A Stochastic Model for Stock Market Price Variation

Abstract

In this paper a differential equation model that could consider environmental effects in decision making of investors in stock exchange market has been developed with a stochastic parameter in the equation. These analyses were logically extended to stochastic vector differential equation that would help in predicting different commodity price processes, and the result obtained by exploring the properties of the principal component analysis solution which is a function of the drift and by imposing a condition on the stochastic part which is a function of the volatility. Furthermore, the results show the level of proportion accounted by first Principal Component Analysis (PCA) a function of the drift. In the same scenario, a non-parametric test discovered by Kolmogorov-Smirnov (KS) was performed; the test revealed that there exist a difference between distributions of volatility and drift.

Keywords: Stock market price, drift, volatility, SDE, PCA and Stochastic analysis

1. Introduction

A stock in a stock market or in an investment represents a share in the ownership of an incorporated company. Stocks are evidence of ownership. Investors buy stocks in the hope that it will yield income from dividends and appreciate, or grow in value. Thus, market price is the current price at which an asset or service can be bought or sold. Economic theory contends that the market prices converge at a point where the forces of supply and demand meet. Market price of stock is the most recent price at which the stock was traded. It is the result of traders, investors and dealers interacting with each other in a market.

In the real world of financial markets, investors and financial analysts are generally more interested in the profit or loss of the stock over a period of time, that is the changes in the price, than in the price self. Therefore, modeling a behavior of a stock exchange market can be made through its relative change of the unstable market variables in time so as to predict stock price fluctuation, advice investors and corporative owners who are working out for convenient ways to do business by issuing of stocks in their cooperation. The basis of this work lies in the observation of [4]. This is so since the path of the stock price process can be linked to his description of the random collision of some tiny particles with the molecules of the liquid he introduced, is what we called Brownian motion.

Now, the market price behavior shows the characteristics as a stochastic process called "Brownian motion" or Wiener process with drift. It is an important example of stochastic processes satisfying a stochastic differential equation.

When randomness are allowed into a differential equation it becomes a stochastic differential equation. The random quantities or variable in the equation can be parameterized into discrete or continuous variables, the parameterized collection of random variables is known as stochastic process that can serve as a model for predicting real-life behavior of any random dynamic problem,[6] In many fields of science and engineering the accurate analysis, design and assessment of system subjected to realistic environment effects must take into account of the potential of “white noise” random forces that would affect the system or error measurements in the system. Randomness is intrinsic to the mathematical formulation of many phenomena, such as, fluctuations in the stock market, noise in population systems, communication networks or irregular fluctuation in observed signals.

SDEs can be used to model various phenomena such as unstable nature of stock prices or physical systems subject to thermal fluctuations. In statistical mechanics, thermal fluctuations are random deviations of a system from its average state that occur in a system at equilibrium. Thermodynamic variables, such as pressure, temperature, or entropy, likewise undergo thermal fluctuations.

It has been observed that stock price is one of the highly volatile variables in a stock exchange market, [2].

The unstable property and other considerable factors such as liquidity on stock return, since the sudden change in share prices occur randomly and frequently. Researchers are kin to study the behavior of the unstable market variable so as to enable investors and owners of cooperation make decisions on the level of their investment in stock market exchange.

The price behavior shows the same characteristics as a stochastic process called “Brownian Motion”. Thus some properties of the stock price process can be derived from those of the Brownian motion process. It is an important example of stochastic processes satisfying a Stochastic Differential Equation (SDE). It is a stochastic process, which assumes that the returns, profits or losses, on the stock are independent and normally distributed. Modeling stock price is concerned with modeling the arrival of new information, which affects the price. Two importance things to note while modeling stock prices are: Probability distribution and Information. These play a major role in the modeling of future stock prices. In other words, the future price of a stock can be predicted within a certain level, if one can anticipate new information about the stock.

Predicting the volatility of the variables in the stock exchange market would require some elements of PCA. PCA is a statistical procedure that uses an orthogonal transformation to convert a set of observations of possibly correlated variables into a set of values of linearly uncorrelated variables called principal components. The number of principal components is less than or equal to the number of original variables. This transformation is defined in such a way that the first principal component has the largest possible variance (that is , accounts for as much of the variability in the data as possible), and each succeeding component in turn has the highest variance possible under the constraint that it is orthogonal to(ie., uncorrelated with) the preceding components. Principal components are guaranteed to be independent if the data set is jointly normally distributed.

