Original Research Article

Stochastic Modeling for the Analysis and Forecasting of Stock Market Trend Using Hidden Markov Model

ABSTRACT

The HMM is generally applied to forecast the hidden system of observation data. In this paper, we deal with the development of HMM for a proper understanding of finance variables in the stock market. Formulation of relationships between and within both the changing share values of Housing Development Finance Corporation Bank Limited (HDFC Bank Ltd) as visible/observed states influenced by the indicators of S&P Bombay Stock Exchange Sensitive Index (Sensex) as invisible/influencing states. Stochastic modeling with hidden Markov models is carried out for exploring various parameters of the model. Deducing mathematical formulation of initial probability vector, transition and observed probability matrices were carried out with the empirical data sets. Furthermore, an attempt was made to estimate the long-run steady-state behavior of both the SENSEX and HDFC Bank share prices. Mathematical derivations for all the required statistical measures are obtained using the method of moments and the method of moment generating function (MGF), probability generating function (PGF) and the characteristic function (CF) for the proposed probability distribution. The findings of these studies will be valid for effective decision-making in portfolio management.

Keywords: Markov chain, Hidden Markov Models, Parameters of HMM, stock market, Portfolio Management.

1. INTRODUCTION

The stock market is a global network that facilitates practically all significant economic activities at a dynamic and effective rate known as the stock value on the basis market stability. The stock market is a common term that now encompasses the entire world's economic activities. In other words, a stock market is a venue where individuals buy and sell equity shares of firms through stockbrokers. The participants of market are investors and traders who are looking for short-term and long-term rewards on their investments. Traders are looking for rapid rewards by keeping an eye on slight changes in the share price. Investors, on the other hand, have a long-term perspective and benefit from capital appreciation. Market pressures cause stock values to fluctuate every day. The legal platform on which equities are transacted is the stock exchange. The Bombay Stock Exchange (BSE) and the National Stock Exchange (NSE) handle the majority of the Indian share market's transactions. Since 1875, the BSE has been in operation. The NSE was established in 1992 and began trading in 1994. Sensex and Nifty are two popular Indian stock market indicators for the two exchanges.

Stock markets are now an important aspect of the world economy. The market's volatility has an impact on our individual and company financial lives, as well as economic progress of a nation. The greater the significance of a stock's variance in relation to market swings, the more volatile it is. To put it in another way, it's a risky investment. In order to mitigate this investment risk, stochastic data analyses can be useful [1]. Due to its large returns, the stock market has traditionally been one of the most commercial and popular investments, according to Kuo. R J et.al in 1996 [2]. Financial analysts rarely have a complete understanding of stock market activity. Both financial analysts and investors, according to Umar MS and Musa TM in 2013, require daily data in predicting the stock prices trend behaviour [3].

Predicting the value of stock offers substantial potential profit opportunities, which is a major driver of study in this area. Information of stock prices, even before a second, results in large profits. Several factors influence stock prices in the stock market, such as political considerations, economic conditions of the state, performance of the company, industry factors, corporate factors, quality of health in the nation and investor psychological issues. The Markov chain model and HMM appear to be quite beneficial in analyzing and predicting future stock price behavior in this financial market. In HMM, these influencing elements that affect stock prices are referred to as hidden states. These states are hidden in an HMM and follow the Markov property, but the states that are dependent on them are observable. In an HMM the observation or the visible state has a probability distribution that corresponds to a probable state at time t. The study is based on Sensex closing share prices (hidden state) influencing HDFC Bank share prices. The major goal of this article is to forecast HDFC Bank's share prices in the near future and to examine its long-term prospects by applying HMM containing three hidden states and three visible states.

1.1. Literature Review

In recent years, a significant amount of research has been published in the hopes of finding an ideal (or nearly optimal) stock market analysis and prediction model. The statistical time series analysis approaches including linear regression, ARMA [4] and multiple regression models have been used in the majority of forecasting research. Traditional statistical prediction models are based on linear time series data.

When evaluating market situations and the transition law between different states, HMM can be used to solve problems involving time series data, regardless of whether the data is linear or nonlinear. HMM can be defined as the statistical Markov chain model where the system being modeled is assumed to be a Markov process with hidden or invisible states. Initially, the concept of Markov chain (MC) was introduced by a Russian mathematician after his name Andrei Andreevic Markov (1856–1922). Later, Leonard E. Baum and other academics researchers published a series of studies [5] on Markov chain model in the late 1960s and early 1970s. In 2014, Juan et al. demonstrated the use of a Markov model in decision process [6] to determine the best strategy for orange farm management. Because it takes fewer calculations to arrive at the best solution, policy iteration is more efficient than linear programming. The change in states of MC that is from one state to another in discrete time is referred to as state transitions and according to Alghamdi in [7], the Markov chain follows memory less property or Markov property, which states that the probability of going from one state to another depends only on its current state and not on its history.

