On The Diophatine Equation $ab(cd+1)+L=u^2+v^2$

Original Research Article

Abstract

The search for integer sums of two squares is still an open problem. A number of research conducted so far has put more attention to Fermat sums of two squares with little attention given to other methods of generating sums of two squares. Let a, b, c, d, u, v and L be positive integers. This study introduces the relation $ab(cd+1)+L=u^2+v^2$ for generating integer sums of two squares. The main objective of this study is to develop general formulae by determining the integer values a, b, c, d, u, v and L such that $ab(cd+1)+L=u^2+v^2$. Moreover, this research provides conjecture for the title equation.

Keywords: Diophantine Equation; Sums of Two Squares

1 Introduction

The classification of integer sums of two squares is still an open problem. A number of research done so far has only provided partial solutions with no universal method of generating all sums of two squares. Let a, b, c, d and K be positive integers. In this study we develop and introduce the relation $ab(cd+1)+K=u^2+v^2$ for generating integer sums of two squares. The theory of sums of two squares has a rich history and goes back to early 16th century pioneered by Fermat 1640.

He stated that an odd prime p is a sum of two squares if and only if $p \equiv 1 mod 4$. This theorem was stated without any formal proof. The major contribution to this theorem was due to Heath-Brown who gave a complete elementary proof in [2] and Zagier[3]. This proof, due to Zagier, is a simplification of an earlier proof by Heath-Brown, which in turn was inspired by a proof of Liouville. The technique of the proof is a combinatorial analogue of the topological principle that the Euler characteristics of a topological space with an involution and of its fixed-point set have the same parity and is reminiscent of the use of sign-reversing involutions in the proofs of combinatorial bijections. In [1], A. David Christopher gave a partition-theoretic proof by considering partitions of the odd prime n having exactly two sizes each occurring exactly and by showing that at least one such partition exists if n is congruent to 1 modulo 4. This research seeks to explore the diophantine equation $ab(cd+1)+L=u^2+v^2$ thereby adding some more general formulae to existing results on integer sums of two squares.

2 On The Diophantine Equation $ab(cd+1)+L=u^2+v^2$.

Next, we present some general results for integer sums of two squares.

Proposition 2.1. *Let a,b,c and d be any positive integers such that $b-a=c-b=d-c=1$. For any $L=b+1$. Then $ab(cd+1)+b+1=b^2+[(a+1)^2+a]^2$.*

Proof. Let $a=m, b=m+1, c=m+2, d=m+3$. Proceeding from L.H.S to the R.H.S we have $ab(cd+1)+b+1=m(m+1)[(m+2)(m+3)+1]+m+1+1=(m^2+m)[m(m+3)+2(m+3)+1]+m+2=(m^2+m)[m^2+5m+7]+m+2=m^2(m^2+5m+7)+m(m^2+5m+7)+m+2=m^4+6m^3+12m^2+8m+2=m^4+6m^3+11m^2+6m+1+m^2+2m+1=[(m+1)^2+m]^2+[m+1]^2$. Since $a=m$ and $b=m+1$ the result easily follows.

□

Proposition 2.2. *Let a,b,c and d be any positive integers such that $b-a=c-b=d-c=1$. For any $L=3b+2$. Then*
$ab(cd+1)+3b+2=[a+2]^2+[(a+1)^2+a]^2=[b+1]^2+[(b-1)+b^2]^2=c^2+[(c-2)+(c-1)^2]^2$.

Proof. Let $a=m, b=m+1, c=m+2, d=m+3$. Proceeding from L.H.S to the R.H.S we have $ab(cd+1)+3b+2=m(m+1)\{(m+2)(m+3)+1\}+3(m+1)+2=(m^2+m)\{m(m+3)+2(m+3)+1\}+3m+3+2=(m^2+m)\{m^2+3m+2m+6+1\}+3m+5=(m^2+m)\{m^2+5m+7\}+3m+5=m^2(m^2+5m+7)+m(m^2+5m+7)+3m+5=m^4+5m^3+7m^2+m^3+5m^2+7m+3m+5=m^4+6m^3+12m^2+10m+5=m^2+4m+4+m^4+6m^3+11m^2+6m+1=[a+2]^2+[(a+1)^2+a]^2$ Since $b=m+1$ and $c=m+2$ the other part of the result easily follows.

