Original Research Article

Walking Mathematics Students Through the Maze of Chi-square Test of Independence and Homogeneity, Test Involving Several Proportions, and Goodness-of-fit Test

ABSTRACT

This study illustrates some practical steps lecturers could use to enable students to apply the chi-square concept. The study relied on a definition and theorem based on the chi-square theoretical model. The participants consisted of seventy (70) (fifty-five (55) males and fifteen (15) females) level 100 mathematics students from a university in Ghana. They were all admitted from the public senior high schools across the country. The students completed the tasks assigned to them in their various groups through active learning, as their lecturer facilitated the process. The lecturer guided the students to complete tasks related to the applications of the chi-square test in solving problems. The results indicated that active learning exposed the students to varied ways to apply the chi-square test. An implication of this study is that lecturers should teach their students about theorems and their related proofs and applications. These theorems are significant in mathematics learning because they are absolute truths. They enable students to develop a deeper understanding of the underlying concepts. The study concludes that by working with concrete examples, students gradually internalized their concept-acquisition skills to the extent that they confidently identified what concept to apply to every question.

Keywords: Theorems, definitions, chi-square theoretical model, absolute truths, applications

1. INTRODUCTION

The chi-square distribution is a continuous asymmetric distribution that has a minimum value of zero (0) and an infinite maximum value. The curve reaches a peak to the right of zero and gradually declines in height. The curve is asymptotic to the horizontal axis, with a unique distribution for each degree of freedom. The mean of the chi-square distribution is the degrees of freedom and the standard deviation is twice the mean. The distribution has a wide dispersion, with a peak farther to the right. The chi-square test is an important test of a hypothesis, appropriate for analysis of a non-parametric data. The chi-square test is used to determine whether there is any significant difference between the observed frequencies and expected frequencies pertaining to any particular phenomenon. A contingency table often shows the frequencies in different cells (categories). The observations must be categorical in nature, but must not follow a normal distribution. The test is applied to assess how likely the observed frequencies would be, assuming the null hypothesis is true. This test is also useful in ascertaining the independence of two random variables based on observations of these variables. This non-parametric test is used extensively for the following reasons: (1) It does not rely on assumptions that the data are drawn from a given parametric probability distributions (2) it is easier to compute and simple enough to understand as compared to the parametric test, and (3) it can be used in situations where the parametric test are not appropriate or measurements prohibit the use of parametric tests. It is defined as $\chi^2 = \sum \frac{(O-E)^2}{E}$, Where O refers to the observed frequencies and E refers to the expected frequencies.

Literature review

Researchers often use statistical tests to perform scientific research, and these affect the outcomes of research conducted. The type of research, based on purpose and objectives, determine the data analysis that is used in every situation [1]. Statistical tests performed research are either parametric or non-parametric. The results of these tests produce a meaningful and suitable summary of the findings to draw conclusions [5].

Parametric tests offer researchers an opportunity to make decisions in accordance with a parameter, a type of distribution and population variance. Parametric statistical test makes assumptions about the population characteristics and the data distributions. As a result, several assumptions are predicated on the data's appropriateness for a normal distribution, the randomness of selection, and their quantitative nature. These tests include Student t-tests and ANOVA tests, which assume that the data be normally distributed.

Non-parametric tests allow researchers to analyse data without using the population characteristics or distribution. The population data do not follow a normal distribution, and the findings are not generalizable [4]. As the shape of the population is unknown, it uses a small sample size, with either nominal or ordinal variables [8].

The chi-square test uses frequency counts or discrete data, to calculate the likelihood of some non-random factors to account for an observed association, by using the independence test. The chi-square test is also used to determine whether non-numerical variables that are used in statistical investigations have any association [6]. The benefit of Pearson's Chi-square distribution is that statisticians can interpret results using statistical techniques independent of normal distribution. A Chi-square table and the appropriate degree of freedom and significance are used to calculate the significance of the Chi-square value [7].

The most significant contributions of Pearson to current statistics theory are the development of independence and homogeneity tests, tests involving several proportions, and goodness of fit tests. These tests in the Karl Pearson family of chi-square tests use the same basic formula in their omnibus forms. When the null hypothesis is rejected, each of these tests has different interpretations and possibilities. The fundamental distinction between these chi-square tests is the proper circumstances in which each should be applied. Both the test of independence and the test of homogeneity have the identical formulation of the omnibus test statistic, but these two tests have different sample assumptions, null hypotheses, and choices in the event of rejection. The primary distinction between them is in the sampling and collection of data. A single sample of data is gathered for the test of independence, and two variables are then compared to discover how they relate to one another. For the results of a Chi-Square test to be reliable, the data must be a random sample from a population about which to inferences can be drawn, the observations must be independent, and, one should not use the Chi-Square if more than 20% of the expected frequencies have a value of less than five (5).

