An Application of Interacting Boson Model (IBM-2) Configuration Mixing in Tin nuclei

Abstract

In this work, the normal and intruder 2p-2h bands in even-even tin isotopes were investigated using IBM-2 configuration mixing calculations. The normal and intruder bands' states have been estimated independently and then allowed to mix using a basic band-mixing Hamiltonian. The energy levels and electric transition probability have been calculated and compared to the available experimental data.

Key Words: Interacting Boson Model, Configuration Mixing, Sn nuclei

1. Introduction

The structure of even-even Sn nuclei has been studied previously using various models, such as the BCS approach with neutron two-quasiparticle excitation [1], broken-pair or generalized seniority schemes [2] and so on. In the Z=50 zone, the presence of a collective band commencing at 0^+ level [3] is a common occurrence. Excitations over the proton shell closure at Z=50 are the source of these intruder states [4]. Wenes *et al*., [5] used a model that included both pure quadrupole vibrational excitations of doubly even nuclei and proton 2p-2h configurations paired with quadrupole vibrational excitations to examine these collective bands in even-even Sn isotopes. The lowest 2_1^+ state in ^{116,118,120}Sn has been defined as vibrational states by Ring and Schuck [6]. According to a Coulomb excitation research in even Sn isotopes, the levels 2_2^+, 2_3^+ and 4_1^+ have a vibrational character [7]. For example, the measured value of $B(E2;4_1^+ \rightarrow 2_1^+)$ for ^{116,118}Sn coincides with the expected value $2B(E2;2_1^+ \rightarrow 0_1^+)$ for a two phonon vibrational state. The crossover transitions $B(E2;2_2^+ \rightarrow 0_1^+)$ and $B(E2;2_3^+ \rightarrow 0_1^+)$ are also significantly delayed. The low-lying structure of Sn isotopes has also been interpreted by Wenes *et al*., as evidence of substantial mixing between the vibrational and rotational structures. We recently attempted to explain the low-lying levels of ^{115,117,119}Sn isotopes in terms of $U^{BF}(5) \otimes SU^F(4)$ dynamical symmetry $U(6/20)$ super Lie group **[8]**. Using a configuration mixing analysis, the intruder bands in nearby Cd [9] and Te [10] nuclei have been well described in IBM-2. According to these investigations, the normal and intruder configurations are strongly mixed. Experimental $B(E2)$ values of various transitions in Sn isotopes also suggest a considerable mixing between the two coexisting forms.

The aim of this study is to look into the ground-state band up to two phonon triplets and the collective bands in even-even $^{112\text{-}118}$Sn nuclei using IBM-2 mixing configuration calculations in which the 2p-2h band is associated with the ground band's anharmonic quadrupole vibration. Of the context of I spin, it's also crucial to look into the consequences in Sn isotopes [11].

2. The Interacting Boson Model-2 (IBM-2)

In this work, the neutron-proton version of the Interacting Boson Model (IBM-2) is employed, which distinguishes between neutron (ν) and proton (π) bosons; a detailed explanation of IBM-2 may be found in **[12]**. In IBM-2, there are three terms to the Hamiltonian operator: one for proton bosons, one for neutron bosons, and one for interactions between unlike bosons. The approach of Duval and Barrett [12] is used in this calculation. The IBM-2 Hamiltonian is given by and it describes both normal $(N_\pi = 0)$ and intruder $(N_\pi = 2)$ written as:

$$H = \varepsilon_d (d_\pi^+ \tilde{d}_\pi + d_\nu^+ \tilde{d}_\nu) + \kappa(\hat{Q}_\pi . \hat{Q}_\nu) + V_{\pi\pi} + V_{\nu\nu} + M_{\pi\nu} \quad \ldots\ldots\ldots (1)$$

Where

$$V_{\rho\rho} = \frac{1}{2} \sum_{J=0.2.4} (2L+1)^{1/2} C_{L\rho} \left[\left(d_\rho^+ \times d_\rho^+ \right)_\pi^{(L)} \left(\tilde{d}_\rho \times \tilde{d}_\rho \right)^{(L)} \right]^{(0)} \ldots\ldots\ldots (2)$$

refers to the interaction between bosons that are similar. The letter ρ stands for π (proton) or ν (neutron) bosons.