□

Proposition 2.3. *Let a,b,c and d be positive integers of the form $2m+1$ such that $b-a=c-b=d-c=2$ and L be any fixed integer such that $L=17$. For any $m\geq 0$ we have*
$ab(cd+1)+17=[a+1]^2+[(b-1)^2+4(b-1)-1]^2=[\frac{a+b}{2}]^2+[(b-1)^2+4(b-1)-1]^2$.

Proof. Let $a=2m+1, b=2m+3, c=2m+5, d=2m+7$. Proceeding from L.H.S to the R.H.S we have $ab(cd+1)+17=(2m+1)(2m+3)\{(2m+5)(2m+7)+1\}+17=2m(2m+3)+1(2m+3)\{2m(2m+7)+5(2m+7)+1\}+17=(4m^2+6m+2m+3)\{4m^2+14m+10m+35+1\}+17=(4m^2+8m+3)\{4m^2+24m+36\}+17=4m^2(4m^2+24m+36)+8m(4m^2+24m+36)+3(4m^2+24m+36)+17=16m^4+96m^3+144m^2+32m^3+192m^2+288m+12m^2+72m+108+17=16m^4+128m^3+348m^2+360m+125=m^2+4m+4+m^4+6m^3+11m^2+6m+1=[a+1]^2+[(b-1)^2+4(b-1)-1]^2$ establishing the results. Since $[a+1]=[\frac{a+b}{2}]$ the other part of the results follows. □

Proposition 2.4. *Let a, b, c and d be positive integers of the form $2m+2$ such that $b-a=c-b=d-c=2$ and L be any fixed integer such that $L=17$. For any $m \geq 0$ we have*
$ab(cd+1)+17=[a+1]^2+[a^2+6a+4]^2=[\frac{a+b}{2}]^2+[a^2+6a+4]^2$.

Proof. Let $a=2m+2, b=2m+4, c=2m+6, d=2m+8$. Proceeding from L.H.S to the R.H.S we have $ab(cd+1)+17=(2m+2)(2m+4)\{(2m+6)(2m+8)+1\}+17=2m(2m+4)+2(2m+4)\{2m(2m+8)+6(2m+8)+1\}+17=(4m^2+8m+4m+8)\{4m^2+16m+12m+48+1\}+17=(4m^2+12m+8)\{4m^2+28m+49\}+17=4m^2(4m^2+28m+49)+12m(4m^2+28m+49)+8(4m^2+28m+49)+17=16m^4+112m^3+196m^2+48m^3+336m^2+588m+32m^2+224m+392+17=16m^4+160m^3+564m^2+812m+409=4m^2+12m+9+16m^4+160m^3+562m^2+800m+400=[2m+3]^2+[(2m+2)^2+6(2m+2)+4]^2$. Since $a=2m+2, b=2m+4$ the result easily follows. □

Proposition 2.5. *Let a, b, c and d be positive integers of the form $2m+1$ such that $b-a=c-b=d-c=2$. For any $L=2(b+8)$ and $m \geq 0$ we have*
$ab(cd+1)+2(b+8)=b^2+[(b-1)^2+4(b-1)-1]^2=[\frac{a+c}{2}]^2+[(b-1)^2+4(b-1)-1]^2$.

Proof. Let $a=2m+1, b=2m+3, c=2m+5, d=2m+7$. Proceeding from L.H.S to the R.H.S we have $ab(cd+1)+2(b+8)=(2m+1)(2m+3)\{(2m+5)(2m+7)+2\}+2(2m+3+8)=2m(2m+3)+1(2m+3)\{2m(2m+7)+5(2m+7)+1\}+2(2m+11)=(4m^2+6m+2m+3)\{4m^2+14m+10m+35+1\}+4m+22=(4m^2+8m+3)\{4m^2+24m+36\}+4m+22=4m^2(4m^2+24m+36)+8m(4m^2+24m+36)+3(4m^2+24m+36)+4m+22=16m^4+96m^3+144m^2+32m^3+192m^2+288m+12m^2+72m+108+4m+22=16m^4+128m^3+348m^2+364m+130=4m^2+12m+9+16m^4+128m^3+344m^2+352m+121=[2m+3]^2+[(2m+3)^2+4(2m+2)-1]^2$.Since $b=2m+3$ we have $ab(cd+1)+2(b+8)=b^2+[(b-1)^2+4(b-1)-1]^2$. whence the other part of the results follows. □