A number of researchers have recently used HMM to analysis and predict the share prices in the stock market. In 2005, Hassan and Nath applied an HMM for the prediction of share prices of interconnected markets in [8]. The HMM is utilized with the four states of observations, such as close, open, high, and low prices, to forecast the future closing price of several airline stocks. In 2006, Guidolin and Timmermann applied HMM with four states and numerous observations to investigate asset allocation decisions on the basis of state-switching in market return [9]. In 2012, Kritzman et al. used a two-state HMM to forecast the inflation rate, volatility in the market and the industrial production index [10]. In 2012, Gupta, A., and Dhingra, B. proposed a Posterior hidden Markov prediction model in [11] for the time series analysis data. The share prices that occur within one day, such as the high and low values of the shares, are used in this model to forecast the next day's share price. Again, in the same year, Nobakht et al. [12] applied an HMM where they used different observation or visible data such as open price, close price, low price, high prices of share in order to forecast its closing price. Tuyen applied a normally distributed HMM in [13] on VN-Index historical data in 2013 to obtain the best Markov model. In 2014, Nguyen used HMM to estimate economic states and share prices using both single and multiple observations in [14] and later in 2015, Nguyen et. al used the HMM to forecast the invisible state of market data and choose stocks on the basis of the projected state [15]. Holzmann et al. observed the states and state space in an HMM in 2016 and discovered the state with the highest volatility, which corresponded the financial crisis [16]. Liu et al. [17] employed a three state HMM in order to explain the time changing distribution of Chinese stock market returns in 2017. In 2018, Nguyen, N discussed the application of HMMs [18] in stock trading based on stock price predictions. To obtain an ideal number of states in HMM, this technique starts by applying the four principles such as the Akaike information criterion AIC, Bayesian information criterion BIC, Hannan Quinn information criterion HQIC, and the Boz Dogan Consistent Akaike Information criterion BDCAIC. Huang et al. [19] investigated non-homogeneous HMMs and proposed a better EM approach for detecting bull and bear market movements in 2019. Suda and Spiteri in 2019 make similar comparisons in [20] to S & P 500 (Standard & Poor's 500), the standard stock index that has been the subject of various finance research. In this regard, a technique for predicting future stock market patterns is being developed. In 2020, Liu et al. proposed an HMM in the context of a transition in the states of an economy and investigated option pricing as the pricing system shifted risks [21].

From the existing literature, it is clearly found that the HMMs are applied for the stock market data. The trend analysis of the stock market is obtained using HMM in this study by taking into account the one-day difference in closing price for a given period. In this approach, HMM is used in order to predict the behavior of the visible states by using the effect of invisible states. The present study is done using HMM, its parameters (A, B, π), and the probability distribution for the sequence of visible states in order to analyze and predict the long-run behavior of stock movement.

2. METHODOLOGY

2.1. Markov Chain Model (Mc Model)

A MC model is a random/stochastic model that describes a series of possible events, with the likelihood of each current event exclusively depends upon its instantaneous previous event and forgets about its history. The Markov chain means that given X_t the state $X_{(t+1)}$ depends only upon X_t but not on $X_{(t-1)}, \ldots, X_1, X_0$. Because of its Markovian features, powerless interest in accurate information, and predicting behavior with many preferences, the Markov model is important in statistics. Mathematically, the Markov property represents if $X_n, n = 0, 1, 2, \ldots t+1$ is a random process with discrete state space S, then $P(X_{t+1}=x_{t+1}/X_t=x_t, X_{t-1}=x_{t-1},...,X_1=x_1,X_0=x_0) = P(X_{t+1}=x_{t+1}/X_t=x_t)$ for all t = 0, 1,2, 3, … and for all states $x_0, x_1,..., x_t, x_{t+1}$. The state-space is defined as the countable set S containing all of X_i's possible values. The condition of the any state of the system may change over time. The transition probability, represented as a_{ij}, is the likelihood that the process will move from state i in the n^{th} step to state j in the $(n+1)^{th}$ step. Hence $a_{ij}=P(X_{n+1}=j \mid X_n=i)$ for all $i, j \in S$ and $n \ge 0$, $1 \le i, j \le N$.

2.2. Hidden Markov Model (Hmm)

Hidden Markov model is a statistical Markov model in which the system under consideration is supposed to be a Markov process, which we refer to X, with hidden, or invisible states. It is assumed that there is another process, we call it Y, whose movement is influenced by the hidden process X. Suppose X_n and Y_n be two stochastic processes such that $n \ge 1$, then the pair $\{X_n, Y_n\}$ is an HMM if the first process X_n follows Markov property whose trend behaviour is not directly visible and $P(Y_m=y_m/X_1=x_1, X_2=x_2,...,X_m=x_m)=P(Y_m=y_m/X_m=x_m)$ for every $n \ge 1$. In our probabilistic model, HMM allows us to discuss about both visible and hidden events that we think of as causal factors. Therefore, the HMM is defined as a stochastic model in which the invisible or hidden states are supposed to follow a Markov Property, and it outperforms the other models in terms of accuracy. The parameters and elements of an HMM (λ) are A, B and π and are determined using the supplied or given input values. The overall HMM is written as λ = (S, V, A, B, π) where the set S ={H_1,H_2, … ,H_N}contains of N possible hidden/invisible states, the set V={V_1,V_2,…,V_M} are M possible visible states, A is a square matrix of dimension N that is called the transition matrix or (TPM), B is also an $N \times M$ rectangular matrix of observation probabilities which is termed as observed probability matrix (OPM) and finally the N dimensional vector π contains the probability of hidden which is known as the initial probability vector (IPV). These parameters A, B and π of HMM satisfy $\sum_{j=1}^{N} a_{ij}=1$, $\sum_{j=1}^{M} b_{ij}=1$ and $\sum_{i=1}^{N} \pi_i=1$, $\pi_i \ge 0$ for $1 \le i \le N$.