In this study, we dwelt on constructivist learning, otherwise known as active learning to discuss three applications of the chi-square test, namely (a) tests for independence and homogeneity (b) test involving several proportions, and (c) goodness-of-fit test. Constructivist learning, allows instructors/lecturers to develop practical applications of theories of learning [3]. The constructivist theory states that concepts follow the actions and new experiences build on an already existing knowledge [2]. Although instructors/lecturers play pivotal roles in student learning, they should act as facilitators and not as a source of knowledge. They must create the environment where students can collaboratively engage in learning with their peers. The purpose of this study was to use appropriate and effective instructional approaches to assist students understand the chi-square concepts. The following research questions guided the study (1) How does the use of SPSS, an instructional tool, enable lecturers to apply the concept of the Chi-square test? (2) What benefits from these applications, enable students to understand the chi-square test?

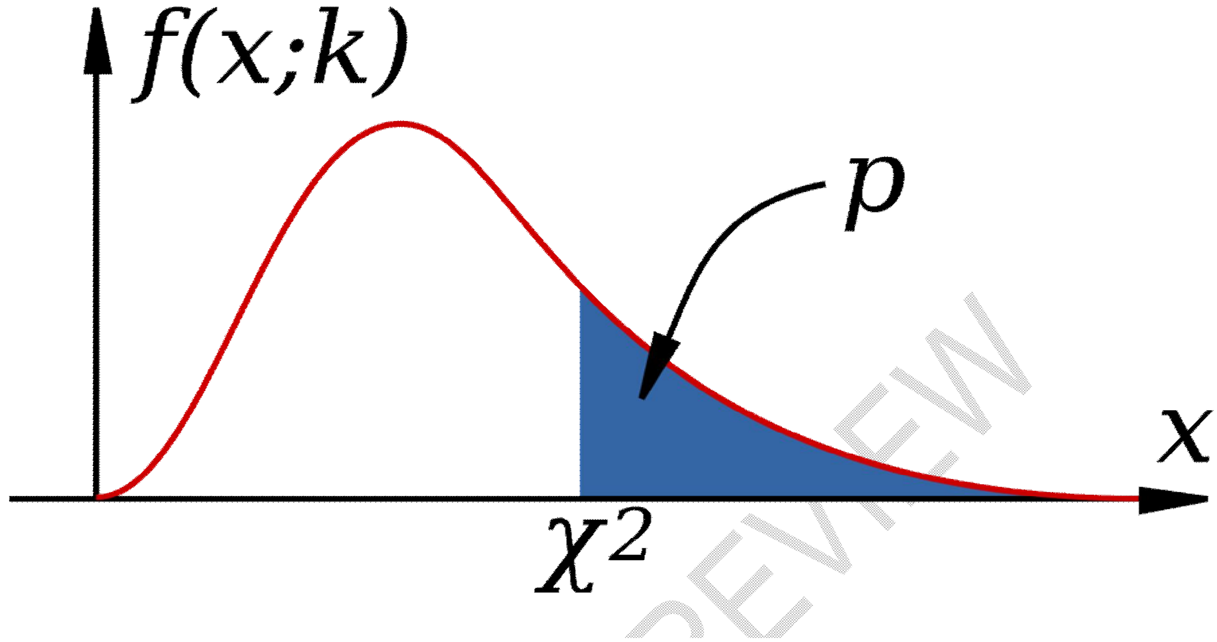
2. MATERIAL AND METHODS

Theoretical model

Definition: The chi-square distribution with *k* degrees of freedom is the distribution with the sum of the squares of *k* independent standard normal random variables. It is a special case of the gamma distribution. We call this distribution $\chi^2(k)$. Thus, if $z_1, z_2 \ldots \ldots \ldots z_k$ standard normal random variables (i.e., each $z_i \sim N(0,1)$), and if they are independent, then ${z_1}^2 + \cdots + {z_k}^2 \sim \chi^2(k)$.

Theorem 1: If random samples of size *n* are selected from a normal population with mean μ and variance σ^2, then the quantity $\chi^2 = \frac{(n-1)s^2}{\sigma^2}$ is a value of a random variable having a chi-square distribution with $n-1$ degrees of freedom. The graph of a typical chi-square (χ^2) distribution is indicated in fig. 1.

Fig.1 A graph of a chi-square (χ^2) distribution



From Figure 1, it is clear that the chi-square distribution is typically skewed to the right. Since the quantity $\chi^2 = \frac{(n-1)s^2}{\sigma^2}$ only contains nonnegative quantities, the values of χ^2 begin on the horizontal axis at zero. The total area under the curve is one. The chi-square distributions of interest is determined by the degrees of freedom–a quantity that is defined as $n-1$, where n represents the sample size.

Participants and Setting

The participants consisted of seventy (70) (fifty-five (45) males and fifteen (15) females) level 100 mathematics students from a university in Ghana. All the students were admitted from the public senior high schools across the country. They have all and completed two (2) introductory statistics courses, and had passed the courses with a grade of C or better. Their average age was twenty (18) years and two (3) months.