[1] Worked on financial modeling using ordinary and stochastic differential equations. This result showed that jump terms, stochastic volatility and Markov switching models in financial markets can be modeled and analyzed in terms of stochastic differential equation. [2] worked on stochastic analysis of stock market price models. They applied stochastic analysis of the behavior of stock prices using a proposed log-normal distribution model in terms of stochastic differential

equation. Their results reveal that the proposed model is efficient for the production of stock prices.

[3] Studied efficient method of moment's estimation of a stochastic volatility model on stock prices. They perform an extensive Monte Carlo study of efficient method of moments (EMM) estimation of a stochastic volatility model. Their results showed that efficient method of moment's procedure provides a very substantial improvement in efficiency relative to simple generalized method of moments (GMM), as the root-mean square error (RMSEs) and reduced uniformly across the simulation design.

[9] on investigating the effect of capital flight on the economy of a Developing nation via the normal-inverse Gaussian (NIG) Distribution, applied Stochastic analysis to concluded that; better policy measures should be instituted to make the domestic economy more attractive for private investment if capital flight is to be confronted and flight capital recaptured. Fama (1965) studied the behavior of stock market prices where he applied stochastic equation and observed that daily changes in log price for a given security follow a stable partisan distribution with characteristic exponent which has far reaching implications. [10] also worked on predicting the value of an option based on an option price, applied the method of SDE in the study. It's described the evolution of price of an option as stochastic volatility based and derived a model equation for predicting the values of option based on the price.

[12] worked on stochastic modeling of stock prices; a method of Brownian motion model was applied to explain the stock price time series. The result showed that as long as a model based upon the white noise is fitted to the market values, the two interpretations will provide different estimates of the parameters, but identical values concerning the predicted stock prices. [7] Studied stock price modeling: theory and practice. Applied the method of log-normal distribution model in terms of SDE and concluded that the geometric Brownian motion shows less accuracy in short time modeling.

[11] Investigated on the measurement of random behavior of stock price changes. They considered the measurement of random behavior of stock market price. Precise conditions were obtained which determine the equilibrium price and growth rate of the stock shares in particular cases.

It is obvious that, [6] have formulated stability analysis of stochastic model for stock market prices in which the unstable natures of stock market forces were analyzed using the properties of fundamental matrix solution. The advantage of this work over the work of [6] is that the present work models the levels of proportion accounted by first Principal Component Analysis (PCA) a function of the stock drift which is the expected annual rate of return and analysis of two key parameters of SDE using Kolmogorov-Smirnov (KS). Our novel contribution complements the previous discovery as it widens the area of application of problem of this nature.

The paper is aimed at studying the stochastic analysis of stock market prices, determining the levels of proportion of total variance by the first PCA as it affects stock market prices and subjecting the two key parameters of stochastic differential equation to Kolmogorov-Smirnov (KS) test to know if they come from a common distribution.

This paper is arranged as follows: Section 2 presents the mathematical preliminaries, formulation of the problem is seen in Subsection 2.1, Subsection 2.3 is PCA of the stock variables, Subsection 2.4 is KS test, Data analysis and results is presented in Section 3 while Section 4 is discussions of results and the paper is concluded in Section 5.

2 Mathematical Preliminaries

let $S(t)$ be the price of some risky asset at time t ,and μ ,an expected rate of returns on the stock and dt as a relative change during the trading days such that the stock price follows a random walk which is governed by a stochastic differential equation.

$$dS(t) = \mu S(t)dt + \sigma S(t)dW_t \qquad (1)$$

Where, μ is drift and σ the volatility of the stock, W_t is a Brownian motion or Wiener's process on a probability space $(\Omega, \xi, \wp), \xi$ is a $\sigma -$algebra generated by $W_t, t \geq 0$.

Definition 1.1: A standard Brownian Motion is simply stochastic process $\{B_t\}_{t\in\tau}$ with the following properties:

i) With probability 1, $B_0 = 0$.

ii) For all $0 \leq t_1 \leq t_2 \leq \cdots \leq t_n$, the increments $B_{t2-}B_{t1}, B_{t3-}B_{t2}, B_{t2}, \cdots, B_{tn} - B_{tn-1}$ are independent.

iii) For $t \geq s \geq 0,\ B_t - B_s \sim N(0, t-s)$.

iv) With probability 1, the function $\rightarrow B_i$ is continuous.
Stock Price Modelling

