Original Research Article

A 3-Person Non-Zero-Sum Game for Sachet Water Companies

Abstract

Sachet water (popularly called pure water in Nigeria) businesses are often competitive due to its high demand. This is so because sachet water is the most affordable form of pure drinking water in Nigeria. Therefore, sachet water firms that aim at excelling in terms of results when facing an ever-competitive market in which competition gets tougher everyday need the knowledge of game theory to identify that strategy which will yield better profit independent of the strategy adopted by other competitors. This paper is aimed to investigate and determine the equilibrium point for three sachet water firms using the Nash equilibrium method as it provides a systematic approach for deciding the best strategy in competitive situation. The result showed two Nash equilibria (promo, promo) and (stay-put, stay-put) with the payoffs (82; 82; 82) and (147; 147; 147) respectively.

Keywords: Game Theory, Sachet Water, 3-Person, Non-zero Sum, Best Response, Nash Equilibrium, Pareto-Optimal, Pareto-Dominated

Introduction

The founding fathers of games theory are considered to be John Von Neumann and Oscar Morgenstern. They authored the book "Theory of Games and Economic Behavior" and discussed in it that a game is a situation in which two or more players participate in the pursuit of competing goals. Games theory is the study of mathematical models of strategic interaction among rational decision makers. According to Nyor *et al*. (2019), it is a tool that can model any situation in which there are people that interact by taking decisions, making moves, etc., in order to attain certain goal.

Games theory is a type of decision theory in which one's choice is determined after considering potential options available to the rivals playing the game. In ordinary decisions under uncertainty, the decision maker is faced with only a random process. The decision maker has not only to analyze alternative course of action available to him but also consider the possible goals, strategies and choices of the competitor. A solution to a game describes the optimal decisions of the players, who may have similar, opposed or mixed interest and the outcome that may result from these decisions.

In the business sector, strategic behavior is common among executives, managers, and investors. They must determine whether to enter new markets, develop new products, invest now or miss out on the opportunity to invest, and set pricing and purchasing policies. Game theoretical models have a lot of possibilities when it comes to assessing business decisions. In game theory models, each player is advised to consider the actions of others when deciding on a strategy, as the player may decide to react to the moves of his opponent. It is very advantageous to a decision maker (Geckil and Anderson, 2010).

Justin *et al.* (2014) cited that sachet water is an important primary source of drinking water in West Africa and explores the relationship between local perceptions of brand quality and bacteriologic quality after controlling for characteristics of vending environment. The results indicate substantial progress in sachet water regulation and quality control.

Sachet water sold in Nigeria and Ghana samples showed pathogenic contamination of as little as 5% among the 60 to 70% of the sample tested. Obiri-Danso *et al.* (2003), Mgbakor *et al*. (2011), and Oyelude & Ahenkorah (2012). Demographic and health sector classify sachet water sold in Nigeria as unimproved probably due to its low pricing according to Sridhar et al. (2017).

Literature Review

Game theory is popular and well known as a vital tool in various fields. The development of game theory is largely expanded. The discussion of game theory is noted to initiate with a letter written in 1713 by James Waldegrave in which Waldegrave obtained solution to a two person card game with mixed strategy. James Madison made what is now known as a game theoretical examination of how states are likely to act under various system of taxation. The study of a dupopoly by Antoine Cournot in 1838 is the most recent example of a formal game theoretical analysis. In his paper, "Researches into the Mathematical Principles of the Theory of Wealth" he provides a solution that is a constrained case of the Nash equilibrium Crider (2012). In 1921 the mathematician Emile Borel proposed a formal theory of games, which was expanded by the John Von Neumann in a 1928 paper titled "theory of parlor games". The Theory of Games and Economic Behavior was published in 1944 by John Von Neumann and Oskar Morgenstern. It established the economic and mathematical foundations for what is now known as "game theory." This book contains a lot of the basic vocabulary and problem-solving techniques that are still used today.