Instructions and Tasks

The study consisted of fourteen (14) groups, each comprising five (5) students. The lecturer met the students and advised them to participate actively in the learning process. Four important principles of active learning were demonstrated in the lecturer-student interactions, the students (1) constructed their own learning (2) built new learning on their prior knowledge (3) enhanced their learning through social interaction with their peers, and (4) developed learning by solving authentic tasks (Cooperstein & Kocevar-Weidinger, 2004). For the purposes of this study, the following pseudonyms were used for the participants: L= Instructor; G1 = A member from group 1; G2 = A member from group 2; G3 = A member from group 3; G4 = A member from group 4; G5 = A member from group 5; G6 = A member from group 6; G7 = A member from group 7; G8 = A member from group 8; G9 = A member from group 9; G10 = A member from group 10, G11: A member from group 11; G12 = A member from group 12 respectively; G13 = A member from group 13, and G14 = A member from group 14 respectively. The vignette below shows the lecturer assisting the students to apply the concept the chi-square concept. Table 1 shows the percentage points of the chi-square distribution.

Table 1 Percentage points of the chi-square distribution

Percentage Points of The Chi- Square Distribution

Degrees of Freedom	Probability of a larger value x^2								
	0.99	**0.95**	**0.9**	**0.75**	**0.5**	**0.25**	**0.1**	**0.05**	**0.01**
1	0.000	0.004	0.016	0.102	0.455	1.32	2.71	**3.84**	6.63
2	0.020	0.103	0.211	0.575	1.386	2.77	4.61	**5.99**	9.21
3	0.115	0.352	0.584	1.212	2.366	4.11	6.25	**7.81**	11.34
4	0.297	0.711	1.064	1.923	3.357	5.39	7.78	**9.49**	13.28
5	0.554	1.145	1.610	2.675	4.351	6.63	9.24	**11.07**	15.09
6	0.872	1.635	2.204	3.455	5.348	7.84	10.64	**12.59**	16.81
7	1.239	2.167	2.833	4.255	6.346	9.04	12.02	**14.07**	18.48
8	1.647	2.733	3.490	5.071	7.344	10.22	13.36	**15.51**	20.09
9	2.088	3.325	4.168	5.899	8.343	11.39	14.68	**16.92**	21.67
10	2.558	3.940	4.865	6.737	9.342	12.55	15.99	**18.31**	23.21
11	3.053	4.575	5.578	7.584	10.341	13.70	17.28	**19.68**	24.72
12	3.571	5.226	6.304	8.438	11.340	14.85	18.55	**21.03**	26.22
13	4.107	5.892	7.042	9.299	12.340	15.98	19.81	**22.36**	27.69
14	4.660	6.571	7.790	10.165	13.339	17.12	21.06	**23.68**	29.14
15	5.229	7.261	8.547	11.307	14.339	18.25	22.31	**25.00**	30.58
16	5.812	7.962	9.312	11.912	15.338	19.37	23.54	**26.30**	32.00
17	6.408	8.672	10.085	12.792	16.338	20.49	24.77	**27.59**	33.41
18	7.015	9.390	10.865	13.675	17.338	21.60	25.99	**28.87**	34.8
19	7.633	10.117	11.651	14.562	18.338	22.72	27.20	**30.14**	36.19
20	8.2600	10.851	12.443	15.452	19.337	23.83	28.41	**31.41**	37.57
22	9.5420	12.338	14.041	17.240	21.337	26.04	30.81	**33.92**	40.29
24	10.856	13.848	15.659	19.037	23.337	28.24	33.20	**36.42**	42.98
26	12.198	15.379	17.292	20.843	25.336	30.43	35.56	**38.89**	45.64
28	13.565	16.928	18.939	22.657	27.336	32.62	37.92	**41.34**	48.28
30	14.953	18.493	20.599	24.478	29.336	34.80	40.26	**43.77**	50.89
40	22.164	26.509	29.051	33.660	39.335	45.62	51.80	**55.76**	63.69
50	27.707	34.764	37.689	42.942	49.335	56.33	63.17	**67.50**	76.15
60	37.485	43.188	46.459	52.294	59.335	66.98	74.40	**79.08**	88.38

3.0 PROCEDURE AND RESULTS

Test for Independence

L: The Chi-square test for independence is used to determine if there is a significant relationship between two nominal (categorical) variables. The frequency of each category for one nominal variable is compared across the categories of the second nominal variable. The data can be displayed in a contingency table where each row represents a category for one variable and each column represents a category for the other variable. For example, a researcher may want to examine the relationship between gender (male vs. female) and empathy (high vs. low). The chi-square test of independence can be used to examine this relationship. The null hypothesis for this test is that there is no relationship between gender and empathy. The alternative hypothesis is that there is a relationship between gender and empathy (e.g., there are more high-empathy females than high-empathy males). A typical question is indicated as follows:

Q 1. A random sample of 200 people was classified according to age (less than 30 or 30 or older) and preference for style of car (sporty or luxury). The results appear in Table 2.