Theorem 1.1: (Ito's formula) Let $(\Omega, \beta, \mu, \mathrm{F}(\beta)$ be a filtered probability space $X = \{X, t \geq 0\}$ be an adaptive stochastic process on $(\Omega, \beta, \alpha, \mathrm{F}(\beta)$ processing a quadratic variation(X) with SDE defined as:

$$dX(t) = g(t, X(t))dt + f(t, X(t))dW(t) \qquad (2)$$

$t \in \Re$ and for $u = u(t, X(t) \in C^{1\times 2}(\Pi \times \mathbb{R}))$

$$du(t, X(t)) = \left\{\frac{\partial u}{\partial t} + g\frac{\partial u}{\partial x} + \frac{1}{2}f^2\frac{\partial^2 u}{\partial x^2}\right\}d\tau + f\frac{\partial u}{\partial x}dW(t) \qquad (3)$$

Using theorem 1.1 and equation (3) comfortably solves the SDE with a solution given below:

$$S(t) = S_0 \ \exp\left\{\sigma dW(t) + \left(\alpha - \frac{1}{2}\sigma^2\right)t\right\}, \forall t \in [0,1]$$

2.1 Formulation of the Problem

Given a finite time horizon $T > 0$,we look at a complete probability space((Ω, f, P) whose stock price dynamics is in(1). We consider stock drift of the expected returns for investors in financial market. From the model equation (1) in Sections 2, we now developed and consider a vector valued SDE, where the securities invested on some bond processes are correlated and to obtain the proportion of total variance accounted by the principal component analysis.
We consider the SDE given by

$$\begin{aligned} dS_1(t) &= \mu_1 S_1(t)dt + \mu_2 S_2(t)dt + S_1(t)\left(\sigma_{11}dw_1(t) + \sigma_{12}dw_2(t)\right) \\ dS_2(t) &= \mu_1 S_1(t)dt - \mu_2 S_2(t)dt + S_2(t)\left(\sigma_{21}dw_1(t) + \sigma_{22}dw_2(t)\right) \\ &\vdots \qquad\qquad \vdots \qquad\qquad \vdots \qquad\qquad \vdots \qquad\qquad \vdots \\ dS_n(t) &= \mu_n S_n(t)dt - \mu_n S_n(t)dt + S_n(t)\left(\sigma_n dw_1(t) + \sigma_{n2}dw_2(t)\right) \end{aligned} \tag{4}$$

Here, it is assumed that $\sigma_{21} = \sigma_{22} \neq 0$

Since both processes S_1,..,S_n are correlated, to develop a vector equation for (4) write the equations in matrix form by

Letting $S = (S_1,\ldots,S_n)^T$

$$A(t) = \begin{pmatrix} \mu_{11} & \mu_{12} \\ \mu_{21} & \mu_{22} \end{pmatrix}, \ \sum B_i(t), \ \ B_1(t) = \begin{pmatrix} \sigma_{11} & 0 \\ 0 & \sigma_{21} \end{pmatrix}, \ B_2(t) = \begin{pmatrix} \sigma_{21} & 0 \\ 0 & \sigma_{22} \end{pmatrix}, \tag{5}$$

A generalized equation for the vector valued SDE can now be put in the form

$$\begin{aligned} dx(t) &= A(t)x(t)dt + \sum_{i=1}^{n} B_i(t, x(t))\, dw_i(t), \\ x(0) &= x_0 \end{aligned} \tag{6}$$

Where $A(t) \in \mathbb{R}^{n\times n}, B_i(t) \in \mathbb{R}^{n\times n}, w_i(t) \in \mathbb{R}^n$ is an n-dimensional Brownian motion, x(t)$\in \mathbb{R}^n$ is a process of price volatility.

It is known in [6], that x(t) for equation (6) is normally distributed because the Brownian motion is just multiplied by time-dependent factors.

Let $A(t) \in \mathbb{R}^{n\times n}$ be Covariance matrix of the homogenous stochastic differential equation (6).

