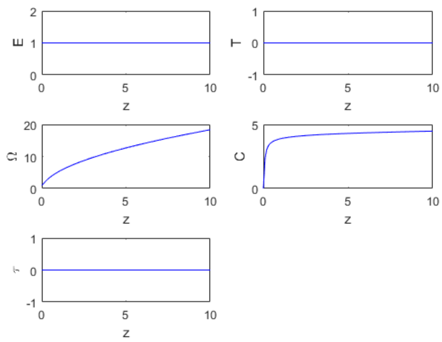
(b) c_1 = 0; c_2 = 0 ; c_3 = 0 ; α = 3.272



(c) c_1 = 0; c_2 = 0 ; c_3 = 0 ; α = 3.273



(d) c_1 = 0; c_2 = 0 ; c_3 = 0 ; α = 10



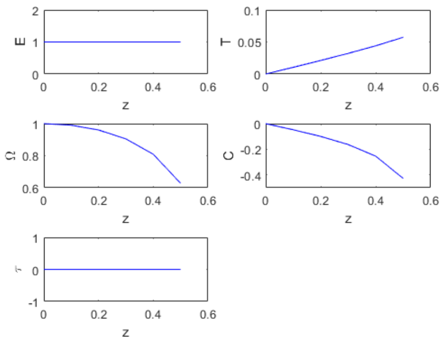
(e) c_1 = 0; c_2 = 0 ; c_3 = 0 ; α = 100

Figure 1: Numerical simulations for case $c_i = 0\ (i \in \{1, 2, 3\})$ and $\alpha \neq 0$

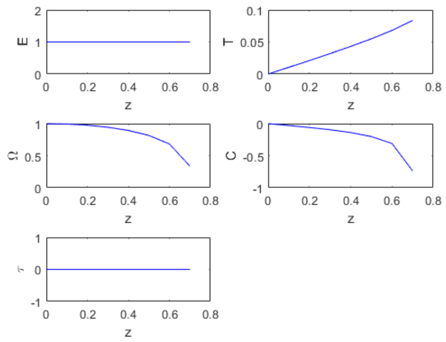
For small values of α $(\alpha \leq 3.272)$, Ω *and* C *decrease. The concavity of* Ω *and* C *changes when* $\alpha \geq 3.273$. *For large values of* α $(\alpha \geq 50)$, Ω *and* C *increase asymptotically. In addition,* E, T *and* τ

remain constant.

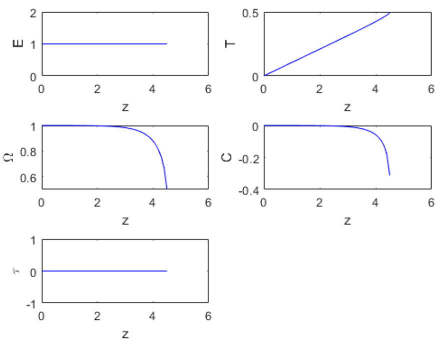
Now, we consider the case $c_i \neq 0$ $(i \in \{1, 2, 3\})$ *and* $\alpha \neq 0$ *showed in Figure 2.*



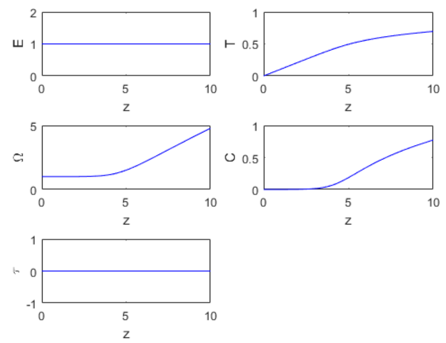
(a) c_1 = 0.01; c_2 = 0.2 ; c_3 = 0.001 ; α = 2



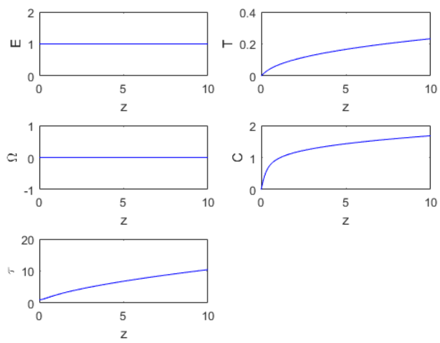
(b) c_1 = 0.01; c_2 = 0.2 ; c_3 = 0.001 ; α = 2.5