Von Neumann and Morgenstern pioneered the idea that economic and social problems can be modeled mathematically as appropriate strategy games. This is confirmed when Nyor *et al*. (2019) used games theory to model Student's Unrest on Nigerian Tertiary Institutions. Their research examined four popular strategies to manage students' crises which provided scientific information to the authorities of Nigerian Higher Institutions in curtailing students' riots on campuses.

In 1950s, John Nash, Harold Kuhn, John Harsanyi and many others developed game theory intensely. John Nash improved the tools and ideas of game theory, according to Von Stengel (2008), introducing universal "cooperative bargaining and non-cooperative theories." In 1951, he established what is now known as the 'Nash equilibrium' of a tactical game by demonstrating that every finite game has an equilibrium point at all times, at which each competitor selects move that are vital knowing fully well the choices of other competitors. Game theory was furthered theoretically and used to evaluate war-like, political and philosophical in the 1950s and 1960s.

Dixit and Nalebuff (1993) have defined game theory as the branch of social science that studies strategic decision making. Another definition is by Hutton (1996) who defines game theory as an intellectual framework for examining what various parties to decision should do given their possession of inadequate information and different objectives. Shoham and Leyton-Brown (2008) states that: game theory is the mathematical study of interaction among independent, self-interested agents.

Bhuiyan (2016) studied game theory and highlighted some applications of game theory in business and economics, politics, philosophy etc. Author pointed out that in business world game theory is applied for determining different strategies. It offers valuable tools for solving strategy problems. Many business strategies are short or long-term plans to achieve sustainable profitability. A business can often successfully position in the market with right strategy and a business will suffer in the long run with wrong strategy.

Issah *et al.* (2020) discussed the solution methods and techniques for solving games. The Nash equilibrium method, maximin-minimax method, dominance method, arithmetic method, matrix method, graphical method and linear programming method where fully discussed.

William (2015) used Microsoft excel in solving a three person game for both total and partial conflict games. Author mentioned that: these solutions find any pure strategy solutions for the players playing alone and without communication, and then every combination of coalitions between players is found and solved.

According to Cigdem and Bulent (2015), it is not possible to talk about 3-player games for zero-sum games. In three player games, there are not only two opponents, but a third party is also involved in the game. Therefore, even if the profit is distributed equally, the loss of one party will be higher than the other parties' gain.

Definition of Terms

Strategy: is the list of all possible action (move, decision alternative, courses of action) that are likely to adopted by a player for every outcome.

Optimal Strategy: is a course of action that puts any player in the most preferred position irrespective of the course of action adopted by the other player

Player: Each participant (interested party) is called a player.

Payoff: is the outcome of playing the game.

Game Theory: is the study of mathematical models of strategic interaction among rational decision makers.

Three Person Game: The game with only three players, say A, B and C is called a Three Person Game.

Simultaneous Games: a game in which each player has only one move and that all moves are made simultaneously.

Nash Equilibrium: An outcome is called a pure Nash equilibrium provided no player can gain a higher payoff by deviating from the move, when all other players stick to their choices.

Pareto Dominance: A Nash equilibrium is Pareto-dominated by another Nash equilibrium if every player's payoff in the first one is smaller or the same as in the second one.

Pareto Optimal: A Nash equilibrium is Pareto-optimal if it is not Pareto-dominated by any other Nash equilibrium, except maybe by some having exactly the same payoffs.

Non Zero Sum Game: is a game in which the sum of the winnings and losses of the various players can be less than or more than zero.

Best Response: In a three player game, the strategy that yields the highest payoff for player A against particular strategies say 'S' and 'R' adopted by the other players is called best response to strategy 'S' and 'R' for player B and C respectively.