3. DESCRIPTION OF THE MODEL

The present study deals with three hidden states, namely the impact states fluctuations such as loss (H_1), no change (H_2), and Gain (H_3) of share prices in Sensex. The impact will result in getting the observed states like Fall (V_1), remain same (V_2), and rise (V_3) in the share prices of HDFC bank. These three invisible states and three visible states for the paper are defined as following in sections 3.1 and 3.2 respectively;

3.1. Hidden States

These three hidden states are defined on the basis of the difference between the next day's and the previous day closing share prices of Sensex. Symbolically, we write these three states as follows.

H_1: When $(x_t - x_{t-1}) < -1$, the Sensex share price is in the state of loss (L).

H_2: When $-1 < (x_t - x_{t-1}) < +1$, the Sensex share price is in the state of no change (N).

H_3: When $(x_t - x_{t-1}) > +1$, the Sensex share price is in the state of gain (G).

Where, x_t is the current and x_{t-1} is the previous closing share price of Sensex.

3.2. Visible States

In the same way, the three visible states are assumed by the difference between the next day and the previous day's closing share prices of HDFC bank. Symbolically, we write these three states as follows.

V_1: When $(y_t - y_{t-1}) < -1$, the HDFC bank share price is in the state of fall (F)

V_2: When $-1 < (y_t - y_{t-1}) < +1$, the HDFC bank share price remains same (S).

V_3: When $(y_t - y_{t-1}) > +1$, the HDFC bank is in the state of rise (R).

Where, y_t is the current and y_{t-1} is the previous closing share price of HDFC bank.

The schematic diagram or state transition diagram of the transitions in hidden states and visible states is displayed below in figure 1. The arrow marks are depicting the connectivity from and to the states.

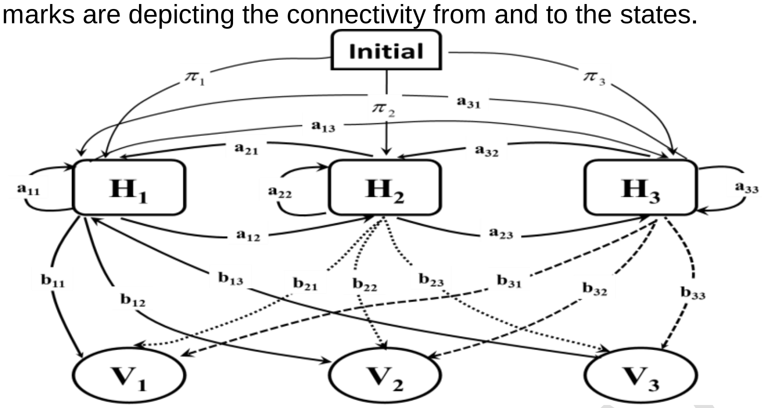

Figure 1: Schematic diagram of HMM

In this paper, H_1=L, H_2=N and H_3=G and Similarly, V_1=F, V_2=S and V_3=R.

The three main questions that must be answered in order to work with the HMM.

I. Given the observation or visible sequence V = {V_t, t = 1,2, 3, …, T} and model λ= (A, B, π); calculate P (V| λ), that is to compute probability of visible sequence $V_1V_2V_3...V_T$.

II. Given the visible sequence V = {V_t, t = 1, 2, 3, …, T} and HMM λ= (A, B, π), choose the most likely hidden sequence H= {H_t, t = 1,2, 3, …, T} that best explains the visible sequence.

III. Given the visible data V = {V_t, t = 1,2, 3, …, T}, how do we adjust the parameters λ= (A, B, π), in order to maximize P (V| λ).

Whare, A, B and π are TPM, OPM, and IPV. Either forward or backward algorithms developed by Baum et al. in 1967 and in 1968 [22,23] can be applied to solve problem (I) whereas both forward and backward algorithms which is also known as Baum–Welch algorithm is used to solve problem (III). The Viterbi algorithms given by Forney, G.D in 1973 and Viterbi, A.J in 1967 solves Problem (II) [24, 25]. In the present paper, the probabilities of the visible state sequences of specific length are obtained using the parameters of HMM.

4. PROBABILITIES OF THE SEQUENCE VISIBLE STATES OF LENGTH ONE AND TWO

In this section, we have obtained the formula for computing the probability of visible states (V_j, j = 1, 2, 3). These probabilities are obtained by the effect of hidden states (H_j, j = 1, 2, 3). Hence the probability expression of visible states is obtained in equation 4.

$$P(V_j) = \sum_{i=1}^{3} \pi_i b_{ij} \geq 0, \; for \; j = 1,2,3 \text{ such that } \sum_{j=1}^{3} P(V_j) = 1 \qquad \text{-------(4)}$$