Then the integral solution of (6) is given by

$$X(t) = X(t)x_0 + \int_0^t X(t)X^{-1}(s)\sum_{i=1}^{n} B_i(t, x(t)dw_i(s)$$

2.2 Principal component Analysis of the stock variables

Definition 1.2: Suppose $\underline{X}$ has a joint distribution which has a variance matrix $\sum$ with eigenvalues $\lambda_1, \geq \lambda_2 \geq \ldots \geq \lambda_p \geq 0$. consider the random variables $y_1 \ldots y_p$ which are linear combination of the $X_1's$ ie:

$$\left.\begin{aligned} y_1 &= \underline{l}_1' \underline{X} = l_{11}X_1 + \ldots + l_{p_1}\lambda_p \\ &\vdots \\ y_p &= \underline{l}_1' \underline{X} = l_1 p X_1 + \ldots + l_{p_1}\lambda_p \end{aligned}\right\} \quad (7)$$

The y_i 's will be PC if they are uncorrelated and the variances of $y_{1,} y_2$ are as large as possible. Recall that if $y_i = l_i' X$. In order to look at the amount of information that is in y_1 .We can consider the proportion of the total population variance due to y_i

$$\frac{\lambda_i}{\lambda_1 + \lambda_2 + \cdots + \lambda_p}, i = 1, \ldots, p \quad (8$$

2.3 Kolmogorov-Smirnov (KS) Test

The KS statistic belongs to nonparametric test in statistics. It is a very useful test for testing normality assumption and based on the largest vertical difference between the hypothesized and empirical distribution, [5]. Besides its use in the test for assumptions of normality, it is popular in the test to verify if two independent samples are drawn from a common distribution. We test as follows:

H_0: The Drift and Volatility stocks are from the same distribution.

H_1: They are not from the same distribution.

3.Data Analysis and Results

In this Section we present the computational results for the problems formulated in Section 2. The problems were solved analytical and graphical solutions obtained using matlab programming language.

To illustrate the stock price fluctuations, we used ten years stock price data extracted from [2] which shows the initial stock prices, drift and volatility with respect to their trading days. We computed the values of the stock drift to obtain covariance matrix solution given as:

$$A(t) = \begin{pmatrix} 0.09377 & 0.04545 \\ 0.04545 & 0.364 \end{pmatrix}$$

Which was extended to compute the levels of proportion accounted by first principal component analysis a function of the drift parameter.

To obtain the values of the volatilities, we use the last term of right hand side of equation(6) which gave the following:

$$\sum B_I = B_1(t) + B_2(t) = \begin{pmatrix} 0.03384 & 0 \\ 0 & 0.04811 \end{pmatrix} + \begin{pmatrix} 0.04811 & 0 \\ 0 & 0.364 \end{pmatrix} = \begin{pmatrix} 0.08195 & 0 \\ 0 & 0.41211 \end{pmatrix}$$

In this paper, we have used the notion of the Brownian motion model to determine the principal component analysis of the dynamics of stock price solution to the stochastic model. The main idea is to use the geometric Brownian motion model of the stock price which is otherwise the solution of the stochastic model to develop and analyze condition that the total variance will be accounted.

Hence, solving the Principal Component Analysis (PCA) of stock drift

$$A(t) = \begin{pmatrix} 0.09377 & 0.04545 \\ 0.04545 & 0.364 \end{pmatrix} \quad (9)$$

Solving for the $|A(t) - \lambda I| = 0$ gives

$$\lambda_1 = 0.0863, \ \lambda_2 = 0.3714$$

To find the eigenvectors for $\lambda_1 = 0.0863$

$$\begin{pmatrix} 0.09377 - \lambda & 0.04545 \\ 0.04545 & 0.364 - \lambda \end{pmatrix}$$

$$\begin{pmatrix} 0.00747 & 0.04545 \\ 0.04545 & 0.2777 \end{pmatrix} \begin{pmatrix} k_1 \\ k_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\Rightarrow 0.00747k_1 + 0.04545k_2 = 0 \quad (10)$$

$$0.04545k_1 + 0.2777k_2 = 0 \quad (11)$$

From (10)

$$0.00747k_1 + 0.04545k_2 = 0$$
$$0.00747k_1 = -0.04545k_2$$
$$k_2 = -0.1644$$

Substituting k_2 in(11) gives

$$0.04545k_1 = -0.2777(-0.1644) = 0.04565$$
$$k_1 = 1.0044$$

Any vector of the form $k_1 = \begin{pmatrix} 1.0044 \\ -0.1644 \end{pmatrix} = \begin{pmatrix} 1.0044c \\ -0.1644c \end{pmatrix}$