Table 2 Classification according to age and preference for style of car

	Car Preference	
Age	Sporty	Luxury
Less than 30	68	42
30 or older	31	59

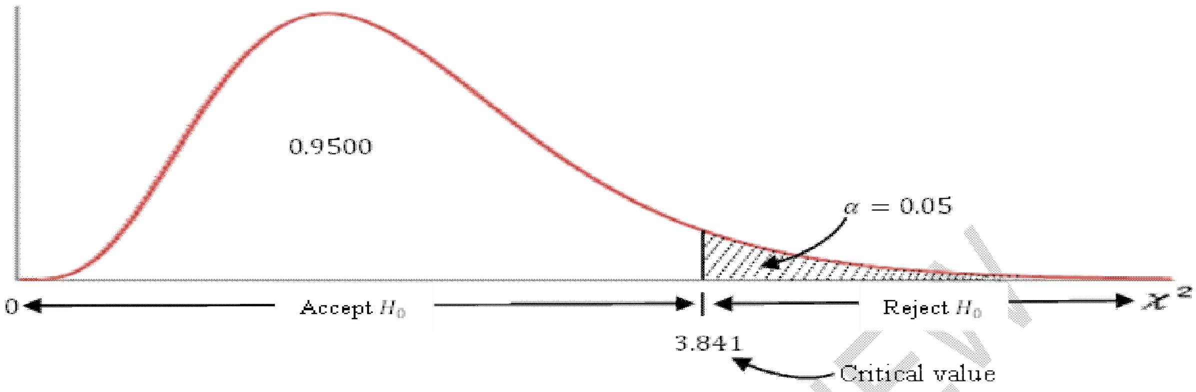
(a) Using a 0.05 level of significance, test the appropriate hypotheses to determine whether car style preference and age are independent.
(b) Test the same hypotheses using SPSS
(c) Provide an APA write-up for the results.

Solution

	Car Preference		
Age	Sporty	Luxury	Total
Less than 30	68 (54.5)	42 (55.6)	110
30 or older	31 (44.6)	59 (45.5)	90
Total	99	101	200

L: Are the variables independent or dependent? To answer this, two hypotheses are set, the null hypothesis, H_0, and the alternative hypothesis, H_1.
H_0: The variables of interest are independent.
H_1: The variables of interest are dependent or associated.
L: What is the population characteristic, G1?
G1: The population characteristic is independence, as stated in the question.
L: Good, based on what we've treated in class, what is the most important decision to take to arrive at the solution, G3?
G3: Our decision will be based on the value of the sum of the relative, squared differences in Theorem 1, $\chi^2 = \sum\left[\frac{(O-E)^2}{E}\right]$.
L: The distribution of this statistic can be approximated by a chi-square distribution with $\upsilon = (r-1) \times (c-1) = (2-1) \times (2-1) = 1$ degree of freedom. From Table 1, with $\alpha = 0.05$, and 1 degree of freedom $\chi^2{}_{0.05} = 3.841$. We will reject H_0 if the chi-square value is greater than 3.841 (see Fig. 2).

Fig. 2 Critical value at 0.05 significance level



L: All the groups should calculate the chi-square value. G2, go ahead and demonstrate this to the class.

G2: The calculated chi-square value is $\chi^2 = \frac{(68-54.5)^2}{54.5} + \frac{(42-55.6)^2}{55.6} + \frac{(31-44.6)^2}{44.6} + \frac{(59-45.5)^2}{45.5} = 14.83$

L: Since $\chi^2 = 14.83 > 3.841$, the value falls in the rejection region. We reject H_0 and conclude that the variables of interest are dependent. Hence a person's car preference to a significant extent depends on his or her age.

L: An alternative solution using SPSS in indicated in table 3.

Table 3 Age × Car Preference Cross tabulation

			Car Preference		
			Sporty	Luxury	Total
Age		Observed Count	68	42	110
	Less than 30	Expected Count	54.5	55.6	110.0
		Observed Count	31	59	90
	30 or older	Expected Count	44.6	45.5	90.0
		Observed Count	99	101	200
Total		Expected Count	99.0	101.0	200.0

Chart 1 : Chi-square tests

	Value	df	Sig.
Pearson Chi-square	14.83	1	.00
N	200		

A chi-square test of independence was conducted to determine whether individuals' preference for the style of car is dependent on their age. The chi-square statistic, age × car preference cross tabulation table and the chi-square tests, indicate that individuals' preference for the style of car is dependent on their age $\chi^2(1, N = 200) = 14.83, p < 0.05$.

