Hinge Total Domination on Some Graph Families

Original Research Article

Abstract

A set S of vertices in a graph $G = (V(G), E(G))$ is a hinge dominating set if every vertex $u \in V \setminus S$ is adjacent to some vertex $v \in S$ and a vertex $w \in V \setminus S$ such that (v, w) is not an edge in $E(G)$. The hinge domination number $\gamma_h(G)$ is the minimum size of a hinged dominating set. A set S is called a total dominating set of G if for every vertex in V, including those in S is adjacent to at least one vertex in S. The cardinality of a minimum total dominating set in G is called the total domination number of G and denoted as $\gamma_t(G)$. In this study, a new parameter called hinged total dominating set was introduced and defined as, a hinge total dominating set of a graph G is a set S of vertices of G such that S is both a hinge dominating set and total dominating set. The hinge total domination number, $\gamma_{ht}(G)$, is the minimum cardinality of a hinge total dominating set of G. We initiate a study of hinge total dominating set and present its characterization. In addition, we also determine the exact values of hinge total domination number on some graph families.

Keywords: Hinge domination number; Total domination number; Hinge total domination number.

2010 Mathematics Subject Classification: 05C12; 05C82; 91D30

1 Introduction

Total domination in graphs was introduced by Cockayne, Dawes and Hedetniemi (9). The literature on this subject has been surveyed and detailed in the two excellent domination books by Haynes, Hedetniemi, and Slater who did an outstanding job of unifying results scattered through some 1200 domination papers. Much interest in total domination in graphs has recently arisen from a computer program Graffiti.pc that has generated several hundred conjectures on total domination (14).

On one hand, hinge domination in graphs is a new parameter for domination that was recently accepted and published last 2018. It was introduced by Kavitha B.N. and Indrani Kelkar and is still open for many possible studies (15).

In this paper, we introduced hinge total dominating set and present its characterization. In addition, we also determine the exact values of hinge total domination number on some graph families.

All graphs under considered here are nontrivial, simple, undirected, and finite. For graph theoretic terminologies not specifically defined nor described in this study, please refer to (7).

2 Preliminary Notes

For formality, we present the definitions of the concepts discussed in this paper.

Definition 2.1. (7) A vertex of degree 0 is referred to as an isolated vertex and a vertex of degree 1 is an end-vertex or a leaf.

Definition 2.2. (11) A support vertex is the neighbor of a leaf. If a vertex v is adjacent to two or more leaves, v is said to be a strong support vertex

Definition 2.3. (2) A graph H is a subgraph of a graph G, denoted by $H \subseteq G$, if the vertex set $V(H)$ of H is contained in the vertex set $V(G)$ of G and all edges of H are edges in G, i.e, $V(H) \subseteq V(G)$ and $E(H) \subseteq E(G)$. For any vertex subset $S \subseteq V(G)$, the induced subgraph by S denoted by $\langle S \rangle_G$ contains all the edges of $E(G)$ whose extremities belong to S.

Definition 2.4. (7) A set S of vertices of G is a dominating set if every vertex in $V(G) \setminus S$ is adjacent to at least one vertex in S. The minimum cardinality among the dominating sets of G is called the domination number of G and is denoted by $\gamma(G)$. A dominating set of cardinality $\gamma(G)$ is then referred to as a minimum dominating set.

Definition 2.5. (16) A set S of vertices in a graph $G = (V(G), E(G))$ is a hinge dominating set if every vertex $u \in V \setminus S$ is adjacent to some vertex $v \in S$ and a vertex $w \in V \setminus S$ such that (v, w) is not an edge in $E(G)$. The hinge domination number $\gamma_h(G)$ is the minimum size of a hinged dominating set.

Definition 2.6. (13) A set S is called a total dominating set of G if for every vertex in V, including those in S is adjacent to at least one vertex in S. The cardinality of a minimum total dominating set in G is called the total domination number of G and denoted $\gamma_t(G)$. (17) In addition,a dominating set S of a graph G is a total dominating set if the induced subgraph $\langle S \rangle$ has no isolated vertices.

Example 1. *In Figure 1, notice that the* $\gamma_t(G) = 4$ *and* $\gamma_h(G) = 2$. *Hence, in* C_6, $\gamma_t(G) > \gamma_h(G)$.