Say is an eigenvector corresponding to $\lambda_1 = 0.0863$

$$\begin{pmatrix} -0.27763 & 0.04545 \\ 0.04545 & -0.0074 \end{pmatrix}\begin{pmatrix} k_1 \\ k_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$-0.27763k_1 + 0.04545k_2 = 0 \quad (12)$$

$$0.04545k_1 - 0.0074k_2 = 0 \quad (13)$$

From (12)

$$-0.27763k_1 = -0.04545k_2$$
$$k_2 = 6.1085$$

Substitute $k_2 = 6.1085$ in (13)

$$0.04545k_1 - 0.0074(6.1085)$$
$$0.04545k_1 = 0.0452029$$
$$k_1 = 0.9946$$

Any vector of the form:

$$k_2 = \begin{pmatrix} 6.1085 \\ 0.9946 \end{pmatrix} = \begin{pmatrix} 6.1085c \\ 0.9946c \end{pmatrix}$$

Say is an eigenvector corresponding to $\lambda_2 = 0.3714$

$$k_1'k_1 = 0.0863$$

$$\begin{pmatrix}1.0044c & -0.1644c\end{pmatrix}\begin{pmatrix}1.0044c \\ -0.1644c\end{pmatrix} = 0.0863,\ 1.00881936c^2 + 0.02702736c^2 = 0.0863$$

$$1.03584672c^2 = 0.0863,\ c = 0.2886,\ e_1 = (0.2886)\begin{pmatrix}1.0044 \\ -0.1644\end{pmatrix} = \begin{pmatrix}0.2899 \\ -0.04756\end{pmatrix}$$

$$k_2'k_2 = 0.3714$$

$$\begin{pmatrix}6.1085c & 0.9946c\end{pmatrix}\begin{pmatrix}6.1085c \\ 0.9946c\end{pmatrix} = 0.3714,\ 37.31377225c^2 + 0.98922916c^2 = 0.3714$$

$$38.30300166c^2 = 0.3714,\ c = 0.09847,\ e_2 = (0.09847)\begin{pmatrix}6.1085 \\ 0.9946\end{pmatrix} = \begin{pmatrix}0.6015 \\ 0.0979\end{pmatrix}$$

The first Principal Component:

$$\left.\begin{aligned} Y_1 &= e_1'k_1 = 0.2899k_1 - 0.04756k_2 \\ Y_2 &= e_2'k_2 = 0.6015k_1 + 0.0979k_2 \end{aligned}\right\} \quad (14)$$

To calculate principal component Analysis accounted for:

$$\lambda_1 = 0.0863,\ \lambda_2 = 0.3714 \quad ,\Rightarrow \frac{\lambda_1}{\lambda_1 + \lambda_2} = \frac{0.0863}{0.4577},\ = 0.1886$$

The proportion of total variance accounted by first principal component is 18% . The two eigen-value measures the variance accounted for by the corresponding principal component.

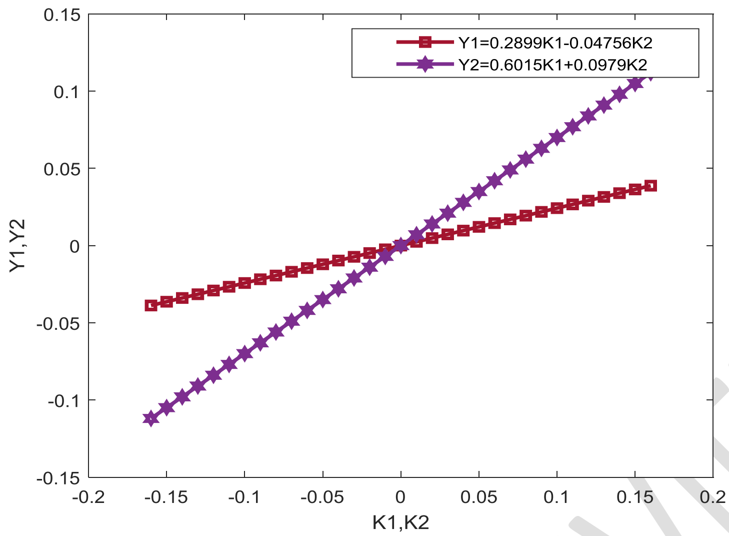

Figure 1: The first Principal Component Analysis of stock market prices

Figure 1 shows the linear combinations of first principal component analysis of stock market prices. It also attests to the proportion of total variance accounted by first principal component. They are uncorrelated and hence an index of financial development which measures the overall development in the financial market.