Proposition 2.6. *Let a, b, c and d be positive integers of the form $2m+2$ such that $b-a=c-b=d-c=2$. For any $L=2(b+8)$ and $m \geq 0$ we have $ab(cd+1)+2(b+8)=b^2+[(a^2+6a+4)]^2$.*

Proof. Let $a=2m+2, b=2m+4, c=2m+6, d=2m+8$. Proceeding from L.H.S to the R.H.S we have $ab(cd+1)+2(b+8)=(2m+2)(2m+4)\{(2m+6)(2m+8)+1\}+2(2m+4+8)=2m(2m+4)+2(2m+4)\{2m(2m+8)+6(2m+8)+1\}+2(2m+12)=(4m^2+8m+4m+8)\{4m^2+16m+12m+48+1\}+4m+24=4m^2(4m^2+28m+49)+12m(4m^2+28m+49)+8(4m^2+28m+49)+4m+24=16m^4+112m^3+196m^2+48m^3+336m^2+588m+32m^2+224m+392+4m+24=16m^4+160m^3+564m^2+816m+416=4m^2+16m+16+16m^4+160m^3+560m^2+800m+400=[2m+4]^2+[(2m+2)^2+6(2m+2)+4]^2=b^2+[(a^2+6a+4)]^2$ establishing the proof. □

Proposition 2.7. *Let a, b, c and d be any positive integers such that $b-a=c-b=d-c=3$ with $L=b+82$ and $a=3m+1$ where $m \geq 0$. Then*
$ab(cd+1)+b+82=[b-1]^2+[(b-1)^2+5(b-1)-5]^2=[\frac{a+b+1}{2}]^2+[(b-1)^2+5(b-1)-5]^2$.

Proof. Let $a=3m+1, b=3m+4, c=3m+7, d=3m+10$. Proceeding from L.H.S to the R.H.S we have $ab(cd+1)+b+82=(3m+1)(3m+4)\{(3m+7)(3m+10)+1\}+b+82=3m(3m+4)+1(3m+4)\{3m(3m+10)+7(3m+10)+1\}+3m+86=(9m^2+12m+3m+4)\{9m^2+30m+21m+70+1\}+3m+86=(9m^2+15m+4)(9m^2+51m+71)+3m+86=9m^2(9m^2+51m+71)+15m(9m^2+51m+71)+4(9m^2+51m+71)+3m+86=81m^4+459m^3+639m^2+135m^3+765m^2+1065m+36m^2+204m+284+3m+86=9m^2+18m+9+81m^4+297m^3+171m^2+297m^3+1089m^2+627m+171m^2+627m+361=[3m+3]^2+[(3m+3)^2+5(3m+3)-5]^2$. Since $b-1=\frac{a+b+1}{2}=3m+3$ we have $ab(cd+1)+b+82=[b-1]^2+[(b-1)^2+5(b-1)-5]^2=[\frac{a+b+1}{2}]^2+[(b-1)^2+5(b-1)-5]^2$. □

Proposition 2.8. *Let a, b, c and d be any positive integers such that $b-a=c-b=d-c=3$ with $L=b+82$ and $a=3m+2$ where $m \geq 0$. Then*
$ab(cd+1)+b+82=[b-1]^2+[(b-2)^2+7(b-2)+1]^2=[\frac{a+b+1}{2}]^2+[(b-2)^2+7(b-2)+1]^2$.