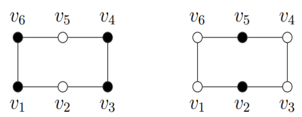

Figure 1: Total Domination and Hinge Domination on C_6

Example 2. *In Figure 2, notice that the* $\gamma_h(G) = 3$ *and* $\gamma_t(G) = 2$. *Hence, in* C_3, $\gamma_h(G) > \gamma_t(G)$.

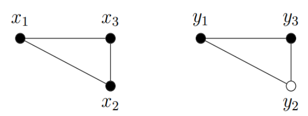

Figure 2: Hinge Domination and Total Domination on C_3

3 Main Results

Definition 3.1. A hinge total dominating set of a graph G is a set S of vertices of G such that S is both a hinge dominating set and total dominating set. The hinge total domination number, $\gamma_{ht}(G)$, is the minimum cardinality of a hinge total dominating set of G.

3.1 Some Realization Results

Theorem 3.1. *Let $G = (V(G), E(G))$ be a nontrivial connected graph with leaf vertices v. If $S \subseteq V(G)$ is a nonempty hinge total dominating set of G, then S contains all the leaf vertices of G.*

Proof. Let $v \in V(G)$ be a leaf vertex of G. On a contrary, assume that $v \notin S$, then $v \in V(G) \setminus S$. Since a leaf vertex is only adjacent to one support vertex, then vertex v cannot be adjacent to a vertex in S and a vertex in $V(G) \setminus S$. This means that S is not a hinge dominating set, a contradiction to the assumption that S is a hinge dominating set of G. Thus, $v \in S$. □

Corollary 3.2. *If S is a nonempty hinge total dominating set of G with leaf vertices, then S contains all support vertices of G.*

Proof. Let $y \in V(G)$ be the support vertices of G. By Theorem 3.1, all leaf vertices are in S. Suppose $y \notin S$, then all leaf vertices are not adjacent to any vertex that is in S. This means that S is not a total dominating set, a contradiction to the assumption that S is a hinge total dominating set of G. Thus, $y \in S$. □

Remark 1. *For any nontrivial, connected graph $G = (V(G), E(G))$, the set $V(G)$ is a hinge total dominating set.*

3.2 Path Graphs

This section presents the results produced on path graphs.

Theorem 3.3. *Let P_n be a path graph, $n \geq 6$, with vertex set $V(P_n)$. Then $S \subset V(P_n)$ is a nonempty hinge total dominating set if and only if the following holds:*

- ***(i)*** *The leaf and support vertices of P_n are in S;*
- ***(ii)*** *The induced subgraph, $\langle V(P_n) \setminus S \rangle$, form a class of P_2;*
- ***(iii)*** *The induced subgraph, $\langle S \rangle$, has no trivial graph.*

Proof. Since P_n, $n \geq 6$, is a nontrivial connected graph, by Theorem 3.1 and Corollary 3.2, the leaf and support vertices of P_n are in S. This proves (i).

Let $v \in \langle V(P_n) \setminus S \rangle$ and S be a nonempty hinge total dominating set of P_n. Then, $|N(v) \cap \langle V(P_n) \setminus S \rangle| = 1$ and $|N(v) \cap S| = 1$. This implies $\deg\,(v) = 1$. Since v is arbitrary, thus for all $v \in \langle V(P_n) \setminus S \rangle$, $\deg\,(v) = 1$. So, the induced subgraph, $\langle V(P_n) \setminus S \rangle$, form a class of P_2. This proves (ii).

Suppose the induced subgraph, $\langle S \rangle$ has an isolated vertex. This implies that there exist a vertex a in the vertex set of $\langle S \rangle$ that is not adjacent to some $b \in S$. This means that S is not a total dominating set, a contradiction since S is a hinge total dominating set. Hence, the $\langle S \rangle$ has no trivial graph. This proves (iii).

The converse immediately follows. □

Remark 2. *For a path P_n of order $2 \leq n \leq 5$, $V(P_n)$ is the hinge total dominating set.*

Theorem 3.4. *If P_n is a path graph of order $n \geq 6$, then*

$$\gamma_{ht}(P_n) = \begin{cases} \frac{n+4}{2} & \text{if } n \equiv 0 \ (\text{ mod } 4)\,; \\ \frac{n+5}{2} & \text{if } n \equiv 1 \ (\text{ mod } 4)\,; \\ \frac{n+2}{2} & \text{if } n \equiv 2 \ (\text{ mod } 4)\,; \\ \frac{n+3}{2} & \text{if } n \equiv 3 \ (\text{ mod } 4)\,. \end{cases}$$

Proof. Let P_n be a path graph of order $n \geq 6$ and $V(P_n) = \{v_0, v_1, ..., v_{n-1}\}$. Assume $S \subset V(P_n)$ is nonempty set hinge total dominating set of P_n. Since P_n is a connected graph, by Theorem 3.1 and Corollary 3.2, $\{v_0, v_1, v_{n-2}, v_{n-1}\}$ is in S.