Since the visible states happen by the influence of hidden/invisible states. Therefore, probability of the sequence of two visible states can occur by the happening of nine different possible combinations of hidden states. Symbolically, we write the probability of (V_uV_v) for all (u, v = 1, 2, 3) can happen jointly with the happening of (H_1H_1), (H_1H_2), (H_1H_3), (H_2H_1), (H_2H_2), (H_2H_3), (H_3H_1), (H_3H_2) and (H_3H_3). The probability of the sequence of two visible states is obtained in following expression 4.1:

$$P(V_uV_v) = \sum_{j=1}^{3} (b_{ju})(b_{jv}) \left\{ \sum_{i=1}^{3} \pi_i a_{ij} \right\}, i, j = 1,2,3 \; \forall \; u, v = 1,2,3 \qquad \text{-------(4.1)}$$

For a specific combination say (V_1V_1), the formula is given in equation 4.2.

$$P(V_1V_1) = \sum_{j=1}^{3} (b_{j1})(b_{j1}) \left\{ \sum_{i=1}^{3} \pi_i a_{ij} \right\}, \; i, j = 1,2,3 \qquad \text{-------(4.2)}$$

Similarly, we can find the probability of all possible sequences of length two such as (V_1V_2), (V_1V_3), (V_2V_1), (V_2V_2), (V_2V_3), (V_3V_1), (V_3V_2) and (V_3V_3) by using equation 4.1.

5. PROBABILITY DISTRIBUTION FOR VARIOUS LENGTHS OF VISIBLE STATES

Let $X_k(\omega)$ be a random variable that represents the number of times the state V_k occurs in a sequence of length T=2. Then $X_k(\omega)=0,1,2,...,T$ where 'T' denotes the length of the visible sequence. Thus, a random variable $X_k(\omega)$ is said to follow the proposed probability distribution if it takes the integral values of 0,1, 2 and its P.M.F is given by

$$P(X_k(\omega)=x_k)=\begin{cases}\sum_{j=1}^{M}\left\{C_x^2 b_{jk}{}^{x}(1-b_{jk})^{2-x}(\alpha)\right\}, & for\ \ x=0,1,2\\ 0 & , \quad otherwise\end{cases} \qquad \text{-------(5)}$$

Where $\alpha=\sum_{i=1}^{N}\pi_i a_{ij}$

This is the general probability distribution for the visible states. The probability distribution for the visible state V_1, V_2 and V_3 are obtained by putting the k=1,2,3. In the present paper, the number of hidden states N=3, the number of visible states M=3 and the length of visible sequence T=2.

5.1. Statistical Measures for the Derived Probability Distribution

In this section, the moments and various statistical parameters are obtained for the derived probability distribution. The r^{th} moment about origan of the above probability distribution is derived using the equation 5.1.

$$\mu_r{}' = 2\sum_{j=1}^{M} b_{jk}\left[\left\{1+\left(2^{r-1}-1\right)b_{jk}\right\}\left(\sum_{i=1}^{N}\pi_i a_{ij}\right)\right], r=1,2,3,4... \qquad \text{-------(5.1)}$$

The formula for the mean and the variance of the probability distribution are derived as follow in equations 5.2:

$$\mu_1{}' = 2\sum_{j=1}^{M}\left[b_{jk}\left(\sum_{i=1}^{N}\pi_i a_{ij}\right)\right]$$

$$\mu_2 = 2\sum_{j=1}^{M}\left[b_{jk}\left(1+b_{jk}\right)(\alpha)\right]-\left[2\beta\right]^2 \qquad \text{------(5.2)}$$

Similarly, to find the shaping and peakedness measures of the distribution, we need to find the third and fourth central moments. The expression for the third and the fourth central moments calculated in equation 5.3;

$$\mu_3 = 2\beta(4\beta)(2\beta-1)+6(2\beta-1)\gamma$$

$$\mu_4 = 2\beta(4\beta-1)\left(-3(2\beta)^2+6\beta-1\right)+\left(12(2\beta)^2-24(2\beta)+7\right)\gamma \qquad \text{-------(5.3)}$$

Hence the measure of skewness (β_1), measure of kurtosis (β_2), γ_1 and γ_2 of the above probability distribution are obtained in the following expressions 5.4 respectively.

$$\beta_1 = \left(2\beta(4\beta)(2\beta-1)+6(2\beta-1)\gamma\right)^2(A)^{-3}$$

$$\beta_2 = \left[2\beta(4\beta-1)\left(-3(2\beta)^2+6\beta-1\right)+\left(12(2\beta)^2-24(2\beta)+7\right)\gamma\right][A]^{-2}$$

$$\gamma_1 = \left(2\beta(4\beta)(2\beta-1)+6(2\beta-1)\gamma\right)(A)^{-\frac{3}{2}} \qquad \text{-------(5.4)}$$

$$\gamma_2 = \left[2\beta(4\beta-1)\left(-3(2\beta)^2+6\beta-1\right)+\left(12(2\beta)^2-24(2\beta)+7\right)\gamma\right][A]^{-2}-3$$

Where $A=2\sum_{j=1}^{M}\left[b_{jk}\left(1+b_{jk}\right)(\alpha)\right]-\left[2\beta\right]^2$, $\alpha=\sum_{i=1}^{N}\pi_i a_{ij}$; $\beta=\sum_{j=1}^{M}b_{jk}(\alpha)$, $\gamma=\sum_{j=1}^{M}b_{jk}{}^2\{\alpha\}$, a_{ij}, b_{ij} and π_i are respectively the elements of TPM, OPM and IPV. In this study, we have N=M=3 and K=1,2,3.

