Original Research Article

AN INVENTORY MODEL FOR DETERIORATING ITEMS WITH THREE PARAMETER WEIBULL DISTRIBUTION UNDER INFLATION

ABSTRACT

This study deals with a deterministic inventory model for deteriorating items with Weibull distribution, considering quadratic demand and trade credit under inflation in which shortages were allowed and partially backlogged. The objective of this study is to minimize the total relevant cost by finding the optimal order quantity and optimal replenishment cycle. The model is developed under two different circumstances; **case 1:** $M < t_2$, that is, if the customer does not pay the supplier by time M, he will earn interest for the outstanding balance and **case 2:** $M \geq t_1$ that is the customer will be able to sell all his stock and earn interest on the sales proceeds until he has settle the account. Numerical examples and sensitivity analysis are given to illustrate the application and the performance of the model. Therefore, this model will be helpful to the retailer to find the optimal replenishment cycle under various situations and provides a new managerial insight that will help the industry to reduce total relevant cost.

Keywords: EOQ, Quadratic Demand, Weibull Distribution, Shortages, Trade Credit, Inflation.

Introduction

Inventory modeling is an important part of operation research which is used in solving variety of warehousing and storing problems. The primary purpose of the inventory modeling is to develop policies that will achieve an optimal inventory investment. It plays a significant role in production and operations function of supply chain management in order to make it applicable and flexible in real life situation and also in the control of inventories of deteriorating items since the introduction of the models on Economic Order Quantity.

The mathematical modeling on inventory control began with the work of Harris (1913), who studied the Economic Order quantity (EOQ) model to determine the optimal order quantity that minimizes the total cost.). One of the basic assumptions of the EOQ model is the infinite life of products, i.e. the value of product does not change. In many inventory systems, deterioration of goods in the form of direct spoilage or gradual physical decay in course of items is a realistic phenomenon and hence it should be considered in inventory modeling. Wee (1995) defined deteriorating item as the item that become decayed, damage, evaporative, expired, invalid, devaluated and so on through time. Inventory problems for deteriorating items have been studied extensively by many researchers. Research in this area of study started first with the work of Whitin (1953), he studied fashion item deteriorating at the end of the storage period. Ghare and Schrader (1963) established an EOQ with a constant rate of deterioration. However, the rate of deterioration increases with time for a few items such as, fruits, vegetables, etc. In addition it has been empirically observed that failure and life expectancy of many items can be expressed in terms of Weibull distribution. Berrotoni (1962) as a matter of fact, in discussing the problem of fitting empirical data to mathematical distributions, found that both leakage failure of dry batteries and life expectancy of ethical drugs could be expressed in terms of Weibull distributions. When used in the context of economic order quantities, Weibull distribution will provide a probability density function that gives the time to deterioration. Covert and Philip (1973) extended the work of Ghare and Schrader (1963) by considering the assumption of constant deterioration rate to represent the distribution of time to deterioration by using a two – parameter Weibull distribution and without shortage. However, in real life situation, the two parameter Weibull distribution deterioration may not be of use because some items start deteriorating after a certain period while storing, but not at the early stage. Rinne (2009) suggested that a three-parameter for some items that do not start to deteriorate in the immediately but only after a certain period of time called life of the item, which may vary from item to item. Philip (1974) generalized the model in Covert and Philip (1973) by considering three parameter Weibull deterioration, no shortages and a constant demand.

There are various inventory items that are not storable for a long time because of their life, e.g. milk, meat, vegetables, etc. Also there are some items that are subjected to constant loss due to their chemical properties or the other inherent conditions, i.e. radioactive materials, volatile liquids etc. As a result of this, the assumption of a constant demand rate may not always be appropriate for many inventory items as the age of inventory has a negative impact on demand due to the loss of consumer confidence on the quality of such products and physical loss of the material. Wee (1995) presented a deteriorating inventory where demand decreases exponentially with time. Jalan et al (1996) reconsider the model of covert and Philip (1973) and extend it to include a time-dependent demand rate and shortages in inventory. However, most of the above model mainly based on time-varying demands like linear and exponential demand. Considering quadratic demand as the next realistic approach, Ghosh and Chaudhuri (2006) considered an economic order quantity (EOQ) model over a finite time-horizon for a deteriorating item with a quadratic, time-dependent demand, allowing shortages in inventory. Amutha and Chandrasekaran (2013) considered an EOQ model for deteriorating items with quadratic demand and time dependent holding cost. Mishra (2015) developed an inventory model for an item with two parameter Weibull distribution and quadratic demand where holding cost was considered as a linear function of time. Smaila and Chukwu (2016) developed an EOQ model with three parameter Weibull quadratic demand and shortages.