Methodology

Formulation of the Problem

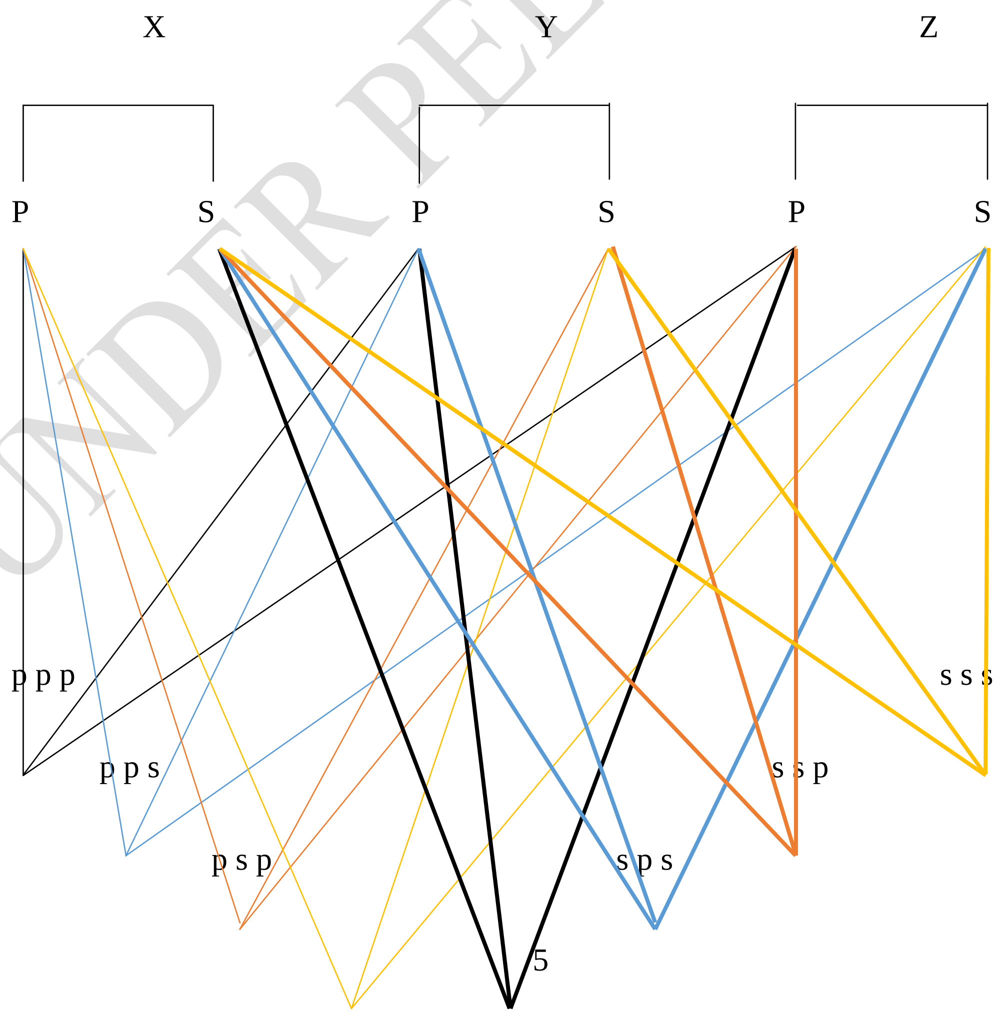
Three sachet water firms playing game with two strategies: promo and stay-put available to each firm. Such game can best be presented in two 2 by 2 matrices; where the first player chooses the matrix, the second player takes the row and the third player goes by column. The first values in each cell are payoffs for first player, middle values for second and last values for the third player.

Let X, Y and Z be three firms (players) in a game with two available strategies Promo (P) and Stay-Put (S) for each player. And let p and s represent the payoff for strategies P and S respectively, then the payoff matrices presentation is as shown below.

Table 1: Payoff Description

Y / Z	**Firm X Offers Promo** (Adopts strategy P)		**Firm X Stays Put** (Adopts strategy S)	
	P	S	P	S
P	p; p; p	p; p; s	s; p; p	s; p; s
S	p; s; p	p; s; s	s; s; p	s; s; s

It can also be shown in a diagram form as:



p s s   s p p

Figure 1: Showing Strategies and Payoffs Available for Firms X, Y and Z

Algorithm for Best Response Method of Three Person Game

Considering the payoff matrices in tables 1, we look at the best response method to search for Nash equilibrium(s) if any for the sachet water firms. The step by step algorithm of the method is shown below.

i. Compare cells having the same position in the different matrices, and underline the highest value player X's payoff (written first in each cell).

ii. Underline in each column the largest middle value in each cell and called it player Y's best response

iii. In each row, underline the biggest value of player Z's payoff (written last in each cell).

iv. If at all each value in a cell is underline, then the outcome is called Nash equilibrium.