The expression $Q_\pi . Q_\nu$, where

$$Q_\rho^{(2)} = (s_\rho^+ \times \tilde{d}_\rho + d_\rho^+ \times s_\rho)^{(2)} + \chi_\rho (d_\rho^+ \times \tilde{d}_\rho)^{(2)} \quad \ldots\ldots\ldots\ldots\ldots\ldots (3)$$

stands for the neutron and proton bosons quadrupole-quadrupole interaction.

$$M_{\pi\nu} = \frac{1}{2} \xi_2 (s_\pi^+ \times d_\nu^+ - d_\pi^+ \times s_\nu^+).(s_\pi \times \tilde{d}_\nu - \tilde{d}_\nu \times s_\pi) - \sum_{K=1,3} \xi_K \left(\left[d_\pi^+ \times d_\nu^+ \right]^{(K)} . \left[\tilde{d}_\pi \times \tilde{d}_\nu \right]^{(K)} \right) \ldots (4)$$

is the Majorana force. In terms of the above Hamiltonian, both the normal and intruder configurations are determined individually. The mixing operator is then used to combine the two setups.

$$V_{mix} = \alpha (s_\pi^+ s_\pi^+ + \tilde{s}_\pi \tilde{s}_\pi)^{(0)} + \beta (d_\pi^+ d_\pi^+ + \tilde{d}_\pi \tilde{d}_\pi) \quad \ldots\ldots\ldots (5)$$

where α and β are strength parameters that can be adjusted for the interaction between the two configurations $H + V_{mix}$ was diagonalized using the basis consisting of the lowest four eigenstates of each configuration in our calculation. The total mixing Hamiltonian is then given by;

$$H_{mix} = H_1 + H_2 + V_{mix} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (6)$$

where $H_1 (H_2)$ is the **IBM-2** Hamiltonian for the first (second) configuration, as given by Eq. (6), and an amount fl has been added to the energies of the second configuration.

The relation [12] is used to compute the energy gap parameter Δ between the two configurations.

$$\Delta = BE(Z,N) - BE(Z-2,N) - B(Z+2,N) + BE(Z,N) + 4V_{ph} \quad \ldots\ldots (7)$$

where BE is the binding energy of the ground state and V_{ph} is the particle-hole interaction energy.

The reduced electric transition probability $B(E2)$ values of observed transitions and quadrupole moments are calculated using mixed wave functions. The E2 transition operator is defined as follows:

$$T^{(E2)} = e_0(e_{\pi 0}Q_{\pi 0} + e_{\nu 0}Q_{\nu 0}) + e_2(e_{\pi 2}Q_{\pi 2} + e_{\nu 2}Q_{\nu 2}) \quad \ldots\ldots\ldots (8)$$

where Q_ρ is defined by Eq.(3) and e_j and $e_{j\rho}$ $(j = 0,2)$ are variables that can be changed. The prefixes 0 and 2 respectively correspond to the normal and intruder setups. The reduced electric transition probability $B(E2)$ is written as:

$$B(E2; J_i^+ \rightarrow J_f^+) = \frac{1}{2J_i + 1} \left| < J_f^+ \left\| T^{(E2)} \right\| J_i^+ > \right|^2 \ldots\ldots\ldots (9)$$

3. Results and Discussion

3.1 Energy Levels

The high energy members of the typical ground-state band in Sn isotopes are extremely difficult to distinguish. Even the two phonon triplets mix significantly with the 2p-2h band and the neutron two quasiparticle levels in these isotopes. We used levels up to the lowest 4_1^+ of the regular band and up to the 10_1^+ state of the invader band in our calculation. Because the Sn isotopes' normal configuration lacks proton bosons, only terms in the Hamiltonian involving neutron bosons will contribute to the determination of the energy value ε_d, $C_{0\pi}$, and $C_{2\pi}$ are the three parameters employed in the normal band calculation, and their adopted values are listed in Table (1).