Test for Homogeneity

L: The chi-square test of homogeneity tests to see whether different columns (or rows) of data in a table come from the same population or not (i.e., whether the differences are consistent with being explained by sampling error alone). For example, in a table showing political party preference in the rows and states in the columns, the test has the null hypothesis that each state has the same party preferences. A typical question is indicated as follows:

Q 2. A large corporation randomly selects 50 executives, 100 employees from middle management, and 150 production-line workers in an attempt to determine the relationship between job category and number of days absent from the job. The results appear in table 4.

Table 4 Classification according to days absent per year and job category

	Job Category		
Days Absent per Year	Executive	Middle Management	Production Line
Fewer than 3	15	21	48
From 3 to 6	27	40	75
More than 6	8	39	27

(a) Using a 0.05 level of significance, test the appropriate hypotheses to determine whether absenteeism is homogeneous with respect to job category.
(b) Test the same hypotheses using SPSS.
(c) Provide an APA write-up for the results.

Solution:

	Job Category			
Days Absent Per Year	Executive	Middle Management	Production line	Total
Fewer than 3	15	21	48	84
	(14)	(28)	(42)	
From 3 to 6	27	40	75	142
	(23.7)	(47.3)	(71)	
More than 6	8	39	27	74
	(12.3)	(24.7)	(37)	
Total	50	100	150	300

L: Are the days absent per year for all workers the same (i.e., proportionate and, hence, homogeneous) for each classification? To answer this, two hypotheses are set, the null hypothesis, H_0, and the alternative hypothesis, H_1.
H_0: For each day of absence, the proportion of executives, employees and middle management personnel are the same.
H_1: For at least a day of absence, the proportion of executives, employees and middle management personnel are not the same.

L: What is the population characteristics, G4?

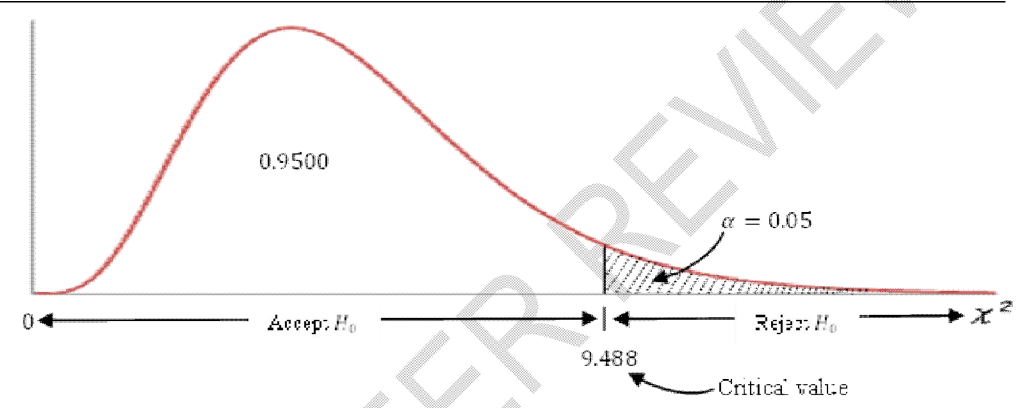
G4: The population characteristic is homogeneity, as indicated in the question.

L: What is the most important decision to take to arrive at the solution, G5?

G5: Our decision will be based on the value of the sum of the relative, squared difference given in Theorem 1: $\chi^2 = \sum\left[\frac{(O-E)^2}{E}\right]$.

L: The sampling distribution of this statistics can be approximated by using a chi-square distribution with $\upsilon = (r-1)\times(c-1) = (3-1)\times(3-1) = 4$ degrees of freedom. From Table 1, with $\alpha = 0.05$, and 4 degrees of freedom, $\chi^2{}_{0.05} = 9.488$. We will reject H_0 if the chi-square value is greater than 9.488 (see Figure 3).

Fig 3 Critical value at .05 significant level



L: All the groups should calculate the chi-square value. G5, go ahead and demonstrate it to the class.

G2: The chi-square value is $\chi^2 = \frac{(15-14)^2}{14} + \frac{(21-28)^2}{28} + \frac{(48-42)^2}{42} + \frac{(27-23.7)^2}{23.7} + \frac{(40-47.3)^2}{47.3} + \frac{(75-71)^2}{71} + \frac{(8-12.3)^2}{12.3} + \frac{(39-24.7)^2}{24.7} + \frac{(27-37)^2}{37} = 0.071 + 1.75 + 0.86 + 0.46 + 1.13 + 0.23 + 1.50 + 8.27 + 2.70 = 16.97$.

L: Since $\chi^2 = 16.97 > 9.49$, the value falls in the rejection region. We reject H_0 and conclude that for each day of absence, the proportion of executives, employees and middle management personnel are not the same

L: An alternative solution using SPSS is indicated in Table 5.