If $n \equiv 0$ (mod 4) , then S contains vertex sets V_{4i} and V_{4i+1} where $i = \{0, 1, 2, ..., \frac{n-4}{4}\}$. Hence, $V_{4i} = \{v_0, v_4, v_8, ..., v_{n-4}\}$ and $V_{4i+1} = \{v_1, v_5, v_9, ..., v_{n-3}\}$. Notice that $|V_{4i}| = \frac{n-4}{4} + 1 = |V_{4i+1}|$. The remaining vertices that is not covered by V_{4i} and V_{4i+1} are v_{n-2} and v_{n-1}. By Theorem 3.1 and Corollary 3.2, $\{v_{n-2}, v_{n-1}\}$ are in S. Thus,

$$\begin{aligned} |S| &= |V_{4i}| + |V_{4i+1}| + |\{v_{n-2}, v_{n-1}\}| = \frac{n-4}{4} + 1 + \frac{n-4}{4} + 1 + 2 \\ &= \frac{n-4}{4} + 1 + \frac{n-4}{4} + 1 + 2 = \frac{n-4+4+n-4+4+8}{4} \\ &= \frac{2n+8}{4} = \frac{n+4}{2}. \end{aligned}$$

If $n \equiv 1$ (mod 4) , then S contains vertex sets V_{4i} and V_{4i+1} where $i = \{0, 1, 2, ..., \frac{n-5}{4}\}$. Hence, $V_{4i} = \{v_0, v_4, v_8, ..., v_{n-5}\}$ and $V_{4i+1} = \{v_1, v_5, v_9, ..., v_{n-4}\}$. Notice that $|V_{4i}| = \frac{n-5}{4} + 1 = |V_{4i+1}|$. The remaining vertices that is not covered by vertex sets V_{4i} and V_{4i+1} are v_{n-3}, v_{n-2} and v_{n-1}. By Theorem 3.1 and Corollary 3.2, $\{v_{n-2}, v_{n-1}\}$ are in S. Now, suppose $v_{n-3} \notin S$. Then, $v_{n-3} \in V(P_n) \setminus S$ is not adjacent to all vertices of $V(P_n) \setminus S$, which is a contradiction since S is a hinge total dominating set. Hence, $v_{n-3} \in S$. Therefore,

$$\begin{aligned} |S| &= |V_{4i}| + |V_{4i+1}| + |\{v_{n-3}, v_{n-2}, v_{n-1}\}| = \frac{n-5}{4} + 1 + \frac{n-5}{4} + 1 + 3 \\ &= \frac{n-5+4+n-5+4+12}{4} = \frac{2n+10}{4} \\ &= \frac{n+5}{2}. \end{aligned}$$

If $n \equiv 2$ (mod 4) , then S contains vertex sets V_{4i} and V_{4i+1} where $i = \{0, 1, 2, ..., \frac{n-2}{4}\}$. Hence, $V_{4i} = \{v_0, v_4, v_8, ..., v_{n-2}\}$ and $V_{4i+1} = \{v_1, v_5, v_9, ..., v_{n-1}\}$. Notice that $|V_{4i}| = \frac{n-2}{4} + 1 = |V_{4i+1}|$. All vertices are covered by vertex sets V_{4i} and V_{4i+1}. Therefore,

$$\begin{aligned} |S| &= |V_{4i}| + |V_{4i+1}| = \frac{n-2}{4} + 1 + \frac{n-2}{4} + 1 \\ &= \frac{n-2+4+n-2+4}{4} = \frac{2n+4}{4} \\ &= \frac{n+2}{2}. \end{aligned}$$

If $n \equiv 3$ (mod 4) , then S contains vertex sets V_{4i} and V_{4i+1} where $i = \{0, 1, 2, ..., \frac{n-3}{4}\}$. Hence, $V_{4i} = \{v_0, v_4, v_8, ..., v_{n-3}\}$ and $V_{4i+1} = \{v_1, v_5, v_9, ..., v_{n-2}\}$. Notice that $|V_{4i}| = \frac{n-3}{4} + 1 = |V_{4i+1}|$.