Besides demand and deterioration rate, other components like allowing shortages are essential for modeling of inventory. Shortages usually take place in two cases, when the shortage items are totally backlogged and the other case when items are partially backlogged. Different types of inventory models with completely backlogging were discussed by; Amutha and Chandrasekaran (2013) considered an inventory model for deteriorating items with three parameter Weibull and price dependent demand. Chaudhary and Sharma (2013) considered an inventory model for deteriorating items with Weibull distribution and time dependent demand. Rai and Sharma (2017) presents an inventory model for deteriorating items with non-linear price dependent demand rate and varying holding cost. But, in real life situation, during the shortage period, the willingness of a customer to wait for items declines with the length of the waiting time. Chang and Dye (1999) investigated EOQ model for deteriorating items with time varying and partial backlogging. . Sana (2010) considered an optimal selling price and lot size with time varying deterioration and partial backlogging.) Roy et al. considered an Economic Order Quantity Model of Imperfect Quality Items with Partial Backlogging.

In classical inventory economical order quantity (EOQ) models, most researchers assume that buyer must pay for the items purchased as soon as the items are received. However, the most prevailing practice is that the supplier may offer a credit period to the buyer to settle his account within the specific settlement period. From a financial standpoint, an inventory represents a capital investment and must compete with other assets for a firm’s limited capital funds. The effects of inflation are not usually considered when an inventory is analyzed because most people think that the inflation would not influence inventory policy to any significant degree.

Most researchers have given valuable results related to the inventory model for deteriorating item with trade credit under inflation without considering shortages, which would make it more applicable in real world. Considering research in this direction; Buzacott (1975) developed an EOQ model with inflation subject to different types of pricing policies. Misra (1979) developed a discount cost model in which the effects of both inflation and time value of money are considered. Chandra and Bahner (1985) developed models to investigate the effects of inflation and time of value of money on optimal order policies. Goyal (1985) developed economic order quantity which the supplier permits a fixed delay in settling the amount owed to him. Aggarwal and Jaggi (1995) developed a model to determine the optimum order quantity for deteriorating items under a permissible delay in payment. Chang et al. (2001) considered an inventory model with varying rate of deterioration and condition of permissible in payment in which the restrictive assumption of constant and take a linear trend in demand into consideration. Chang and Dye (2002) discussed inventory model for deteriorating items under permissible delay in payments, considering the phenomena of deterioration of physical goods and a vendor who may offer a fixed credit period to customers to settle the account. Singh and Panda (2015) studied an inventory model for generalized Weibull deteriorating items with price dependent demand and permissible delay in payment under inflation.

In this study, effort has been made to determine an inventory model for deteriorating items with three-parameter Weibull distribution, considering quadratic demand and trade credit under inflation in which shortages were allowed and partially backlogged.

2. Assumptions

The followings are the assumptions used in deriving the model.

- The demand rate is deterministic and quadratic function of time.

- The lead time is zero.
- The deterioration rate is three-parameter Weibull distribution.
- Replenishment is instantaneous and infinite.
- Inflation rate is constant.
- Shortages are allowed and partially backlogged.
- No repair or replacement of deteriorated items during cycle.
- During the time, the account is not settled as well as the buyer pays off all units sold and start paying for the interest charges on the items in stock.