5.2. Generating Functions

Various generating functions such as the moment generating function (MGF), the probability generating function (PGF), and the characteristic functions (C.F.) of the derived probability distribution are also obtained. The MGF, PGF and CF are given in the expressions (I), (II), and (III) respectively:

The moment generating function is given in the following expression.

$$M_x(t) = \sum_{j=1}^{M} \left[\left\{ \left(1-b_{jk}\right)\left(1-b_{jk}+2e^{t}b_{jk}\right)+e^{2t}b_{jk}{}^{2} \right\} \alpha \right] \qquad \text{-------(I)}$$

The PGF is denoted by P(s) and is obtained as in the following expression.

$$P(s) = 2\sum_{j=1}^{M} \left[b_{jk}\left(1-b_{jk}\right)\{\alpha\} \right] + 2s\sum_{j=1}^{M} \left[b_{jk}\{\alpha\} \right] \qquad \text{-------(II)}$$

Characteristic function of the proposed probability model is also obtained in the following equation.

$$\phi_x(t) = \sum_{j=1}^{M} \left[\left\{ \left(1-b_{jk}\right)\left(1-b_{jk}+2e^{it}b_{jk}\right)+e^{2it}b_{jk} \right\} \alpha \right], i=\sqrt{-1} \text{ where, } \alpha = \sum_{i=1}^{N} \pi_i a_{ij} \text{ and k=1,2,3} \qquad \text{-------(III)}$$

6. EMPIRICAL DATA MODELLING AND STATISTICAL ANALYSIS

In the present paper, the daily day's share prices of Sensex were used as hidden states, which influence the market value of the closing share price of HDFC bank. We have collected 494 observations of both the Sensex and HDFC bank from 3 April 2017 to 29 March. 2019, i.e., for almost three years of data from the BSE (*www.bseindia.com* & *www.yahoofinance.com*). The data is of a discrete time-discrete state. The summary of the given date of HDFC bank and Sensex is given in table-1.

Table 1: Summary Table of Sensex closing prices and HDFC bank share values.

	Closing share Prices					
	Minimum	Mean	Variance	Maximum	CV	Correlation
HDFC Bank	1433.4	1915.6	34699.41	2316.5	9.725	0.936
SENSEX	29319.1	29788	5186222	38896.63	7.646	

From the above table, we can see that the correlation between the two states, i.e., Invisible (Sensex) and the visible states (HDFC bank), is 0.936. This means that the two states are highly positively correlated, i.e., the change in Sensex influences the likelihood HDFC bank.

The daily data regarding the closing share prices of Sensex and HDFC bank are converted in to states such as L, N, G and F, S R defined in section 3.1 and 3.2 in MS excel using IF command. Frequency table for all the three hidden states L, N, G and F, S, R is collected by COUNTIF function in MS excel and is displayed in table 2.

Table 2: Frequency table of the F, S, R, L, N, G and the combination of two

States	Frequency	States	Frequency	States	Frequency	States	Frequency
L	255	LL	133	NL	1	GL	113
F	212	LF	144	NF	0	GF	70
N	2	LN	1	NN	0	GN	1
S	31	LS	14	NS	0	GS	17
G	267	LG	110	NG	1	GG	153
R	251	LR	67	NR	2	GR	180

6.1. Transition Probability Matrix

The TPM of a MC model gives the probabilities of moving from one state to another in a single time unit. Since we have applied the HMM, the TPM is the matrix of transition probabilities of reaching to hidden state j from hidden state i. The elements of TPM are denoted by a_{ij} and is defined as $a_{ij} = P(X_{t+1} = j \mid X_t = i)$. Note that all the elements of the TPM are positive and the rows of any state TPM must sum to one.

The transition frequency matrix is given by.

$$P = \begin{matrix} & L & N & G & \\ L & 133 & 1 & 110 & 244 \\ N & 1 & 0 & 1 & 2 \\ G & 113 & 1 & 153 & 267 \end{matrix}$$

TPM from X_{n-1} to X_n is obtained in matrix A.

$$A = \begin{matrix} & L & N & G \\ L & 0.545082 & 0.004098 & 0.45082 \\ N & 0.5 & 0 & 0.5 \\ G & 0.423221 & 0.003745 & 0.573034 \end{matrix}$$

In the above matrix A, it can easily be seen that there will be loss, no change and gain with respective probability 0.54, 0.004 and 0.57 in the share prices of Sensex in the next day when it was in the state of loss in the previous day. In the third row, we can say that the share prices of Sensex will be in the state of L, N, or G with probabilities 0.42, 0.003 and 0.57 respectively in the next day given that it was in the state of G in the previous day.