The remaining vertex that is not covered by remark 3 is v_{n-1}. By Theorem 3.1, v_{n-1} is in S. Therefore,

$$\begin{aligned}|S| &= |V_{4i}| + |V_{4i+1}| + |\{v_{n-1}\}| = \frac{n-3}{4} + 1 + \frac{n-3}{4} + 1 + 1 \\ &= \frac{n-3+4+n-3+4+4}{4} = \frac{2n+6}{4} \\ &= \frac{n+3}{2}.\end{aligned}$$

Furthermore, let x be an arbitrary element in S. Suppose we have $S \setminus \{x\}$. Then the induced subgraph $\langle V(P_n) \setminus S\rangle$ now contains either P_1 or P_3, a contradiction to the second condition of Theorem 3.3. This means that S is not a hinge total dominating set. Hence, S is a hinge total dominating set with minimum cardinality, $|S| = \gamma_{ht}(P_n)$. Therefore, the results follow. □

3.3 Cycle Graph

This section presents the results produced on cycle graphs.

Theorem 3.5. *Let C_n be a cycle graph, $n \geq 4$, with vertex set $V(C_n)$. Then $S \subset V(C_n)$ is nonempty set hinge total dominating set if and only if the following holds:*

- ***(i)*** *The induced subgraph, $\langle V(C_n) \setminus S\rangle$, form a class of P_2;*
- ***(ii)*** *The induced subgraph, $\langle S\rangle$, has no trivial graph.*

Proof. Let $v \in \langle V(C_n) \setminus S\rangle$ and S be a nonempty hinge total dominating set of C_n. Then, $|N(v) \cap \langle V(C_n) \setminus S\rangle| = 1$ and $N(v) \cap S| = 1$. This implies $deg\ (v) = 1$. Since v is arbitrary, thus for all $v \in \langle V(C_n) \setminus S\rangle$, $\deg\ (v) = 1$. So, the induced subgraph, $\langle V(C_n) \setminus S\rangle$, form a class of P_2. This proves (ii).

Suppose the induced subgraph, $\langle S\rangle$ has a trivial graph. This implies that there exist a vertex a in the vertex set of $\langle S\rangle$ that is not adjacent to some $b \in S$. This means that S is not a total dominating set, a contraction since S is a hinge total dominating set. Hence, the $\langle S\rangle$ has no trivial graph. This proves (ii).

The converse immediately follows. □

Remark 3. *For a cycle graph C_3, $\gamma_{ht}(C_3) = 3$.*

Theorem 3.6. *If C_n is a cycle graph of order $n \geq 4$, then*

$$\gamma_{ht}(C_n) = \begin{cases} \frac{n}{2} & \text{if } n \equiv 0 \ (\ mod\ 4)\,; \\ \frac{n+1}{2} & \text{if } n \equiv 1 \ (\ mod\ 4)\,; \\ \frac{n+2}{2} & \text{if } n \equiv 2 \ (\ mod\ 4)\,; \\ \frac{n+3}{2} & \text{if } n \equiv 3 \ (\ mod\ 4)\,. \end{cases}$$

Proof. Let C_n be a path graph of order $n \geq 4$ and $V(C_n) = \{v_0, v_1, ..., v_{n-1}\}$. Assume $S \subset V(C_n)$ to be nonempty hinge total dominating set of C_n.

If $n \equiv 0 \ (\bmod 4)$, then S contains vertex sets V_{4i} and V_{4i+1} where $i = \{0, 1, 2, ..., \frac{n}{4}\}$. Hence, $V_{4i} = \{v_0, v_4, v_8, ..., v_n\}$ and $V_{4i+1} = \{v_1, v_5, v_9, ..., v_{n+1}\}$. Notice that

$|V_{4i}| = \frac{n}{4} + 1 = |V_{4i+1}|$. Since $v_0 = v_n$ and $v_1 = v_{n+1}$, it necessary to subtract 2 in the cardinality of S to avoid counting the same vertex twice. Therefore,

$$\begin{aligned}|S| &= |V_{4i}| + |V_{4i+1}| - 2 = \frac{n}{4} + 1 + \frac{n}{4} + 1 - 2 \\ &= \frac{n+4+n+4-8}{4} = \frac{2n}{4} \\ &= \frac{n}{2}.\end{aligned}$$