Proof. Let $a=3m+2, b=3m+5, c=3m+8, d=3m+11$. Proceeding from L.H.S to the R.H.S we have $ab(cd+1)+b+82=(3m+2)(3m+5)\{(3m+8)(3m+11)+1\}+b+82=3m(3m+5)+2(3m+5)\{3m(3m+11)+8(3m+11)+1\}+3m+87=(9m^2+21m+10)\{9m^2+33m+24m+88+1\}+3m+87=9m^2(9m^2+57m+89)+21m(9m^2+57m+89)+10(9m^2+57m+89)+3m+87=81m^4+513m^3+801m^2+189m^3+1197m^2+1869m+90m^2+570m+890+3m+87=9m^2+24m+16+81m^4+351m^3+279m^2+351m^3+1521m^2+1209m+279m^2+1209m+961=[3m+4]^2+[(3m+3)^2+7(3m+3)+1]^2$.Since $b-1=3m+4$ and $b-2=3m+3$ the results easily follows. ☐

Proposition 2.9. *Let a, b, c and d be any positive integers such that $b-a=c-b=d-c=3$ with $L=b+82$ and $a=3m+3$ where $m \geq 0$. Then*
$ab(cd+1)+b+82=[b-1]^2+[(b-3)^2+9(b-3)+9]^2=[\frac{a+b+1}{2}]^2+[(b-3)^2+9(b-3)+9]^2$.

Proof. Let $a=3m+3, b=3m+6, c=3m+9, d=3m+12$. Proceeding from L.H.S to the R.H.S we have $ab(cd+1)+b+82=(3m+3)(3m+6)\{(3m+9)(3m+12)+1\}+b+82=3m(3m+6)+3(3m+6)\{3m(3m+21)+9(3m+12)+1\}+3m+88=(9m^2+18m+9m+18)\{9m^2+36m+27m+109\}+3m+88=9m^2(9m^2+63m+109)+27m(9m^2+63m+109)+18(9m^2+63m+109)+3m+88=81m^4+567m^3+981m^2+243m^3+1701m^2+2943m+162m^2+1134m+1962+3m+88=9m^2+30m+25+81m^4+405m^3+405m^2+405m^3+2025m^2+2025m+405m^2+2025m+2025=[3m+5]^2+[(3m+3)^2+9(3m+3)+9]^2=[b-1]^2+[(b-3)^2+9(b-3)+9]^2=[\frac{a+b+1}{2}]^2+[(b-3)^2+9(b-3)+9]^2$ since $b-1=\frac{a+b+1}{2}=3m+5$ and $b-3=3m+3$ the results easily follows. ☐

Proposition 2.10. *Let a, b, c and d be positive integers of the form $3m+1$ such that $b-a=c-b=d-c=3$. For any $L=3a(a-1)+81$ and $m \geq 0$. Then*
$ab(cd+1)+3a(a-1)+81=[2a]^2+[(b-1)^2+5(b-1)-5]^2$.

Proof. Let $a=3m+1, b=3m+4, c=3m+7, d=3m+10$. Proceeding from L.H.S to the R.H.S we have $ab(cd+1)+3a(a-1)+81=(3m+1)(3m+4)\{(3m+7)(3m+10)+1\}+3(3m+1)(3m+1-1)+81=3m(3m+4)+1(3m+4)\{3m(3m+10)+7(3m+10)+1\}+(9m+3)3m+81=(9m^2+12m+3m+4)\{9m^2+30m+21m+70+1\}+27m^2+9m+81=(9m^2+15m+4)(9m^2+51m+71)+27m^2+9m+81=9m^2(9m^2+51m+71)+15m(9m^2+51m+71)+4(9m^2+51m+71)+27m^2+9m+81=81m^4+459m^3+639m^2+135m^3+765m^2+1065m+36m^2+204m+284+27m^2+3m+81=36m^2+24m+4+81m^4+297m^3+171m^2+297m^3+1089m^2+627m+171m^2+627m+361=[2(3m+1)]^2+[(3m+3)^2+5(3m+3)-5]^2$.Since $a=3m+1, b=3m+4$ we have $ab(cd+1)+3a(a-1)+81=[2a]^2+[(b-1)^2+5(b-1)-5]^2$. ☐

Proposition 2.11. *Let a, b, c and d be positive integers of the form $3m+2$ such that $b-a=c-b=d-c=3$. For any $L=3a(a-1)+81$ and $m \geq 0$. Then*
$ab(cd+1)+3a(a-1)+81=[a+b-3]^2+[(b-2)^2+7(b-2)+1]^2$.