6.2. Observed Probability Matrix

OPM is also the matrix of observed or emission probabilities from hidden states to visible states. The elements of OPM are the conditional probabilities of observed states given hidden states. In the present paper, the hidden states are L, N and G of Sensex which influence the visible states such as F, S and R of HDFC bank. The elements of OPM are denoted by b_{ij} and are defined as $b_{ij} = P(Y_t = j \mid X_t = i)$. Observed frequency table from the invisible states of Sensex to the visible states of HDFC bank are obtained in the following matrix O.

$$O = \begin{matrix} & F & S & R & \\ L & 144 & 14 & 67 & 225 \\ N & 0 & 0 & 2 & 2 \\ G & 70 & 17 & 180 & 267 \end{matrix}$$

Therefore, OPM from state X_n of the Invisible state to state Y_n of the visible state is also calculated using MS Excel and R and is given by

$$B = \begin{matrix} & F & S & R \\ L & 0.64 & 0.062222 & 0.297778 \\ N & 0 & 0 & 1 \\ G & 0.262172 & 0.06367 & 0.674157 \end{matrix}$$

This matrix shows us how and to what extent the change in invisible states will influence the visible state. In this study, HMM is used to predicting the share prices of HDFC bank by identifying the change in the share prices of Sensex by obtaining TPM and OPM.

6.3. Initial Probability Vector

In order to calculate the IPV, we need to obtain the initial frequency table. The initial frequency table is the frequency of invisible states which is presented in table 3.

Table 3: Initial Frequency Table

States	Loss (L)	No change (N)	Gain (G)	Total
Frequency	267	2	225	494

IPV is the probabilities of the hidden states. Therefore, the vector π is the probability of loss, no change and gain in the closing prices of Sensex.

$$\begin{array}{ccc} L & N & G \end{array}$$

$$\pi = \begin{bmatrix} 0.540486 & 0.004049 & 0.455466 \end{bmatrix}$$

This vector reveals that there are 54% chances of losing and 45% chances gaining the share price of Sensex. The schematic diagram of our model parameters is presented empirically in figure 2. The numbers on the respective arrows represents the probabilities from one state to another state.

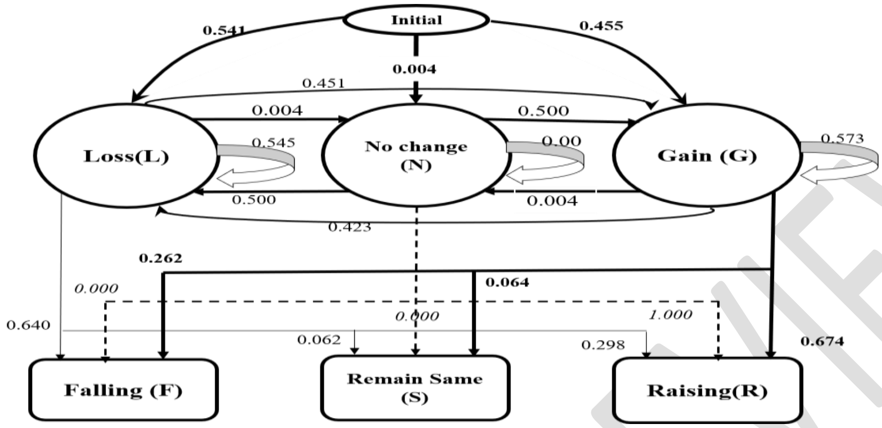

Figure 2: Schematic Diagram of the HMM.

6.4. Probabilities of the sequence of visible stats

Probabilities of the visible states that are the probabilities of the fall (F) in the share values of HDFC bank, the probability that the share value remains same(S) and the probability of rising(R) the share values of the HDFC bank are obtained using the equation 4 from section 4 are.

Table 4: Probabilities of the sequence of single visible states.

P(F)=0.465321	P(S)=0.06263	P(R)=0.472049

From above, we have observed that the probability of falling and raising the share values of HDFC bank are almost same which represents the volatility of share market.

Using the equation from section 4, the probability of the sequence of two visible states are also obtained in the following table 6.

Table 5:Probabilities of the sequence of two visible states.

P (F, F) = 0.235283	P (S, F) = 0.02795	P (R, F) = 0.182822
P (F, S) = 0.027947	P (S, S) = 0.003949	P (R, S) = 0.030817
P (F, R) = 0.182822	P (S, R) = 0.030817	P (R, R) = 0.277598

P (F, F) is the probability of fall in the share prices of HDFC bank for the two consecutive days. From the table-6, it is observed that the rising share prices of HDFC bank for the two consecutive days is maximum. It reveals that there is maximum probability of rising the share prices of HDFC bank tomorrow, if the share values of HDFC bank is in the state of rise today.

6.4.1. Descriptive Statistical Measures

The values of the parameters like mean and variance of the derived probability distribution for the real time data using the equations 5.2 derived in section 5.1 are obtained. In this section, we are studying the fall state. The mean $\mu'_1 = 0.57889$ and $\mu_2 = 0.714343$ are obtained using R and Excel. The coefficient of skewness and the coefficient kurtosis of the probability distribution of falling state (F) is calculated in table-6.

Table 6: Statistical Parameters of the Model.