The intruder configuration is predicted to have a gamma soft structure. The intruder band was chosen for this calculation to have an $E(4_1^+)/E(2_1^+)$ ratio of roughly 2.0, which is near to the SU(5) limit. The parameter ε_d adopted values show a smooth variation with the neutron number, reaching a minimum at mid-shell. From $N = 62$ to $N = 64$, the value of κ increases, then progressively decreases as the neutron number increases. In the calculation of Te isotopes, a similar type of neutron number dependency has been reported for the parameter κ. In Sn isotopes, the value of χ_π has been kept constant in all intruder bands. The intruder configuration parameter χ_ν has been changed. This parameter's adopted values are somewhat less than zero. The Majorana force parameters ξ_1, ξ_2, and ξ_3 are derived from calculations on nearby Cd isotopes **[9]**. The mixing operator V_{mix} admixes the two

configurations, which are calculated separately. There is only one free parameter for a U(5)-O(6) mixing using Eq.(5), according to Jolie and Lehmann **[13]**, because the matrix components of the two terms containing the two types of bosons are not independent.

As a result, we've only used one parameter α, and we haven't varied it with the mass number. Eq.(7) has been used to compute the energy-gap parameter Δ between the two configurations (Eq.(7)). The V_{ph} value for the $Z = 50$ area was discovered to be 2.540 MeV **[14]**. The Ref. **[15]** was used to compute the required experimental binding energy values. The rest of the settings are set to zero.

Table (1a): The IBM-2 Hamiltonian Parameters in (MeV units) for Normal Configuration $(N_\pi = 0)$**.**

Parameters	^{112}Sn	^{114}Sn	^{116}Sn	^{118}Sn
ε_d	1.25	1.30	1.25	1.20
κ	-0.167	-0.140	-0.147	-0.158
χ_ν	0.73	0.65	1.90	0.85
χ_π	0.40	0.40	0.40	0.40
$C_{0\nu}$	-0.31	-0.31	-0.35	-0.31
$C_{2\nu}$	-0.15	-0.15	-0.20	-0.12
$C_{4\nu}$	-0.05	0.0	0.0	0.0
ξ_2	0.04	0.04	0.04	0.04

$\xi_1 = \xi_3 = 0.24$ MeV**,** $C_{0\pi} = C_{2\pi} = C_{4\pi} = 0.0$ MeV

Table (1b): The IBM-2 Hamiltonian Parameters in (MeV units) for Intruder Configuration $(N_\pi = 2)$**.**

Parameters	^{112}Sn	^{114}Sn	^{116}Sn	^{118}Sn
ε_d	0.64	0.62	0.59	0.63
κ	-0.167	-0.140	-0.147	-0.158
χ_ν	-0.21	-0.17	-0.12	-0.07
χ_π	0.40	0.40	0.40	0.40
$C_{0\nu}$	0.0	-0.30	0.0	-0.30
$C_{2\nu}$	-0.15	-0.15	-0.20	-0.12
$C_{4\nu}$	-0.05	0.0	0.0	0.0
ξ_2	0.04	0.04	0.04	0.04

$\xi_1 = \xi_3 = 0.24$ MeV**,** $C_{0\pi} = C_{2\pi} = C_{4\pi} = 0.0$ MeV

The energy levels of $^{112\text{-}118}$Sn even-even isotopes were compared to experiment values **[16,17,18,19]** are listed in Table (2). The estimated and actual energy spectra of ^{112}Sn and ^{114}Sn nuclei have also been shown to agree. The wave function has little effect on e energy values. As a result, the transition probabilities must be calculated, which are highly dependent on the system's wave function.

Table (2): Comparison between experimental and IBM-2 energy levels for Sn isotopes (in MeV units)