3. Notations

The followings notations are taken in developing the model:

1. A : Ordering cost of inventory per order.
2. $R(t)$: The quadratic demand rate, i.e., $R(t) = a + bt + ct^2, a > 0, b \neq 0, c \neq 0$ where a, b and c are the initial demand rate , increasing demand rate and change demand rate respectively .
3. $Z(t) = \alpha\beta(t-\gamma)^{\beta-1}, 0 < \alpha < 1, \beta > 0 \,\&\, 0 < \gamma < 1$. Here $\alpha, \beta \,\&\, \gamma$ are called the scale parameter, the shape parameter and the location parameter respectively.
4. $B(t)$: The backlogging rate which was given as $B(t) = e^{-\delta(T-t)}, \delta > 0$ where δ is called the backlogging parameter.
5. δ : The constant backlogging parameter where $0 \leq \delta < 1$.
6. R : Inflation rate
7. M : Permissible delay in settling the account.
8. T : The fixed length of each ordering cycle:

9. $I_1(t)$: On-hand inventory at time t when $t \geq 0$.

10. C_p : The purchase cost per unit.

11. C_h : Total holding cost per cycle.

12. t_1 : The time when the inventory level reaches zero.

13. TRC : The total relevant cost per unit time.

14. Q: Ordering quantity per cycle.

4. Model formulation

The inventory I(t) at time t $(0 \leq t \leq T)$, is describe as shown in the figure 1.

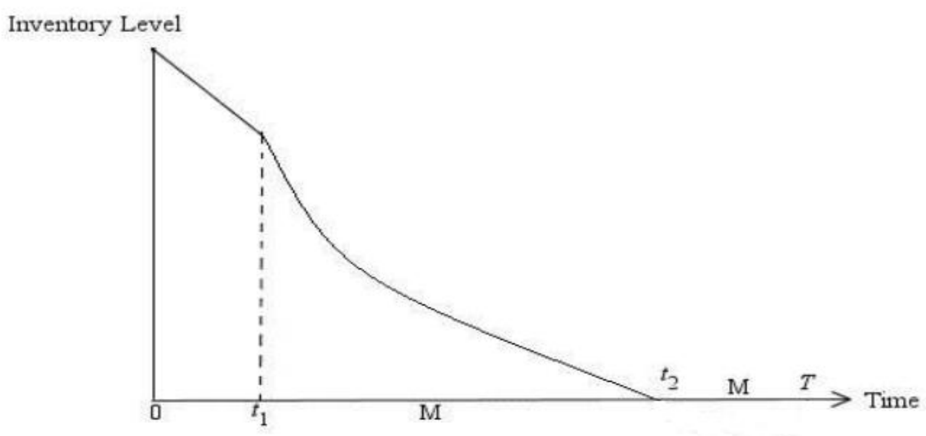

Figure 1: Inventory Pattern

The inventory system goes as follow: at $t = 0$, a lot size of certain units enter the system. In the interval $[0, t_1]$, the inventory level gradually decreases due to demand and it vanishes at time $t = t_1$. Then, shortages are allowed to occur during the interval $[t_1, T]$ and shortages decreases partly due to deterioration and all the demand during the shortage period $[t_1, T]$ is partially backlogged.

Thus, the differential equations which describes the instantaneous states of $I(t)$ at any time t over the period $[0,t_1]$ is given by

$$\frac{dI_1(t)}{dt} = -\left(a+bt+ct^2\right), 0 \le t \le t_1 \tag{1}$$

with the boundary condition $I_1(0) = Q_1$

The solution of equation (1) becomes

$$I_1(t) = Q_1 - (at + \frac{bt^2}{2} + \frac{ct^3}{3}) \ , \ 0 \le t \le t_1 \tag{2}$$

The differential equation that describes inventory level which decreases as a result of the effect of the demand and deterioration was given by

$$\frac{dI_2(t)}{dt} + Z(t)I_2(t) = -R(t), t_1 \le t \le t_2 \tag{3}$$

with boundary conditions at $I(t_2) = 0$ and $I(T) = -Q_2$

The solution of equation (3) becomes

$$I_2(t) = K - \alpha\left(a(t_2 - t) + \frac{b(t_2{}^2 - t^2)}{2} + \frac{c(t_2{}^3 - t^3)}{3}\right)(t-\gamma)^{\beta}, \tag{4}$$

considering the continuity of $I(t)$ at $t = t_1$, it follows from the equation (1) and (3)

$$I_1(t_1) = I_2(t_1)$$

$$\Rightarrow Q_1-(at_1+\frac{bt_1^{\,2}}{2}+\frac{ct_1^{\,3}}{3})= K-\alpha\left(a(t_2-t_1)+\frac{b(t_2^{\,2}-t_1^{\,2})}{2}+\frac{c(t_2^{\,3}-t_1^{\,3})}{3}\right)(t_1-\gamma)^{\beta} \qquad (5)$$