Parameters	Values	Parameters	Values	Parameters	Values
μ_3	0.14983	β_1	0.061587	γ_1	0.248167
μ_4	1.0242	β_2	2.007112	γ_2	0.99289

6.5. Long or Steady State Behavior in Sensex Prices

Forecasting of long run behavior of Sensex is very meaningful for investors. The long run behavior of Sensex is observed by determining the higher order TPM. The TPM obtained in section 6.1 is given by

$$A = \begin{bmatrix} 0.545082 & 0.004098 & 0.45082 \\ 0.5 & 0 & 0.5 \\ 0.423221 & 0.003745 & 0.573034 \end{bmatrix}$$

This TPM is an Ergodic Markov chain, which means it is irreducible, positively recurrent, aperiodic, and time homogeneous. That is for an ergodic Markov chain, $\lim_{n\to\infty} a_{ij}^{(n)}$ exists and is independent of i. This assumption of ergodicity aids in the predicting of the share prices in the long-term behaviour. Here the stationary matrix for the share values of Sensex is obtained as follows.

$$A^8 = \begin{matrix} & L & N & G \\ L & 0.4822932 & 0.003900978 & 0.5138058 \\ N & 0.4822932 & 0.003900978 & 0.5138058 \\ G & 0.4822932 & 0.003900978 & 0.5138058 \end{matrix}$$

From the above matrix, we see that the stationary is reached at 8th step that is the steady-state is obtained at A^8 which represents that after the 8th trading days since 494 trading days, the TPM tends to the state of equilibrium or steady state. After then the TPM remains unchanged for the onwards consecutive trading days. This steady state TPM of Sensex reveals the following information.

1. The probability of loss in the closing price of Sensex in near future and in the long run irrespective of its initial states weather it was in L, N or G is 0.48.
2. There are 0.003 likelihood that Sensex prices will have no change in future irrespective of its initial.
3. The probability of gain in the closing price of Sensex future no matter what the initial states is weather L or N or G is 0.51.

we can conclude from the above information that probability of gaining (G) in the closing prices of Sensex is having greater likelihood irrespective of its initial state it was, whether Loss, No change or Gain.

6.6. Steady State Behavior of HDFC Bank

Since the matrix B is also an irreducible that is independent of where we start from, if we let the chain run for a long period of time, then the distribution of Yn will converge to a constant. The OPM is obtained in section 6.2 as

$$B = \begin{matrix} & F & S & R \\ L & 0.64 & 0.062222 & 0.297778 \\ N & 0 & 0 & 1 \\ G & 0.262172 & 0.06367 & 0.674157 \end{matrix}$$

Similarly, as in section 6.5, the stationary behavior of the share prices of HDFC Bank are obtained after multiplying 19 number of times as given in the following matrix.

$$B^{19} = \begin{matrix} & F & S & R \\ L & 0.396386 & 0.05931947 & 0.5442946 \\ N & 0.396386 & 0.05931947 & 0.5442946 \\ G & 0.396386 & 0.05931947 & 0.5442946 \end{matrix}$$

The steady-state is obtained at B^{19} and it represents that after the 19th trading days since 494 trading days, OPM tends to the state of equilibrium. The OPM remains unchanged after then, for the onwards consecutive trading days. The above stationary distribution reveals the following information.

1. There are 40% chances that closing share prices of HDFC bank will fall (F) in the near future and in the long run irrespective of its initial states whether there is L, N or G in the price of Sensex.
2. There are 5% chances that share prices of HDFC bank will remain in the same state (S) in the future irrespective of its initial states.

3. There are 55% probability that the closing share prices of HDFC bank will be in rise (R) in future no matter whether there is the loss, no change or gain in the share value of Sensex.

The above information clearly reveals that rising (R) in the share prices of HDFC bank in future has maximum probability. It can hence be recommended that investing in HDFC Bank for the long run could be the best choice for the investors and share value of HDFC Bank is on rising from the 19th day onwards and, it may be suggested for selling of HDFC Bank share after the 19th day.

7. FINDINGS AND CONCLUSION

Existing literature shows that HMMs are applied in order to forecast the stock market trend, and the calculation is done up to the HMM parameters (A, B, π). The present work is based on HMM along with the development of probability distribution obtained in section-5. The expression for all the statistical parameters is obtained in section 5.1 and the MGF, PGF and C.F are also obtained in section 5.2 for the derived probability distribution.

Observing the data on the share values of HDFC Bank and the closing price of Sensex for two finance years from April 2017- Mar 2019, we have around 495 observations which revealed the significant correlation (95%) between HDFC Bank share values and Sensex prices. The coefficient of variation for HDFC bank share values and Sensex closing prices are 9.724 and 7.645, respectively presented in table 1. Results of TPM is disclosed in section 6.1 regarding the transits of Sensex closing prices. The OPM regarding the visible states of HDFC bank share values is disclosed in section 6.2. The IPV presented in section 6.3 reveals that the state of loss has 54.05% likelihood and the state of gain percentage is 45.55% observed with Sensex prices. The probabilities of visible sequences with one value disclosed in table 4 section 6.4 reveal that the chance for rise and fall in the share prices of HDFC are 0.47, and 0.46 respectively. As per the results in table-5 in section 6.4, It is observed that there is the maximum likelihood of rising the share prices of HDFC bank in consecutive two days.

Stationary probabilities, presented in section 6.5, reveals that transition probabilities become stationary with order 8 (8th step transition). Steady state TPM of Sensex concludes that there is 48% likelihood of losing and 51% of gaining the prices of Sensex in future irrespective of its initial state. The steady-state for OPM is obtained at B^{19} in section 6.6. This stationary distribution of OPM for the share prices of HDFC bank reveals that there are 40% chances that share values of HDFC Bank will fall and 55% probability that share values of HDFC bank will rise in the near future irrespective of its initial state. It may be interpreted that share value of HDFC bank is on rising from the 19th day onwards and it may be suggested that investing in HDFC bank in the long run could be the best choice for investors.