Levels	^{112}Sn			^{114}Sn		
	Exp. [16]	IBM-2		Exp. [17]	IBM-2	
		$N_\pi = 0$	$N_\pi = 2$		$N_\pi = 0$	$N_\pi = 2$
0_1^+	0.0	0.0	0.0	0.0	0.0	0.0
2_1^+	1.256	1.256	1.255	1.299	1.299	1.301
4_1^+	2.247	2.244	2.787	2.187	2.200	2.521
6_1^+	2.549	2.611	2.891	3.149	3.183	3.542
8_1^+	4.770	4.801	5.011	3.871	3.901	4.341
10_1^+	4.880	4.971	5.189	4.139	4.220	4.341
2_2^+	2.151	2.220	2.592	2.238	2.267	2.890
0_2^+	2.190	2.258	2.491	1.953	1.843	2.225
4_2^+	2.521	2.618	3.782	2.614	2.724	3.110
3_1^+	2.913	3.141	3.543	3.025	3.125	3.321
0_3^+	2.618	2.731	3.109	2.156	2.254	2.980
2_3^+	2.476	2.510	3.311	2.2454	2.280	2.897
Levels	^{116}Sn			^{118}Sn		
	Exp. [18]	IBM-2		Exp. [19]	IBM-2	
		$N_\pi = 0$	$N_\pi = 2$		$N_\pi = 0$	$N_\pi = 2$
0_1^+	0.0	0.0	0.0	0.0	0.0	0.0
2_1^+	1.293	1.297	1.345	1.229	1.250	1.341
4_1^+	2.390	2.397	2.619	2.280	2.289	2.543
6_1^+	3.032	3.110	3.431	2.999	2.311	3.761
8_1^+	3.492	3.500	3.754	2.889	2.956	3.650
10_1^+	3.547	3.521	3.750	3.108	3.209	3.761
2_2^+	2.112	2.211	2.311	2.042	2.152	2.675
0_2^+	1.756	1.987	2.118	1.758	1.950	2.530
4_2^+	2.529	2.610	2.980	2.408	2.507	3.329
3_1^+	2.996	3.001	3.622	2.725	2.825	3.761
0_3^+	2.027	2.110	2.563	2.058	2.118	2.581
2_3^+	2.225	2.229	2.462	2.120	2.129	2.619

3.2 Electric Transition Probability

To calculate the electric operator, we relied on Eq.(8). The identification of proton and neutron bosons effective charges e_π and e_ν is crucial for an E2 transition. These isotopes lying in SU(5) limit (vibrational symmetry), therefore, the relationship

between (e_π , e_ν) and the reduced transition probability B(E2) for vibrational limit is given in the form **[20]:**

$$B(E2;2_1^+ \rightarrow 0_1^+) = \frac{5(e_\pi N_\pi + e_\nu N_\nu)^2}{N} \text{..... .....(10)}$$

where $B(E2;2_1^+ \rightarrow 0_1^+)$ is the experimental reduced transition probability from the first excited states 2_1^+ to the ground state 0_1^+, N is the total number of bosons. The relation (10) was used to estimate the effective boson charges for proton and neutron bosons. In this calculation we use the following criteria to determine the effective charges. $e_\pi = 0.12$ $e.b$ is a constant throughout the whole isotopic chain and the $e_\nu = 0.09, 0.085, 0.070$ and 0.075 $e.b$ for ^{112,114,116,118}Sn respectively and the ration ratio $e_2 / e_0 = 1.2$.

Between the states, $B(E2)$ values were obtained by evaluating matrix members of the $T^{(E2)}$ operator. Only the neutron component of the first term of the $T^{(E2)}$ operator contributes to the typical configuration of Sn isotopes. Only e_ν and χ_ν are used to evaluate the matrix elements of the first term of the $T^{(E2)}$ operator. For these Sn isotopes, matrix elements of the second term of this operator were estimated in the space of two proton bosons and a variable number of neutron bosons. The parameters χ_ν and χ_π are the same as in the computation of the energy value. The effective neutron bosonic charge values used exhibit a smooth variation with neutron number, becoming minimum for $N = 66$. In the calculations on neighboring Te isotopes **[10]**, a similar type of variance was detected. For both configurations, the effective bosonic charge for neutron bosons has remained unchanged. The ratio of the parameters e_2 / e_0, which has been kept constant at 1.2 for all $^{112\text{-}118}$Sn isotopes, has a large impact on the E2 matrix elements.

Table (3) compares experimental and computed $B(E2)$ values for the isotopes $^{112\text{-}118}$Sn. Some of the discrepancies between experimental findings and theoretical predictions are discussed. The E2 transition probability of the $2_3^+ \rightarrow 2_1^+$ and $0_3^+ \rightarrow 2_1^+$ transitions in ^{116}Sn are over-predicted. This suggests that the current model may have overestimated the intruder band's contributions to the 2_3^+ and 0_3^+ states. There aren't a lot of experimental data on ^{112,114}Sn isotopes. The experimental value of the transition $4_1^+ \rightarrow 2_1^+$ is surprisingly tiny when compared to surrounding ^{116,118}Sn isotopes, and cannot be explained using the current model. One reason could be that there is a strong contribution from the neutron two quasiparticle structures in this 4_1^+ state,