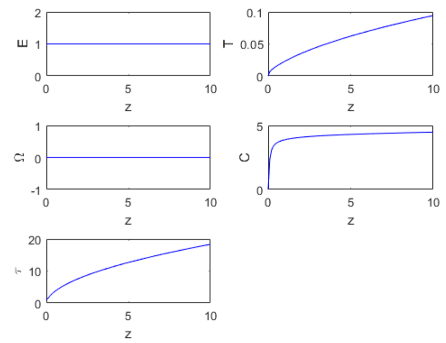
(c) c_1 = 0.01; c_2 = 0.2 ; c_3 = 0.001 ; α = 3.272



(d) c_1 = 0.01; c_2 = 0.2 ; c_3 = 0.001 ; α = 3.273



(e) c_1 = 0.01; c_2 = 0.2 ; c_3 = 0.001 ; α = 10



(f) c_1 = 0.01; c_2 = 0.2 ; c_3 = 0.001 ; α = 100

Figure 2: Numerical simulations for case $c_i \neq 0$ $(i \in \{1, 2, 3\})$ and $\alpha \neq 0$

Ω, C, E and T have the same behaviour as the case above. But, we notice that T is a line with positive slope.

3 Discussion

Since $\frac{dE}{dz} = 0$ and $\frac{d\Omega}{dz} = 0$, therefore the pulse energy and frequency remain constant when the pulse propagates along the fiber.

The equation (1.12) shows that the variation of T is not affected by quintic coefficient α.

The variation of C is affected by all parameters of the pulse (1.23).

Only the coefficient of the third order dispersion c_1 affected the variation of τ (1.26).

Finally, the equation (2.2) is the model of nonlinear dynamics. In fact, we have a system of ODEs and it requires to set initial conditions : $E(0) = 1$; $T(0) = 0$; $\Omega(0) = 1$; $C(0) = 0$ and $\tau(0) = 0$.

We use fourth order Runge-Kutta method for integration. In order to examine the effect of quintic coefficient α on the propagation, we have studied pulse parameters for the case $c_i = 0$; $i \in \{1, 2, 3\}$ and $\alpha \neq 0$, particularly $\alpha = 2$; $\alpha = 3.272$; $\alpha = 3.273$; $\alpha = 10$ and $\alpha = 100$; and the case $c_i \neq 0$; $i \in \{1, 2, 3\}$ and $\alpha \neq 0$; particularly :

- ✓ $c_1 = 0.01$; $c_2 = 0.2$; $c_3 = 0.001$; $\alpha = 2$
- ✓ $c_1 = 0.01$; $c_2 = 0.2$; $c_3 = 0.001$; $\alpha = 2.5$
- ✓ $c_1 = 0.01$; $c_2 = 0.2$; $c_3 = 0.001$; $\alpha = 3.272$
- ✓ $c_1 = 0.01$; $c_2 = 0.2$; $c_3 = 0.001$; $\alpha = 3.273$
- ✓ $c_1 = 0.01$; $c_2 = 0.2$; $c_3 = 0.001$; $\alpha = 10$
- ✓ $c_1 = 0.01$; $c_2 = 0.2$; $c_3 = 0.001$; $\alpha = 100$

In the work of Shwetanshumala and Biswas (2008) with the same equation using collective variables approach, it shows that the influence of self steepening c_2 is the largest on the beam center.

4 Conclusion

In this paper, we have applied the moment method approach to investigate the dynamics of a femtosecond pulse propagation in optical fibers under higher order effects. In order to achieve this, Gaussian ansatz was chosen and fourth order Runge-Kutta method was used to integrate the system of ordinary differential equations. We used initial conditions. For the set of model parameters, we computed the results that are depicted in figures. The variations of each parameter of system show the effect of the quintic coefficient α. The parameter α governs the dynamics of the pulse propagation. The further work will be interested to generalize this equation by using the same method.

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