Table 5 days absent per year × Job Category Cross tabulation

			Job Category			
			Executive	Middle management	Production line	Total
Days absent per year	Fewer than 3	Count	15	21	48	84
		Expected count	14.0	28.0	42.0	84.0
	From 3 to 6	Count	27	40	75	142
		Expected	23.7	47.3	71.0	142.0

		count				
	More than 6	Count Expected count	8 12.3	39 24.7	27 37.0	74 74.0
Total		Count Expected count	50 50.0	100 100.0	150 150.0	300 300.0

Table 6 Chi square value

Chi- Square	df	Sig.
17.06	4	0.003

A chi-square test of homogeneity was conducted to determine whether absenteeism is homogeneous with respect to job category. The chi-square statistic, job category × days absent per year cross tabulation table and the chi-square tests, indicate that job category is not homogeneous with days absent per year, $\chi^2(4, N = 300) = 17.06, p < 0.05$ (see Table 6). Thus, for at least a day of absence, the proportion of executives, employees and middle management personnel are not the same.

Chi-square Test Involving Several Proportions

L: Suppose we have more than two binomial experiments and wish to test hypotheses where the null hypothesis has the form $H_0: p_1 = p_2 = \cdots = p_k$. By recording the number of successes that occur for each of these binomial experiments, we can produce a 2 x k contingency table, and then use theorem 2 that says: The sampling distribution of the statistic $\chi^2 = \sum\left[\frac{(o-E)^2}{E}\right]$ can be approximated by a chi-square distribution with $(r-1) \times (c-1)$ degrees of freedom, where r is the number of rows and c is the number of columns in the r x c contingency table, to determine whether $p_1 = p_2 = \cdots = p_k$. The contingency table is always a 2 x k table, because one row contains the number of observed successes and the other row contains the number of observed failures for each of the k binomial experiments. For $r = 2$, the degrees of freedom can be calculated as $(r-1) \times (c-1) = (2-1) \times (k-1) = k-1$. Statistically, we must have k independent binomial random samples, and the numbers of successes and failures for each random sample make up the entries in the contingency table. A typical question is indicated as follows:

Q3. A large hospital randomly selects 30 mothers who were less than 25 years old when they gave birth to their first child, 40 mothers who were between 26 and 35 years old when they gave birth to their first child, and 30 mothers who were older than 35 when they gave birth to their first child, to determine whether the proportion of normal births is the same regardless of the age of the mother. The results appear in Table 7.

Table 7 Classification according to types of birth and age of mother

	Age of Mother		
Types of Birth	Less than 25	26 to 35	Older than 35
Normal	22	23	9
Abnormal	8	17	21

1. Using a 0.05 level of significance, test whether the proportion of normal births is the same for each category of the mother.
2. Test the same hypotheses using SPSS
3. Provide an APA write-up for the results.

Solution:

	Age of Mother			
Types of Birth	Less than 25	26 to 35	Older than 35	Total
Normal	22	23	9	54
	16.2	21.6	16.2	
Abnormal	8	17	21	46
	13.8	18.4	13.8	
Total	30	40	30	100

L: Are the three probabilities of mothers giving normal birth the same? To answer this, two hypotheses are set, the null *hypothesis*, H_0, and the alternative hypothesis, H_1.
H_0: $p_1 = p_2 = p_3$
H_1: p_1, p_2, and p_3 are not all equal.

L: What is the population characteristic, G6?

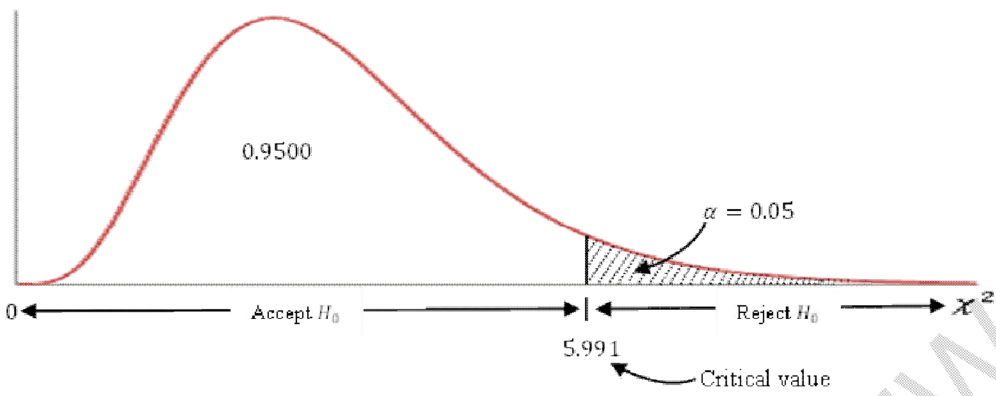
G6: The population characteristic is proportion, as stated in the question. The parameters involved are p_1, p_2, and p_3 for the three binomial experiments.

L: Great, G4, what is the most important decision to take to arrive at the solution?

G4: Our decision will be based on the value of the sum of the relative, squared difference given in Theorem 2: $\chi^2 = \sum\left[\frac{(O-E)^2}{E}\right]$.