Description of the Problem

Three sachet (pure) water companies in a particular location share a market and are currently competing for sales. Each company is now to decide whether to offer a promo of one bag per 10 bags purchased by the retailers. It is expected that an average of 8000 bags of pure water are sold per month from each company. Whenever the three firms adapt to promo strategy, the market will be shared equally and the relative cost of the strategy is in vein. In a case where two firms adapt to promo, the third firm loses half of its market share to the two firms offering promo. Also, in a situation where two firms decide not to offer promo, the third firm gains one third from each of their market share. Research shows that each firm has the capacity of producing 14000 bags per month. Determine the optimal strategy for the companies.

Data Collection and Procedure

The data used in this paper was collected in March 2021 through an interview with the manager Tarayya table water. Tarayya, Ideal and Asa table water companies co-exist in the same locality having the same target customers. The aim of the three firms is to provide healthy drinking water and employment opportunity to the people living in Gangara and the neighboring villages. A town located in Giwa L.G.A. of Kaduna State in Nigeria.

Raw Data

As revealed by the manager, the cost of production are as follows: 1 kg of leather roll cost ₦1650, a super pack of packing bag (which contains 10 packs and each pack contains 100 pieces of packing bags) costs ₦6450, an average of 35 bags are produced per kg, one gallon of gas which cost ₦1200 is burned by the generator in two hours, two Dingli packing machines produce average of 100 bags per hour each and it takes 30mins to raise the temperature of the machines before packing starts, 1 gallon of petrol which cost ₦1000 is used in cars for the distribution of 200 bags of sachet water, ₦400 is given as salary for the production of 100 bags of sachet water and ₦600 for selling the same number of bags. The cost of maintenance is not unique hence it is assumed to cost the company ₦5 per bag.

Analysis of Data

The table below contains the list of materials that constitute the cost of production of the sachet water.

Table 2: Showing Analysis of Cost of Production

Production Materials	**Initials**	**Cost per bag(naira)**
Leather Roll	L	₦47.15
Packing bag	P	₦6.45
Generator fuel	G	₦8
Distribution Car fuel	C	₦5
Producers Salary	R	₦4
Distributors Salary	D	₦6
Maintenance	M	₦5

Computations for Payoffs

The analyzed data is used to compute the payoffs for each of the firms

a. If all the firms adapt to promo, then each produce (8000 + 800) = 8800 bags since one bag is given as promo per 10 bags.

$$\text{Income per Month} = \text{price per bag } \times \text{bags sold per month}$$

$$100 \times 8000 = 800{,}000$$

$$\text{Expenses per Month} = (\text{L} + \text{P} + \text{G} + \text{C} + \text{R} + \text{D} + \text{M}) \times \text{ bags produced per month}$$

$$(47.15 + 6.45 \ + \ 5 \ + \ 8 \ + \ 4 \ + \ 6 \ + \ 5) \ \times \ 8800$$

$$81.6 \ \times 8800 = 718{,}080$$

$$\text{Profit per Month} = \text{Income} - \text{Expenses}$$

$$800{,}000 - 718{,}080 = \ 81{,}920$$

b. If two firms offer promo, and one firm stay-put then, the two companies each have:

$$\text{Income per Month} = \text{price per bag } \times \text{bags sold per month}$$

$$100 \times 10000 = 1{,}000{,}000$$

Expenses per Month:

Note: 11000 bags are produced (one bag as promo for each 10 bags of 10000 bags)

$$(\text{L} + \text{P} + \text{G} + \text{C} + \text{R} + \text{D} + \text{M}) \times \text{ bags produced per month}$$

$$(47.15 + 6.45 \ + \ 5 \ + \ 8 \ + \ 4 \ + \ 6 \ + \ 5) \ \times \ 11{,}000$$

$$81.6 \ \times 11000 = 897{,}600$$

The payoff is: $1{,}000{,}000 - 897{,}600 = \ 102{,}400$

Now, for the stay-put company

$$\text{Income per Month} = \text{price per bag } \times \text{bags sold per month}$$

$$100 \times 4000 = 400{,}000$$

Expenses per Month:

$$(\text{L} + \text{P} + \text{G} + \text{C} + \text{R} + \text{D} + \text{M}) \times \text{ bags produced per month}$$

$$(47.15 + 6.45 \ + \ 5 \ + \ 8 \ + \ 4 \ + \ 6 \ + \ 5) \ \times \ 4000$$

$$81.6 \ \times 4000 = 326{,}400$$

The payoff is: $400{,}000 - 326{,}400 = 73{,}600$

c. If only one firm adapt to the promo strategy, then the firm sales 12,000 bags hence

$$\text{Income per Month} = \text{price per bag } \times \text{bags sold per month}$$

$$100 \times 12{,}000 = 1{,}200{,}000$$

Expenses per Month:

Note: 13,200 bags are produced (since promo of one per ten bags is offered)

$$(\mathrm{L} + \mathrm{P} + \mathrm{G} + \mathrm{C} + \mathrm{R} + \mathrm{D} + \mathrm{M}) \times \text{ bags produced per month}$$

$$(47.15 + 6.45 + 5 + 8 + 4 + 6 + 5) \times 13{,}200$$

$$81.6 \times 13{,}200 = 1{,}077{,}120$$

$$\text{Profit per Month} = \text{Income} - \text{Expenses}$$

$$1{,}200{,}000 - 1{,}077{,}120 = 122{,}880$$

For the other firms that stay-put

$$\text{Income per Month} = \text{price per bag } \times \text{bags sold per month}$$

$$100 \times 6000 = 600{,}000$$

$$\text{Expenses per Month} = (\mathrm{L} + \mathrm{P} + \mathrm{G} + \mathrm{C} + \mathrm{R} + \mathrm{D} + \mathrm{M}) \times \text{ bags produced per month}$$

$$(47.15 + 6.45 + 5 + 8 + 4 + 6 + 5) \times 6000$$

$$81.6 \times 6000 = 489{,}600$$

$$\text{Profit per Month} = \text{Income} - \text{Expenses}$$

$$600{,}000 - 489{,}600 = 110{,}400$$

d. Lastly, in a case where all firms decide to stay-put, then

$$\text{Income per Month} = \text{price per bag } \times \text{bags sold per month}$$

$$100 \times 8000 = 800{,}000$$

$$\text{Expenses per Month} = (\mathrm{L} + \mathrm{P} + \mathrm{G} + \mathrm{C} + \mathrm{R} + \mathrm{D} + \mathrm{M}) \times \text{ bags produced per month}$$

$$(47.15 + 6.45 + 5 + 8 + 4 + 6 + 5) \times 8000$$

$$81.6 \times 8000 = 652{,}800$$

$$\text{Profit per Month} = \text{Income} - \text{Expenses}$$

$$800{,}000 - 652{,}800 = 147{,}200$$

The Payoff Matrix

The payoffs computed above are used to formulate the two payoff matrices. Tarayya (T) firm chooses the matrix, Ideal (I) choose the row and Asa (A) chooses the column. The payoffs are approximated to the nearest thousand

Table 3: Formulated Payoff Matrix

I / A	Firm T Offers Promo (Adopts strategy P)		Firm T Stays Put (Adopts strategy S)	
	P	S	P	S
P	82; 82; 82	102; 102; 73	73; 102; 102;	110; 122; 110
S	102; 73; 102	122; 110; 110	110; 110; 122	147; 147; 147

Result

We first find the best response moves for the three firms. For firm 'T', we consider the first entries and compare cells having the same position in different matrices, and underline the highest value