The maximum inventory level for each cycle is given by

$$Q_1=(at_1+\frac{bt_1^{\,2}}{2}+\frac{ct_1^{\,3}}{3})+K-\alpha\left(a(t_2-t_1)+\frac{b(t_2^{\,2}-t_1^{\,2})}{2}+\frac{c(t_2^{\,3}-t_1^{\,3})}{3}\right)(t_1-\gamma)^{\beta} \qquad (6)$$

Putting equation (6) into equation (2), we have

$$I_1(t)=\begin{bmatrix} (at_1+\frac{bt_1^{\,2}}{2}+\frac{ct_1^{\,3}}{3})+K \\ -\alpha\left(a(t_2-t_1)+\frac{b(t_2^{\,2}-t_1^{\,2})}{2}+\frac{c(t_2^{\,3}-t_1^{\,3})}{3}\right)(t_1-\gamma)^{\beta} \\ -(at+\frac{bt^2}{2}+\frac{ct^3}{3}) \end{bmatrix} 0\le t\le t_1 \qquad (7)$$

During the shortage interval $\left[t_1,T\right]$, the demand at time $'t'$ is partially backlogged at rate

$e^{-\delta(T-t)}$. The differential equation governing the amount of demand backlogged is given by

$$\frac{dI_3(t)}{dt}=-e^{-\delta(T-t)}\left(a+bt+ct^2\right), t_2\le t\le T \qquad (8)$$

with boundary conditions at $I(t_2)=0$ and $I(T)=-Q_2$

The solution of equation (8) becomes

$$I_3(t)=\begin{bmatrix} \left(a(t_2-t)+\frac{b(t_2^{\,2}-t^2)}{2}+\frac{c(t_2^{\,3}-t^3)}{3}\right)-a\delta\left(T(t_2-t)-\frac{(t_2^{\,2}-t^2)}{2}\right) \\ -b\delta\left(T\frac{(t_2^{\,2}-t^2)}{2}-\frac{(t_2^{\,3}-t^3)}{3}\right)-c\delta\left(T\frac{(t_2^{\,3}-t^3)}{3}-\frac{(t_2^{\,4}-t^4)}{4}\right) \end{bmatrix}, \; t_2\le t\le T \qquad (9)$$

Substituting $t = T$ in equation (9), we derive the maximum amount of demand backlogged per cycle as:

$$Q_2 = \left[\begin{array}{l} \left(a(t_2 - T) + \dfrac{b(t_2^{\,2} - T^2)}{2} + \dfrac{c(t_2^{\,3} - T^3)}{3} \right) - a\delta \left(T(t_2 - T) - \dfrac{(t_2^{\,2} - T^2)}{2} \right) \\ -b\delta \left(T\,\dfrac{(t_2^{\,2} - T^2)}{2} - \dfrac{(t_2^{\,3} - T^3)}{3} \right) - c\delta \left(T\,\dfrac{(t_2^{\,3} - T^3)}{3} - \dfrac{(t_2^{\,4} - T^4)}{4} \right) \end{array} \right]$$

$$= \left[\begin{array}{l} a(1 - \delta T)(t_2 - T) + \dfrac{1}{2}(b + a\delta - b\delta T)(t_2^{\,2} - T^2) \\ + \dfrac{1}{3}(c + b\delta - c\delta T)(t_2^{\,3} - T^3) + \dfrac{1}{4} c\delta (t_2^{\,4} - T^4) \end{array} \right] \qquad (10)$$