L: The sampling distribution of this statistics can be approximated by using a chi-square distribution with $\upsilon = (r-1) \times (c-1) = (2-1) \times (3-1) = 2$ degrees of freedom. From Table 4, with $\alpha = 0.05$, and 2 degrees of freedom, $\chi^2_{0.05} = 5.991$. We will reject H_0 if the chi-square value is greater than 5.991 (see Fig. 4).

Fig. 4 Critical value at 0.05 significance level



L: All the groups should calculate the chi-square value. G5, go ahead and demonstrate this to the class.

G5: The chi-square value is $\chi^2 = \frac{(22-16.2)^2}{16.2} + \frac{(23-21.6)^2}{21.6} + \frac{(9-16.2)^2}{16.2} + \frac{(17-18.4)^2}{18.4} + \frac{(21-13.8)^2}{13.8} + \frac{(8-13.8)^2}{13.8} = 2.077 + 0.091 + 3.2 + 0.107 + 3.757 + 2.438 + 1.50 =$ 11.67

L: Since $\chi^2 = 11.67 > 5.991$, the value falls in the rejection region. We reject H_0 and conclude that the three probabilities are not all equal.

L: An alternative solution using SPSS is indicated in Table 8.

Table 8 Age of mother × types of birth cross tabulation

			Age of Mother			
			Less than 25	26-35	Older than 35	Total
Types of Birth	Normal	Count	22	23	9	54
		Expected Count	16.2	21.6	16.2	54.0
	Abnormal	Count	8	17	21	46
		Expected Count	13.8	18.4	13.8	46.0
Total		Count	30	40	30	100
		Expected Count	30.0	40.0	30.0	100.0

Table 9 Chi-square value

Chi- Square	df	Sig.
11.67	4	0.000
N	100	

A chi-square test of several proportions was conducted to determine whether the three probabilities of mothers giving normal birth are the same. The chi-square statistic, age of mother × types of birth cross tabulation table and the chi-square tests, indicate that the three probabilities of mothers giving normal birth are not the same, $\chi^2(1, N = 100) = 66.80, p < 0.05$ (see Table 9).

Chi-Square Goodness-of-fit Test

L: A test of goodness-of-fit establishes whether an observed frequency distribution differs from a theoretical distribution. For example, we could test the hypothesis that a random sample with 44 men and 56 women has been drawn from a population in which men and women are equal in frequency, i.e., with the theoretical distribution of 50 men and 50 women.

Q 4. A computer programmer wishes to determine whether the random number generator on his personal computer does, in fact, randomly generate the digits 1, 2, 3, 4, 5 one-fifth of the time. He randomly generates 500 of these digits with the results given in Table 10.

Table 10 Outcomes and frequencies

Outcome	1	2	3	4	5
Frequency	87	112	82	96	123

(a) Using $\alpha = 0.05$, test to see whether the programmer's random number generator does, in fact,
(b) generate each of these digits with probability of $\frac{1}{5}$.
(c) Test the same hypotheses using SPSS
(d) Provide an APA write-up for the results.

Solution:

L: The fact that each of the digits should be generated approximately 100 times follows from the fact that the theoretical probability distribution for the random variable x representing the digit generated is given by

x	1	2	3	4	5
$P(x)$	1/5	1/5	1/5	1/5	1/5

L: We already have the observed frequencies, and the expected frequencies in each cell can be found by calculating $\frac{1}{5} \times 500 = 100$. The table below lists the observed (and the expected) frequencies and the totals:

	Outcome					
Frequency	1	2	3	4	5	Total
Observed	87	112	82	96	123	500
Expected	(100)	(100)	(100)	(100)	(100)	(500)

L: Do the data follow the specified probability distribution? To answer this question, two hypotheses are set, the null hypothesis, H_0, and the alternative hypothesis, H_1.
H_0: The data follow the specified probability distribution.
H_1: The data do not follow the specified probability distribution.

L: What is the population characteristic, G6?

G6: The population characteristic is the probability distribution indicated.

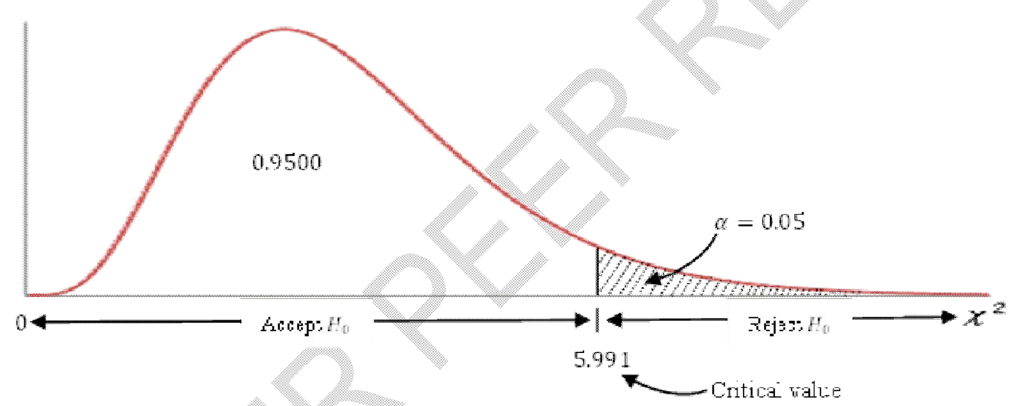
x	1	2	3	4	5
$p(x)$	1/5	1/5	1/5	1/5	1/5

L: Great, G4, what is the most important decision to take to arrive at the solution?