Table (3): Comparison between experimental and IBM-2 calculated $B(E2)$ values for $^{112\text{-}118}$Sn isotopes in e^2b^2 units

Isotopes	Transitions	Exp.	IBM-2
	$2_1^+ \rightarrow 0_1^+$	0.0512(12)	0.0050
	$2_2^+ \rightarrow 0_1^+$	0.00013(4)	0.00018

^{112}Sn	$0_2^+ \rightarrow 2_1^+$	≤ 0.029	0.031
	$2_2^+ \rightarrow 2_1^+$	0.037(150)	0.038
	$2_3^+ \rightarrow 0_1^+$	< 0.00019	0.00022
	$2_3^+ \rightarrow 2_1^+$	< 0.0014	0.0019
	$4_1^+ \rightarrow 2_1^+$	0.018(30)	0.020
^{114}Sn	$2_1^+ \rightarrow 0_1^+$	0.05(100)	0.055
	$2_2^+ \rightarrow 0_1^+$	0.07(300)	0.074
	$0_2^+ \rightarrow 2_1^+$	0.0003	0.00041
	$2_2^+ \rightarrow 2_1^+$	0.016	0.018
	$2_3^+ \rightarrow 0_1^+$	0.013	0.0142
	$2_3^+ \rightarrow 2_1^+$	0.021(60)	0.022
	$4_1^+ \rightarrow 2_1^+$	< 0.01	0.011
	$4_2^+ \rightarrow 2_1^+$	0.25	0.028
^{116}Sn	$2_1^+ \rightarrow 0_1^+$	0.0417(13)	0.0420
	$2_2^+ \rightarrow 0_1^+$	0.0002(8)	0.00025
	$0_2^+ \rightarrow 2_1^+$	0.0605(101)	0.066
	$2_2^+ \rightarrow 2_1^+$	0.0131(50)	0.0135
	$2_3^+ \rightarrow 0_1^+$	0.00017(10)	0.0002
	$2_3^+ \rightarrow 2_1^+$	0.0101(54)	0.012
	$4_1^+ \rightarrow 2_1^+$	0.1277(706)	0.129
	$4_2^+ \rightarrow 2_1^+$	$> 5 \times 10^{-6}$	5.5×10^{-6}
	$4_2^+ \rightarrow 2_2^+$	> 0.0403	0.047
^{118}Sn	$2_1^+ \rightarrow 0_1^+$	0.0416(17)	0.044
	$2_2^+ \rightarrow 0_1^+$	0.00026(4)	0.0003
	$0_2^+ \rightarrow 2_1^+$	0.0653(103)	0.071
	$2_2^+ \rightarrow 2_1^+$	0.0237(34)	0.033
	$0_3^+ \rightarrow 2_1^+$	> 0.00072	0.00088
	$4_1^+ \rightarrow 2_1^+$	0.0585(103)	0.061
	$4_2^+ \rightarrow 2_1^+$	< 0.0096	0.0107

Experimental data are taken from Refs. [16,17,18,19]

resulting in a lower transition probability. The experimental $B(E2)$ value of the transition $0_2^+ \rightarrow 2_1^+$ in the nucleus of ^{112}Sn is approximately half that of ^{114,116,118}Sn isotopes, a fact that our calculation does not recreate. Except for these discrepancies, practically every other transition can be described using this simple model. One of the

biggest sources of mistake, we believe, may be traced back to the omission of the contributions of two quasiparticle states.

The electric quadrupole moment of the first excited state was calculated based on the following equation:

$$Q_J = \left[\frac{16\pi}{5}\right]^{1/2} \begin{bmatrix} J & 2 & J \\ -J & 0 & J \end{bmatrix} < J_J \| T^{(E2)} \| J_J > \quad \ldots\ldots. (11)$$

We estimated the static quadrupole moment of the first 2_1^+ state in addition to the transition probabilities. In Table (4), the calculated values are compared to the experimental results. Obviously, the $Q(2_1^+)$ values are negative, which means that the nucleus has a prolate shape in this first excited state.