The total order quantity Q per cycle is obtained by adding (6) and (10)

$$Q = (at_1 + \frac{bt_1^{\,2}}{2} + \frac{ct_1^{\,3}}{3}) + K - \alpha \left(a(t_2 - t_1) + \frac{b(t_2^{\,2} - t_1^{\,2})}{2} + \frac{c(t_2^{\,3} - t_1^{\,3})}{3} \right)(t_1 - \gamma)^{\beta}$$

$$+ \left[\begin{array}{l} a(1 - \delta T)(t_2 - T) + \dfrac{1}{2}(b + a\delta - b\delta T)(t_2^{\,2} - T^2) \\ + \dfrac{1}{3}(c + b\delta - c\delta T)(t_2^{\,3} - T^3) + \dfrac{1}{4} c\delta (t_2^{\,4} - T^4) \end{array} \right] \qquad (11)$$

The associated cost of the inventory system includes the following;

- Ordering cost
- Holding cost
- Shortage cost
- Deterioration cost

We derive the associated cost as follows:

Ordering Cost $\left(OC\right) = A$ (12)

Holding cost $\left(HC\right)$:

$$HC = \int_0^{t_2} I(t)e^{-Rt}dt = \int_0^{t_1} I_1(t)e^{-Rt}dt + \int_{t_1}^{t_2} I_2(t)e^{-Rt}dt \quad \text{(See appendix)} \tag{13}$$

Shortages Cost:

$$SC = -c_2\int_{t_2}^{T} I(t)e^{-Rt}dt = -c_2\int_{t_2}^{T} I_3(t)e^{-Rt}dt$$

$$= -c_2\int_{t_2}^{T}\left[\left[\begin{array}{l}\left(a(t_2-t)+\dfrac{b(t_2{}^2-t^2)}{2}+\dfrac{c(t_2{}^3-t^3)}{3}\right)-a\delta\left(T(t_2-t)-\dfrac{(t_2{}^2-t^2)}{2}\right)\\ -b\delta\left(T\,\dfrac{(t_2{}^2-t^2)}{2}-\dfrac{(t_2{}^3-t^3)}{3}\right)-c\delta\left(T\,\dfrac{(t_2{}^3-t^3)}{3}-\dfrac{(t_2{}^4-t^4)}{4}\right)\end{array}\right]\right]e^{-Rt}dt$$

$$= \frac{(T-t_2)^2c_2}{40}\left[\begin{array}{l}\dfrac{-10}{3}Rc\delta t_2^4+\left(\dfrac{-8}{3}Rc\delta T+(-4Rb+8c)\delta-4Rc\right)t_2^3\\ +(3c\delta RT^2+((-4c-3Rb)\delta-8Rc)T+(10b-5Ra)\delta-5Rb+10c)t_2^2\\ +\left(\begin{array}{l}2c\delta RT^3+\left(\left(\dfrac{14}{3}Rb-\dfrac{8}{3}c\right)\delta-\dfrac{16}{3}Rc\right)T^2\\ +\left(\left(-\dfrac{20}{3}b-\dfrac{10}{3}Ra\right)\delta+\dfrac{20}{3}c-10Rb\right)T-\dfrac{20}{3}Ra+\dfrac{40}{3}b+\dfrac{40}{3}a\delta\end{array}\right)t_2\\ +c\delta RT^4+\left(\left(\dfrac{7}{3}Rb-\dfrac{4}{3}c\right)\delta-\dfrac{8}{3}Rc\right)T^3+\left(\left(\dfrac{25}{3}Ra-\dfrac{10}{3}b\right)\delta+\dfrac{10}{3}c-5Rb\right)T^2\\ +\left(-\dfrac{40}{3}a\delta+\dfrac{20}{3}b-\dfrac{40}{3}Ra\right)T+20a\end{array}\right] \tag{14}$$