Proof. Let $a=3m+2, b=3m+5, c=3m+8, d=3m+11$. Proceeding from L.H.S to the R.H.S we have $ab(cd+1)+3a(a-1)+81=(3m+2)(3m+5)\{(3m+8)(3m+11)+1\}+3(3m+2)(3m+2-1)+81=3m(3m+5)+2(3m+5)\{3m(3m+11)+8(3m+11)+1\}+(9m+6)(3m+1)+81=(9m^2+15m+6m+10)(9m^2+33m+24m+88+1)+9m(3m+1)+6(3m+1)+6(3m+1)+81=(9m^2+21m+10)(9m^2+57m+89)+27m^2+9m+18m+6+81=9m^2(9m^2+57m+89)+21m(9m^2+57m+89)+10(9m^2+57m+89)+27m^2+27m+87=81m^4+513m^3+801m^2+189m^3+1197m^2+1869m+90m^2+570m+890+27m^2+27m+87=36m^2+48m+16+81m^4+351m^3+279m^2+351m^3+1521m^2+1209m+279m^2+1209m+961=[6m+4]^2+[(3m+3)^2+7(3m+3)+1]^2$.Since $a+b-3=6m+4, b-2=3m+3$ we have $ab(cd+1)+3a(a-1)+81=[2a]^2+[a+b-3]^2+[(b-2)^2+7(b-2)+1]^2$. ☐

Proposition 2.12. *Let a, b, c and d be positive integers of the form $3m+3$ such that $b-a=c-b=d-c=3$. For any $L=3a(a-1)+81$ and $m \geq 0$. Then*
$ab(cd+1)+3a(a-1)+81=[a+b-3]^2+[(b-3)^2+9(b-3)+9]^2$.

Proof. Let $a=3m+3, b=3m+6, c=3m+9, d=3m+12$. Proceeding from L.H.S to the R.H.S we have $ab(cd+1)+3a(a-1)+81=(3m+3)(3m+6)\{(3m+9)(3m+12)+1\}+3(3m+3)(3m+3-1)+81=3m(3m+6)+3(3m+6)\{3m(3m+12)+9(3m+12)+1\}+(9m+9)(3m+2)+81=(9m^2+18m+9m+18)\{9m^2+36m+27m+108+1\}+9m(3m+2)+9(3m+2)+81=(9m^2+27m+18)\{9m^2+63m+109\}+27m^2+18m+27m+18+81=9m^2(9m^2+63m+109)+27m(9m^2+63m+109)+18(9m^2+63m+109)+27m^2+45m+99=81m^4+567m^3+981m^2+243m^3+1701m62+2943m+162m^2+1134m+1962+27m^2+45m+99=36m^2+72m+36+81m^4+405m^3+405m^2+405m^3+2025m^2+2025m+405m^2+2025m+2025=[6m+6]^2+[(3m+3)^2+9(3m+3)+9]^2$.Since $a+b-3=6m+6, b-2=3m+3$ we have $ab(cd+1)+3a(a-1)+81=[2a]^2+[a+b-3]^2+[(b-2)^2+7(b-2)+1]^2$.

□

Proposition 2.13. *Let a, b, c and d be positive integers of the form $3m+1$ such that $b-a=c-b=d-c=3$. For any $L=3a^2+a+82$ and $m \geq 0$. Then*
$ab(cd+1)+3a^2+a+82=[a+b-2]^2+[(b-1)^2+5(b-1)-5]^2$.

Proof. Let $a=3m+1, b=3m+4, c=3m+7, d=3m+10$. Proceeding from L.H.S to the R.H.S we have $ab(cd+1)+3a^2+a+82=(3m+1)(3m+4)\{(3m+7)(3m+10)+1\}+3(3m+1)^2+(3m+1)+82=3m(3m+4)+1(3m+4)\{3m(3m+10)+7(3m+10)+1\}+3(3m+1)^2+3m+1+82=(9m^2+12m+3m+4)\{9m^2+30m+21m+70+1\}+27m^2+21m+86=(9m^2+15m+4)(9m^2+51m+71)+27m^2+21m+86=9m^2(9m^2+51m+71)+15m(9m^2+51m+71)+4(9m^2+51m+71)+27m^2+21m+86=81m^4+459m^3+639m^2+135m^3+765m^2+1065m+36m^2+204m+284+27m^2+21m+86=36m^2+36m+9+81m^4+297m^3+171m^2+297m^3+1089m^2+627m+171m^2+627m+361=[6m+3]^2+[(3m+3)^2+5(3m+3)-5]^2$.Since $a+b-2=6m+3, b-1=3m+3$ we have $ab(cd+1)+3a(a-1)+81=[a+b-2]^2+[(b-1)^2+5(b-1)-5]^2$. □

Proposition 2.14. *Let a, b, c and d be positive integers of the form $3m+2$ such that $b-a=c-b=d-c=3$. For any $L=3a^2+a+82$ and $m \geq 0$. Then*
$ab(cd+1)+3a^2+a+82=[a+b-2]^2+[(b-2)^2+7(b-2)+1]^2$.