The results in section-6.4.1 reveal that, on average, the HDFC bank share value falls having mean time 0.57889 with variance 0.714343, which indicates that the share value is always in the raising state. As per the values of $\mu_3 = -0.14983$, $\beta_1 = 0.061587$, which implies that there exists negative skewness on the price value distribution. Hence, we can infer that the average rising value is always less than the model raising values. The coefficient of β_2 and γ_2 reveals that the consistency or peakedness of the distribution curve is just below the normal.

COMPETING INTERESTS DISCLAIMER:

Authors have declared that they have no known competing financial interests OR non-financial interests OR personal relationships that could have appeared to influence the work reported in this paper.

REFERENCES

[1] Yu J and Meyer R. Multivariate Stochastic Volatility Models: Bayesian Estimation and Model Comparison. Research Collection School of Economics. 2006; 25(2-3): 361-384.

[2] A Kuo R J, Lee L C and Lee C F. Integration of Artificial NN and Fuzzy Delphi for Stock market forecasting. IEEE International Conference on Systems, Man, and Cybernetics. 1996; 2: 1073-1078.

[3] Umar Ms and Musa Tm. Stock prices and firm earning per share in Nigeria. JORIND. 2013; 11(2):187-192.

[4] Kimoto T, Asakawa K, Yoda M and Takeoka M. Stock market prediction system with modular neural networks. Proc. International Joint Conference on Neural Networks, San Diego. 1990; 1: 1-6.

[5] Rabiner LR. A tutorial on hidden Markov models and selected applications in speech recognition. Proc IEEE. 1989; 77(2): 257–286.

[6] Landeta, J. M. I, Cortés, C. B. Y and Azúa, H. M. Markovian decision process to find optimal policies in the management of an orange farm. Investigación Operacional. 2014; 35(1): 68-77.

[7] Alghamdi, R. Hidden Markov models (HMMs) and security applications. International Journal of Advanced Computer Science and Applications. 2016; 7(2):39-47.

[8] Hassan, M. R., and Nath, B. Stock market forecasting using hidden Markov model: a new approach. In 5th International Conference on Intelligent Systems Design and Applications IEEE, (ISDA'05). 2005 SEPTEMBER; 192-196.

[9] Guidolin, Massimo and Allan Timmermann. The architecture of complex weighted networks. SSRN FRB of St. Louis Working Paper No. 2005-002C, FRB of St. Louis, MO, USA; 2006.

[10] Kritzman, M, Page S and Turkington D. Regime shifts: Implications for dynamic strategies (corrected). Financial Analysts Journal. 2012; 68(3): 22-39.

[11] Gupta, A, and Dhingra B. Stock market prediction using hidden Markov models. In 2012 Students Conference on Engineering and Systems IEEE. 2012 (March); 1-4.

[12] Nobakht, B, C. E. Joseph, and B. Loni. Stock market analysis and prediction using hidden Markov models. Paper presented at the 2012 Students Conference on IEEE Engineering and Systems (SCES), Allahabad, Uttar Pradesh, India, March 16–18, 2012; 1–4.

[13] Tuyen, L. T. Markov financial model using hidden Markov model. International Journal of Applied Mathematics and Statistics. 2013; 40(10): 72-83.

[14] Nguyen, Nguyet Thi. Probabilistic Methods in Estimation and Prediction of Financial Models. (Doctoral dissertation, The Florida State University; 2014.

[15] Nguyen, N and Nguyen, D. Hidden Markov Model for Stock Selection. Risks. 2016; 3:455–473.

[16] Holzmann, H and Schwaiger, F. Testing for the number of states in hidden Markov models. Computational Statistics & Data Analysis. 2016; 100:318-330.

[17] Liu Z and Wang S. Decoding Chinese stock market returns: Three-state hidden semi-Markov model. Pacific-Basin Finance Journal. 2017; 44: 127-149.

[18] Nguyen, N. Hidden Markov model for stock trading. International Journal of Financial Studies. 2018; 6(2): 36.

[19] Huang, M, Huang Y and HE, K. Estimation and testing non-homogeneity of Hidden Markov model with application in financial time series. Statistics and Its Interface. 2019; 12(2): 215-225.

[20] Suda, D and Spiteri, L. Analysis and Comparison of Bitcoin and S and P 500 Market Features Using HMMs and HSMMs. Information. 2019; 10(10): 322.

[21] Liu, D. Markov modulated jump-diffusions for currency options when regime switching risk is priced. International Journal of Financial Engineering. 2019; 6(04): 1950038.

[22] Baum, L.E. and Egon, J.A. An inequality with applications to statistical estimation for probabilistic functions of Markov process and to a model for ecology. Bulletin of the American Mathematical Society. 1967; 73(3): 360-363.

[23] Baum L.E. and Sell, G.R. Growth functions for transformations on manifolds. Pac. J. Math. 1968; 27(2): 211-227.

[24] Forney, G.D. The Viterbi algorithm. IEEE. 1973 61: 268–278.

[25] Viterbi, A.J. Error bounds for convolutional codes and an asymptotically optimal decoding algorithm. IEEE transactions on Information Theory. 1967; 13(2): 260-269.