Deterioration Cost:

$$DC = c_p\left[Q_1 - \int_0^{t_2}(a+bt+ct^2)e^{-Rt}dt\right] = c_p\left[Q_1 - \int_0^{t_1}(a+bt+ct^2)e^{-Rt}dt - \int_{t_1}^{t_2}(a+bt+ct^2)e^{-Rt}dt\right]$$

$$= c_p\begin{bmatrix} a\left((1-\alpha t_1^{\beta})t_2 + \alpha t_1^{\beta+1}\right) + \frac{1}{2}b\left((1-\alpha t_1^{\beta})t_2^2 + \alpha t_1^{\beta+2}\right) \\ \frac{1}{3}c\left((1-\alpha t_1^{\beta})t_2^3 + \alpha t_1^{\beta+3}\right) + \frac{a\alpha(t_2^{\beta+1} - t_1^{\beta+1})}{\beta+1} + \frac{b\alpha(t_2^{\beta+2} - t_1^{\beta+2})}{\beta+2} \\ + \frac{c\alpha(t_2^{\beta+3} - t_1^{\beta+3})}{\beta+3} + \frac{1}{4}cRt_1^4 - \frac{1}{3}(-bR+c)t_1^3 - \frac{1}{2}(-aR+b)t_1^2 \\ -at_1 + \frac{1}{4}cR(t_2^4 - t_1^4) - \frac{1}{3}(-bR+c)(t_2^3 - t_1^3) \\ -\frac{1}{2}(-aR+b)(t_2^2 - t_1^2) - a(t_2 - t_1) \end{bmatrix} \quad (15)$$

In this study, we considered two cases; interest paid and interest earned. Case I: $(0 \le M \le t_2)$ and Case II: $(0 \le t_2 \le M)$.

Case I: $(0 \le M \le t_2)$: The retailers earns interest from revenue generated from sales up to M. interest is earned during the period M to t_2. The interest earned was obtained as;

Interest earned per cycle:

$$IE_1 = pI_e \int_0^M t\,\mathrm{e}^{-Rt}\left(a+bt+ct^2\right)dt$$

$$= pI_e\left[\frac{-1}{5}cRM^5 + \frac{1}{4}(-bR+c)M^4 + \frac{1}{3}(-aR+b)M^3 + \frac{1}{2}aM^2\right] \quad (16)$$

Interest payable per cycle after the due period M:

$$IP_1 = cI_p \int_M^{t_2} I(t)\,\mathrm{e}^{-Rt}\,dt \quad \text{(See appendix)} \quad (17)$$

The total cost per unit during a cycle, $C_1(t_2,T)$ consist of the following:

$$C_1(t_2,T)=\frac{1}{T}\left[OC+HC+DC+SC+IP_1-IE_1\right] \tag{18}$$

Substituting equation (13 - 17) into equation (18) the total cost for case I was obtained.

Differentiating equation (18) with respect to T and t_1 and equate to zero:

$$\frac{\partial C_1(t_2,T)}{\partial T}=0,\ \ \frac{\partial C_1(t_2,T)}{\partial t_1}=0 \tag{19}$$

By solving equation (19) for T and t_2, the optimal cycle length was obtained as $T=T^*$ and $t_2=t_2^*$ provided it satisfy the equation

$$\frac{\partial^2 C_1(t_2,T)}{\partial T^2}>0,\quad \frac{\partial^2 C_1(t_2,T)}{\partial t_1^{\,2}}>0 \text{ and } \frac{\partial^2 C_1(t_2,T)}{\partial T^2}\left[\frac{\partial^2 C_1(t_2,T)}{\partial t_1^{\,2}}\right]-\left[\frac{\partial^2 C_1(t_2,T)}{\partial T\ \partial t_1}\right]>0 \tag{20}$$

Case II: $(0\le t_1\le M)$ interest is earned up to the permissible delay period and no interest is payable during this period.

Interest earned up to the permissible delay period:

$$IE_2\ =pI_e\left[\int_0^{t_1} t\,\mathrm{e}^{-Rt}\left(a+bt+ct^2\right)dt+\int_{t_1}^{t_2} t\,\mathrm{e}^{-Rt}\left(a+bt+ct^2\right)dt+\left(a+bt+ct^2\right)t_2(M-t_2)\right]$$

$$=pI_e\begin{bmatrix}\frac{-1}{6}cRt_1^{\,5}+\frac{1}{5}(-bR+c)t_1^{\,5}+\frac{1}{4}(-aR+b)t_1^{\,4}+\frac{1}{3}at_1^{\,3}\\ \frac{-1}{5}cR(t_2^{\,5}-t_1^{\,5})+\frac{1}{4}(-bR+c)(t_2^{\,4}-t_1^{\,4})+\frac{1}{3}(-aR+b)(t_2^{\,3}-t_1^{\,3})\\ \frac{1}{2}a(t_2^{\,2}-t_1^{\,2})+\left(a+bt+ct_2^2\right)t_2(M-t_2)\end{bmatrix} \tag{21}$$