G4: Our decision will be based on the value of the sum of the relative, squared difference given in theorem 1: $\chi^2 = \sum\left[\frac{(O-E)^2}{E}\right]$

L: The sampling distribution of this statistics can be approximated by using a chi-square distribution with $\upsilon = k - 1 = 5 - 1 = 4$ degrees of freedom, where k is the number of the column cells. From Table 4, we have ${\chi^2}_{0.05} = 9.488$ with $\upsilon = 4$ degrees of freedom. We will reject H_0 if the chi-square value is greater than 9.488 (see fig.5).

Fig 5 Critical value at 0.05 significance level



L: All the groups should calculate the chi-square values. G7, go ahead and demonstrate this to the class.

G7: The calculated chi-square value is: $\chi^2 = \frac{(87-100)^2}{100} + \frac{(112-100)^2}{100} + \frac{(82-100)^2}{100} + \frac{(96-100)^2}{100} + \frac{(123-100)^2}{100} = 11.82$.

L: Since $\chi^2 = 11.82 > 9.488$, the χ^2 value falls in the rejection region. We reject H_0 and conclude that the random number generator does not generate the digits 1, 2, 3, 4, 5 according to the theoretical probability distribution.

L: An alternative solution using SPSS is indicated in Table 11.

Table 11 Observed and expected frequencies

Digit	Observed Frequency	Expected Frequency	Residual
1	87	100	-13
2	112	100	12
3	82	100	-18

4	96	100	-04
5	123	100	23
Total	500	500	

Table 12 Chi- square value

Chi- Square	df	Sig.
11.82	4	0.019
N	500	

A chi-square test of goodness-of-fit was performed to determine whether the five digits randomly generated equal number of digits. The results indicated that the number generated for each digit was not equally distributed in the population, $\chi^2(4, N = 500) = 11.82, p < 0.05$ (see Table 12)

4. DISCUSSION

The use of an active classroom or constructive learning technique provided the students with a great opportunity to engage with the content since they explored ideas among themselves before arriving at conclusions [3]. They built consensus through dialogue and open debate when working in their respective groups. As they engaged with themselves, they nurtured confidence and expressed themselves competently. This learning method enabled the lecturer to elicit responses from the students based on their thinking and creativity, with the lecturer largely facilitating the learning process [3]. These strategies included but are not limited to, making mind maps and allowing the students to make presentations.

By applying the definitions and theorems, the students understood the basics of the axiomatic structure characterizing the chi-square concept and its applications. These definitions and theorems embody the essence of its understanding and specify the latent meanings in the concept. They also helped to develop a deeper understanding of the underlying concepts. Facilitating the application of the chi-square concept by the lecturer helped reduce the memory load the students would have experienced if they had gone through the entire process.

Implications to teaching and learning

Lecturers should teach their students about theorems and their related proofs and applications. Theorems are significant in mathematics learning because they are absolute truths. They enable students to develop a deeper understanding of the underlying concepts. Most theorems are proved from axioms using some rules of inference, which means that the knowledge about axioms in mathematics learning is also crucial.

Lecturers should teach their students to understand definitions since inadequate knowledge about definitions creates a problem in mathematics learning. Definitions build conflict between the structure of mathematics, as conceived by professional mathematicians, and the cognitive processes of concept acquisition. It seems that no one in the mathematical community disagrees with the claim that mathematics is a deductive theory. It starts with primary notions about axioms and is grounded in understanding definitions.

Lecturers should facilitate the learning process so that students would have more opportunities to express their thinking through group work and exhibit a deeper mathematical understanding of the underlying concepts, which will enable them to have better retention. Students' strategies improve when they can incorporate the ideas of colleagues as well in their explanations. Classrooms enable students to practice the skills of reasoned argument and collective knowledge toward a common goal. Lecturers could use group discussions as opportunities for students to support one another to advance the aim of the classroom community.

4. CONCLUSION

This study has demonstrated that, by working with examples, the students gradually internalized their concept-acquisition skills to the extent that they confidently recognized what concept to apply to every question. Lecturer-student interaction has an impression on classroom management and affects student learning. The establishment of a positive teacher-student relationship aids a student's cognitive, social and emotional growth and enhances his/her mental well-being. The lecturer-student relationship impacts positively students' self-esteem and improves his/her skills. Lecturer-teacher interactions are vital for students' academic self-concept and for enhancing their enthusiasm and success.

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