Proof. Let $a=3m+2, b=3m+5, c=3m+8, d=3m+11$. Proceeding from L.H.S to the R.H.S we have $ab(cd+1)+3a^2+a+82=(3m+2)(3m+5)\{(3m+8)(3m+11)+1\}+3(3m+2)^2+(3m+2)+82=3m(3m+5)+2(3m+5)\{3m(3m+11)+8(3m+11)+1\}+3(3m+2)^2+(3m+2)+82=(9m^2+15m+6m+10)(9m^2+33m+24m+88+1)+3(9m^2+12m+4)+3m+84=(9m^2+21m+10)(9m^2+57m+89)+27m^2+39m+96=9m^2(9m^2+57m+89)+21m(9m^2+57m+89)+10(9m^2+57m+89)+27m^2+39m+96=81m^4+513m^3+801m^2+189m^3+1197m^2+1869m+90m^2+570m+890+27m^2+39m+96=36m^2+60m+25+81m^4+351m^3+279m^2+351m^3+1521m^2+1209m+279m^2+1209m+961=[6m+5]^2+[(3m+3)^2+7(3m+3)+1]^2$.Since $a+b-2=6m+5, b-2=3m+3$ we have $ab(cd+1)+3a^2+a+82=[a+b-2]^2+[(b-2)^2+7(b-2)+1]^2$.

□

Proposition 2.15. *Let a, b, c and d be positive integers of the form $3m+3$ such that $b-a=c-b=d-c=3$. For any $L=3a^2+a+82$ and $m \geq 0$. Then*
$ab(cd+1)+3a^2+a+82=[a+b-2]^2+[(b-3)^2+7(b-3)+9]^2$.

Proof. Let $a=3m+3, b=3m+6, c=3m+9, d=3m+12$. Proceeding from L.H.S to the R.H.S we have $ab(cd+1)+3a^2+a+82=(3m+3)(3m+6)\{(3m+9)(3m+12)+1\}+3(3m+3)^2+(3m+3)+82=3m(3m+6)+3(3m+6)\{3m(3m+12)+9(3m+12)+1\}+27m^2+54m+27+3m+85=(9m^2+18m+9m+18)\{9m^2+36m+27m+108+1\}+27m^2+57m+112=(9m^2+27m+18)\{9m^2+63m+109\}+27m^2+57m+112=9m^2(9m^2+63m+109)+27m(9m^2+63m+109)+$

$18(9m^2+63m+109)+27m^2+57m+112 = 81m^4+567m^3+981m^2+243m^3+1701m62+2943m+162m^2+1134m+1962+27m^2+57m+112 = 36m^2+84m+49+81m^4+405m^3+405m^2+405m^3+2025m^2+2025m+405m^2+2025m+2025 = [6m+7]^2+[(3m+3)^2+9(3m+3)+9]^2$.Since $a+b-2 = 6m+7, b-3 = 3m+3$ we have $ab(cd+1)+3a^2+a+82 = [a+b-2]^2+[(b-3)^2+7(b-3)+9]^2$.
□

The next result provides conjecture for the title equation.

Conjecture 2.1. *Let a, b, c and d be positive integers and L some constant. Then the equation $ab(cd+1)+L = u^2+v^2$ has many integral solution but no general solution without the restriction $b-a = c-b = d-c = \pm u$ where u is an integer .*

Conclusion
This paper has researched on diophantine equation $ab(cd+1)+L = u^2+v^2$. The study revealed some novel formulas for generating sums of two squares. This was achieved by determining integers a, b, c, d, u, v and L together with factorization argument and application of modular arithmetic. To this far, research in this area is still minimal and we recommend other researchers to carry out more studies regarding the title equation.

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