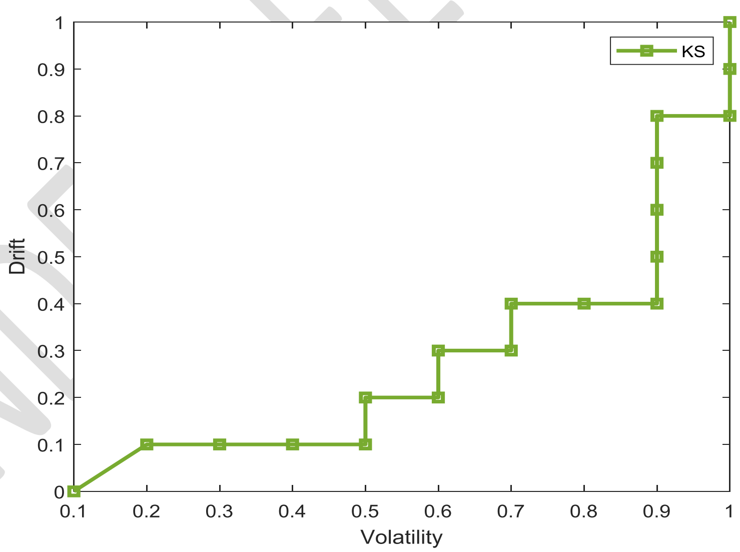

Figure 2: Graphical representation of KS test for Volatility and Drift stock variables

In Figure 2, shows levels of stair-case functions between Volatility and Drift, it portrays significance levels of the stock variables since both of them do not have same characteristics in stock market business.

Following the hypothesis testing of Section 2.3 using the Kolmogorov-Smirnov test, $\text{P-value} = 0.0486$ and $\text{KSSTAT} = 0.4211$ all are greater 0.05 implies that we should accept H_1 and therefore conclude that there is significant difference between Drift and Volatility. That is to say that both stock variables do not belong to a common distribution.

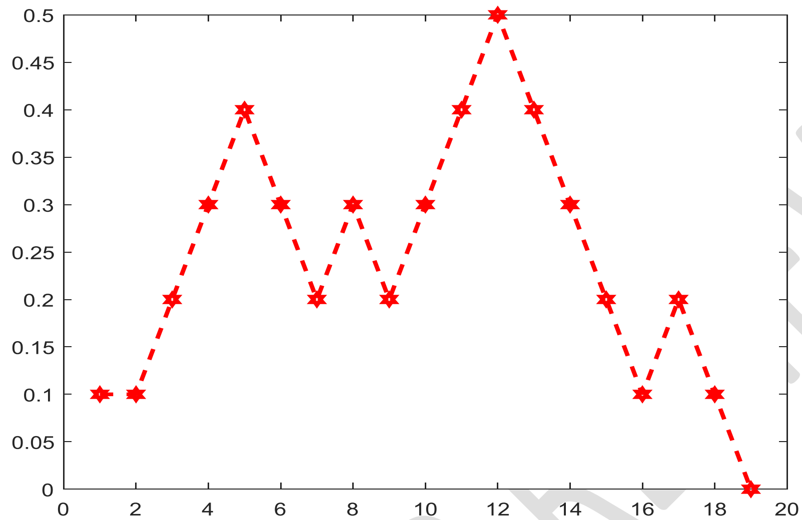

Figure 3: The difference in ranked KS test for Volatility and Drift stock variables

The two stock variables are displaying stock price fluctuations which cause panic buying in real-life trading. They are uncorrelated and follow a normal distribution. This also shows that the two stock variables do not belong to a common distribution, see Figure 3.

4 Conclusions

In this paper, we studied the problem of stock price fluctuations using stochastic differential equations, principal component analysis and KS goodness of fit test. The analytical solutions were detailed; the computational and graphical results were presented and discussed respectively.

The analyses were logically extended to stochastic vector differential equation that would help in predicting different commodity price processes, and the result obtained by exploring the properties of the principal component analysis. Furthermore, the results show the level of proportion accounted by first Principal Component Analysis (PCA) a function of the drift is 18%. In the same scenario, a non-parametric test discovered by Kolmogorov-Smirnov (KS) was performed; the test revealed that there exist a difference between distributions of volatility and drift as it affects stock market.

We therefore conclude that, this model will help investors, economist, policy makers and opinion leaders who are working assiduously in making sure of maximizing profit and minimizing lost.

COMPETING INTERESTS DISCLAIMER:

Authors have declared that no competing interests exist. The products used for this research are commonly and predominantly use products in our area of research and country. There is absolutely no conflict of interest between the authors and producers of the products because we do not intend to use these products as an avenue for any litigation but for the advancement of knowledge. Also, the research was not funded by the producing company rather it was funded by personal efforts of the authors.

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