Table (4): Electric quadrupole moments for first excited states $Q(2_1^+)$ in *eb* units

isotopes	^{112}Sn		^{114}Sn	
	Exp.[16]	**IBM-2**	**Exp.**	**IBM-2**
$Q(2_1^+)$	-0.03(11)	-0.033	-	-0.141
isotopes	^{116}Sn		^{118}Sn	
$Q(2_1^+)$	**Exp.[18]**	**IBM-2**	**Exp.[19]**	**IBM-2**
	-0.17(4)	-0.18	-0.05(14)	-0.057

4. Concluding Remarks

In this paper, we used IBM-2 mixed configuration calculation to look at both normal and intruder configurations. This calculation accurately reproduces both normal ($N_\pi = 0$) and invader ($N_\pi = 2$) bands. The discrepancies between some experimental B(E2) values and theoretical expectations, especially in the two phonon triplets in the Sn isotopes, indicate a considerably more complex structure for the low lying states. The neutron two-quasiparticle structure was not taken into account in this analysis, which could have influenced the results.

References

[1] W. F. Van Gunsteren, E. Bocker, and K. Allaart" Nuclear Structure of Sn Nuclei" Z. Phys. **267 (1974)** 87.

[2] G. Bonsignori, M. Savoia, K. Allaart, A. Van Egmond, and G. Te Velde "Microscopic approache to magnetic excitations in IBM-2", Nucl. Phys. **A432** (1985) 389.

[3] J. Bron *et al*., "Shape Coexistence and its Symmetries" Nucl. Phys. **A318** (1979) 335.

[4] K. Heyde, P. Van Isacker, M. Waroquier, G. Wenes, and M.

Sambataro" Description of the Low-Lying Levels in $^{112-114}$Cd" , Phys. Rev. C **25 (1982)** 3160.
[5] G. Wenes, P. Van Isacker, M. Waroquier, K. Heyde, and J. Van Maldeghem, "Collective bands in doubly even Sn nuclei: Energy spectra and electromagnetic decay properties" Phys. Rev. C **23 (1981)** 2291.
[6] P. Ring and P. Schuck "The Nuclear Many-body Problem" Springer-Verlag, New York, (1980) p. 218.
[7] N. G. Jonsson, A. Backlin, J. Kantele, R. Julin, M. Luontama, and A. Passoja, " 0^+ states and E0 and E2 Transition rates in Even-Even Sn Nuclei" Nucl. Phys. **A371 (1981)** 333.
[8] A. K. Singh, G. Gangopadhyay, and D. Banerjee "U(6/20) Supersymmetry in $^{115-119}$Sn Isotopes" Phys. Rev. C **53** (1996) 2524.
[9] M. Deleze *et al*., "Study of the mixed ground-state and intruder bands in ^{110,112,114}Cd Systematics" Nucl. Phys. **A551 (1993)** 269.
[10] J. Rikovska, N. J. Stone, P. M. Walker, and W. B. Walters, Nucl. Phys. **A505**, (1989) 145.
[11] C. De Coster, K. Heyde, B. Decroix, P. Van Isacker, J. Jolie, H. Lehmann, and J. L. Wood "Particle-hole excitations in the Interacting Boson Model (I) General Structure and Symmetries" Nucl. Phys. **A600 (1996)** 251.
[12] P. D. Duval, and B. R. Barrett" Quantitative Description Configuration Mixing In the Interacting Boson Model", Nucl. Phys. **A376 91982)** 213.
[13] J. Jolie and H. Lehmann" The U(5)-O(6) model : An Analytic Approach to Shape Coexistence" , Phys. Lett. B **342** (1995) 1.
[14] E. R. Flynn and P. D. Kunz" Absolute E0 and E2 transition rates and Collective States", Phys. Lett. **68B (1977)** 40.
[15] C. M. Lederer and V. S. Shirley and Wiley "Table of Isotopes", edited by, New York, 1978.
[16] D. De Frenne and E. Jacobs, Nuclear Data Sheets 79 (1996) 725.
[17] Jean Blachot, Nuclear Data Sheets 97 (2002) 663.
[18] Jean Blachot, Nuclear Data Sheets 111 (2010) 783.
[19] K. Kito, Nuclear Data Sheets 75 (1995) 118.
[20] P. Van Isacker, K. Heyde, J. Jolie, and A. Sevrin" The F-spin Symmetric Limits of the Neutron-Proton Interacting Boson Model" Ann. Phys. (NY) 171 (1986) 253.