If $n \equiv 1 \pmod 4$, then S contains vertex sets V_{4i} and V_{4i+1} where $i = \{0, 1, 2, ..., \frac{n-1}{4}\}$. Hence, $V_{4i} = \{v_0, v_4, v_8, ..., v_{n-1}\}$ and $V_{4i+1} = \{v_1, v_5, v_9, ..., v_n\}$. Notice that $|V_{4i}| = \frac{n-1}{4} + 1 = |V_{4i+1}|$. Since $v_0 = v_n$, it necessary to subtract 1 in the cardinality of S to avoid counting the same vertex twice.Therefore,

$$\begin{aligned}|S| &= |V_{4i}| + |V_{4i+1}| - 1 = \frac{n-1}{4} + 1 + \frac{n-1}{4} + 1 - 1 \\ &= \frac{n-1+4+n-1+4-4}{4} = \frac{2n+2}{4} \\ &= \frac{n+1}{2}.\end{aligned}$$

If $n \equiv 2 \pmod 4$, then S contains vertex sets V_{4i} and V_{4i+1} where $i = \{0, 1, 2, ..., \frac{n-2}{4}\}$. Hence, $V_{4i} = \{v_0, v_4, v_8, ..., v_{n-2}\}$ and $V_{4i+1} = \{v_1, v_5, v_9, ..., v_{n-1}\}$. Notice that $|V_{4i}| = \frac{n-2}{4} + 1 = |V_{4i+1}|$. Since no vertex is repeated, therefore,

$$\begin{aligned}|S| &= |V_{4i}| + |V_{4i+1}| = \frac{n-2}{4} + 1 + \frac{n-2}{4} + 1 \\ &= \frac{n-2+4+n-2+4}{4} = \frac{2n+4}{4} \\ &= \frac{n+2}{2}.\end{aligned}$$

If $n \equiv 3 \pmod 4$, then S contains vertex sets V_{4i} and V_{4i+1} where $i = \{0, 1, 2, ..., \frac{n-3}{4}\}$. Hence, $V_{4i} = \{v_0, v_4, v_8, ..., v_{n-3}\}$ and $V_{4i+1} = \{v_1, v_5, v_9, ..., v_{n-2}\}$. Notice that $|V_{4i}| = \frac{n-3}{4} + 2$ and $|V_{4i+1}| = \frac{n-3}{4} + 1$. In addition, the vertex v_{n-1} is also in S since $S \setminus \{v\}$ would make S not a hinge dominating set, a contradiction. Now, notice that $v_1 = v_{n+1}$, so, it necessary to subtract 1 in the cardinality of S to avoid counting the same vertex twice. Therefore,

$$\begin{aligned}|S| &= |V_{4i}| + |V_{4i+1}| + |\{v_{n-1}\}| - 1 = \frac{n-3}{4} + 2 + \frac{n-3}{4} + 1 + 1 - 1 \\ &= \frac{n-3+8+n-3+4+4-4}{4} = \frac{2n+6}{4} \\ &= \frac{n+3}{2}.\end{aligned}$$

Furthermore, let x be an arbitrary element in S. Suppose we have $S \setminus \{x\}$. Then the induced subgraph $\langle V(C_n) \setminus S \rangle$ now contains either P_1 or P_3, a contradiction to the second condition of Theorem 3.3. This means that S is not a hinge total dominating set. Hence, S is a hinge total dominating set with minimum cardinality, $|S| = \gamma_{ht}(C_n)$. Therefore, the results follow. □

3.4 Star Graph

This section presents the results produced star graphs.

Theorem 3.7. *For any star graph $K_{1,n}$, $n \geq 1$, $\gamma_{ht}(K_{1,n}) = n + 1$.*

Proof. Let $K_{1,n}$ be a star graph with $V(K_{1,n}) = \{x, v_0, v_1, ..., v_{n-1}\}$. Since $K_{1,n}$ are composed of leaf vertices $\{v_0, v_1, ..., v_{n-1}\}$ and support vertex $\{x\}$, by Theorem 3.1 and Corollary 3.2, $V(K_{1,n})$ must be in the hinge total dominatimg set S. Therefore, $\gamma_{ht}(K_{1,n}) = |S| = n + 1$. □

3.5 Complete Bipartite Graph

This section presents the results produced on complete bipartite graphs.