Table 4: Showing Best Response for Firm T

I / A	Firm T Offers Promo (Adopts strategy P)		Firm T Stays Put (Adopts strategy S)	
	P	S	P	S
P	82; 82; 82	102; 102; 73	73; 102; 102;	110; 122; 110
S	102; 73; 102	122; 110; 110	110; 110; 122	147; 147; 147

For firm 'I' we underline the highest value second entry in each column in both matrices

Table 5: Showing Best Response for Firm I

I / A	Firm T Offers Promo (Adopts strategy P)		Firm T Stays Put (Adopts strategy S)	
	P	S	P	S
P	82; 82; 82	102; 102; 73	73; 102; 102;	110; 122; 110
S	102; 73; 102	122; 110; 110	110; 110; 122	147; 147; 147

Lastly, for firm A, we underline the highest value third entry in each row in both matrices.

Table 6: Showing Best Response for Firm A

I / A	Firm T Offers Promo (Adopts strategy P)	Firm T Stays Put (Adopts strategy S)

	P	S	P	S
P	82; 82; $\underline{82}$	102; 102; 73	73; 102; 102;	110; 122; $\underline{110}$
S	102; 73; 102	122; 110; $\underline{110}$	110; 110; 122	147; 147; $\underline{147}$

Putting the best response moves together, we have

Table 7: Showing Nash Equilibrium

I / A	**Firm T Offers Promo** (Adopts strategy P)		**Firm T Stays Put** (Adopts strategy S)	
	P	S	P	S
P	$\underline{82;}$ $\underline{82;}$ $\underline{82}$	102; 102; 73	73; 102; 102;	$\underline{110;}$ 122; $\underline{110}$
S	102; 73; 102	122; $\underline{110;}$ $\underline{110}$	$\underline{110;}$ $\underline{110;}$ 122	$\underline{147;}$ $\underline{147;}$ $\underline{147}$

Discussion of Results

The result showed in table 7 contains two Nash equilibria (promo, promo) and (stay-put, stay-put) with the payoffs $(\mathbf{82};\ \mathbf{82};\ \mathbf{82})$ and $(\mathbf{147}; \mathbf{147}; \mathbf{147})$ respectively. Games with more than one pure Nash equilibria are sometimes called "coordination games". The strategy (promo, promo) is Pareto dominated by (stay-put, stay-put), thus (stay put, stay put) is Pareto optimal. That is to say both firms should stay put, but that is likely not to happen except if negotiations can take place in advance. Firm 'T' fears that firms 'I' and 'A' might give promo if it stays put lowering firm 'T's payoff to $\mathbf{73}$. Also, firm 'I' fears that firms 'T' and 'A' might give promo if it stays put and hence reducing the payoff of firm 'I' to $\mathbf{73}$. Similarly, firm 'A's payoff reduces to $\mathbf{73}$ when 'T' and 'I' give promo and it stays put. This is so because the game is symmetric in nature. Therefore, both firms should give promo if the rules of the game do not allow negotiations.

Conclusion and Recommendation

In this paper, we discussed three person game in general; consisting of a tabular and diagrammatic representation of the payoffs, formulating the payoff matrix and step by step algorithm for best response (Nash equilibrium) method. The result shows that each firm should give promo, except if negotiation is possible in advance, then it is advised that each firm should stay-put. Hence, in non-zero sum setting; negotiation, partnership and alliances may yield a win-win for the three firms. Therefore, we may now conclude that the aim of this project to determine the optimal strategy has been achieved.

Based on the observations and results obtained, author recommend that the three sachet water companies: Tarayya, Ideal and Asa should consider this research and apply the principles of game theory in determining that particular strategy that yields the greatest gain and negotiate if necessary. In fact, business firms should consult and seek the advice of specialist in Operations Research and Particularly Game Theory when situations of competitiveness arise.

COMPETING INTERESTS DISCLAIMER:

Authors have declared that no competing interests exist. The products used for this research are commonly and predominantly use products in our area of research and country. There is absolutely no conflict of interest between the authors and producers of the products because we do not intend to use these products as an avenue for any litigation but for the advancement of knowledge. Also, the research was not funded by the producing company rather it was funded by personal efforts of the authors.

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