Theorem 3.8. *Let $K_{m,n}$ be a complete bipartite graph, $m, n \geq 2$ with partite sets A and B such that $|A| = m$ and $|B| = n$. Then, $S \subset V(K_{m,n})$ is nonempty hinge total dominating set if and only if the following holds.*

(i) $1 \leq |S \cap A|$ *and* $1 \leq |S \cap B|$*;*

(ii) $|S \cap A| \leq m - 1$ *and* $|S \cap B| \leq n - 1$*.*

Proof. Without loss of generality, suppose all vertices from partite set A are not in S. Then, all vertices in B that is in S are not adjacent to vertices in S. This means that S is not a total dominating set of $K_{m,n}$, a contradiction to the assumption that S is a hinge total dominating set. Hence, atleast one vertex from A must be in S. Therefore, $1 \leq |S \cap A|$. The same is true for partite set B, so, $1 \leq |S \cap B|$. This proves (i). Now, suppose all vertices from partite set A are in S. Then, all vertices in B that is not in S are not adjacent to vertices in $K_{m,n} \setminus S$. This means that S is not a hinge dominating set of $K_{m,n}$, a contradiction to the assumption that S is a hinge total dominating set. Hence, at least one vertex from A must not be in S. Therefore, $|S \cap A| \leq m - 1$. The same is true for partite set B, so, $|S \cap B| \leq n - 1$. This proves (ii).

The converse immediately follows. □

Theorem 3.9. *If $K_{m,n}$ is a complete bipartite graph with $m, n \geq 2$, then $\gamma_{ht}(K_{m,n}) = 2$.*

Proof. Let $K_{m,n}$ be a graph with partite sets A and B such that $m, n \geq 2$. Assume S to be a hinge total dominating set of $K_{m,n}$.

Suppose $\gamma_{ht}(K_{m,n}) = 1$ where $a \in A$ is the only element of S. Then, vertices $A \setminus \{a\}$ is not adjacent to any vertex in S. This implies that S is not a hinge dominating set. In addition, vertex $a \in S$ i not adjacent to any vertices in S which implies that S is not a total dominating set. This is a contradiction since S is a hinge total dominating set. Hence, $\gamma_{ht}(K_{m,n}) \neq 1$ or $\gamma_{ht}(K_{m,n}) \geq 2$. Since we can find vertices $a \in A$ and $b \in B$ that makes the set $S = \{a, b\}$ a hinge total dominating set, hence, $\gamma_{ht}(K_{m,n}) \leq 2$. Therefore, $\gamma_{ht}(K_{m,n}) = 2$. □

3.6 Complete Graph

This section presents the results produced on complete graphs.

Theorem 3.10. *For any complete graph K_n, $n \geq 2$, $\gamma_{ht}(K_n) = n$.*

Proof. Assume S to be a hinge total dominating set of K_n. Suppose $\gamma_{ht}(K_n) < n$. Then there exists a vertex $x \in V(K_n) \setminus S$. Since all the vertices in K_n are adjacent, then no vertex x can make S a hinge dominating set. This is a contradiction since S is a hinge dominating set. Hence, $\gamma_{ht}(K_n) \geq n$. By Remark 1, all vertices of K_n can make S a hinge total dominating set. Hence, $\gamma_{ht}(K_n) \leq n$. Therefore, $\gamma_{ht}(K_n) = n$. □

3.7 Wheel Graph

This section presents the results produced on wheel graphs.

Remark 4. *For a wheel graph* W_3, $\gamma_{ht}(W_3) = 4$.

Theorem 3.11. *For any wheel graph* W_n, $n \geq 4$, $\gamma_{ht}(W_n) = \gamma_{ht}(C_n)$.

Proof. By definition, wheel graph is a join of a cycle graph C_n and a trivial graph K_1. Assume $S \subseteq V(W_n)$ is a nonempty hinge total dominating set of W_n. Now, by Theorem 3.6, the $\gamma_{ht}(C_n)$ was determined. Suppose $x \in K_1$ is not in S, then no condition was defied in the assumption of S. The same result holds if $x \in K_1$ is in S. With this, the least number of vertices in W_n is the same as the least number of vertices in C_n. Therefore, $\gamma_{ht}(W_n) = \gamma_{ht}(C_n)$. □

4 Conclusion

Hinge domination has been used to create better network communication (16) while Total dominations has been used to represent network design in minimizing trade-off between resource allocation and redundancy (13). In this study, we combined these types of domination and introduced Hinge Total domination. We present some characterizations and exact values for a hinge total domination number on some graph families namely; path graphs, cycle graphs, star graphs, complete bipartite graphs, complete graphs, and wheel graphs. Moreover, some vertices that is always present in any hinge total dominating set was also identified.

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