The total cost per unit during a cycle, $C_2(t_2,T)$ consist of the following:

$$C_2(t_2,T) = \frac{1}{T}\left[OC + HC + DC + SC - IE_2\right] \tag{22}$$

Substituting the equations (13, 14, 15, and 21) into equation (22) the total cost for case II was obtained.

Differentiating equation (22) with respect to T and t_2 and equate it to zero:

$$\frac{\partial C_2(t_2,T)}{\partial T} = 0, \text{ and } \frac{\partial C_2(t_2,T)}{\partial t_2} = 0 \tag{23}$$

By solving equation (23) for T and t_1 , the optimal cycle length was obtained $T = T^*$ and $t_2 = t_2^*$

Provided it satisfy the equation

$$\frac{\partial^2 C_2(t_2,T)}{\partial T^2} > 0,\ \frac{\partial^2 C_2(t_2,T)}{\partial t_2^{\,2}} > 0 \text{ and } \frac{\partial^2 C_2(t_2,T)}{\partial T^2}\left[\frac{\partial^2 C_2(t_2,T)}{\partial t_2^{\,2}}\right] - \left[\frac{\partial^2 C_2(t_2,T)}{\partial T\ \partial t_2}\right] > 0 \tag{24}$$

5. Numerical Example

To illustrate the models numerically, examples are presented for the models developed in chapter three. Considering the inventory data taken from Raman and Veer (2013), with the following parameters: A = 100, c_p =25, p =40, I_p =0.15, I_e =0.12, M =0.08 years, a =1000, b =0.05, c =0.01, C_2 = 8, C_3 = 2, R =0.01, backlogging parameter δ = 0.8, and t_1 = 0.05. In addition to the data we let scale parameter α = 0.04, shape parameter β = 2, location parameter γ = 0.5.

Case I:

For this case, the optimal value of t_2^* = 0.6052, T^* =1.0843, the optimal total cost $C_1(t_2,T)$ = 865.1502 and the optimum order quantity Q^* = 301.523 are obtained.

Case II:

For this case, the optimal value of t_2^* = 0.6843, T^* =1.0451, the optimal total cost $C_2(t_2,T)*$ = 624.541 and the optimum order quantity Q^* = 204.21are obtained.

6. Sensitivity Analysis

The sensitivity analysis is performed in respect of distinct associated parameters. This is done by varying one parameter and keeping the other parameters constant. The sensitivity analysis was performed only for two cases.

Table 1: Sensitivity Analysis for Case I - $M < t_2$

Parameters	%	t_2	T	Cost	Q
p	+50%	0.0532	1.049	844.5	280.6
	+20%	0.1253	1.0729	798.3	295.3
	-20%	0.1556	1.0924	793.6	295.4
	-50%	0.1753	1.1525	678.5	300.4
α	+50%	0.0456	1.0271	890.4	320.4
	+20%	0.1552	1.0294	856.3	302.5
	-20%	0.0765	1.0458	798.6	301.2
	-50%	0.0835	1.0734	769.2	300.7
β	+50%	0.1172	1.0353	672.2	245.3
	+20%	0.1178	1.0247	740.2	265.5
	-20%	0.1182	1.0287	798.5	268.2
	-50%	0.1192	1.0399	845.8	285.3
M	+50%	0.1263	1.1875	709.5	301.2
	+20%	0.1248	1.1801	742.5	315.5
	-20%	0.5130	1.1824	765.4	322.3
	-50%	0.5342	1.1860	780.2	331.7
R	+50%	0.1986	1.1370	824.1	336.5
	+20%	0.1820	1.1401	822.3	325.4
	-20%	0.2462	1.1440	800.3	320.1
	-50%	0.4562	1.4820	794.5	306.3

Table 2: Sensitivity Analysis for Case II- M ≥ t_1

Parameters	%	t_2	T	Cost	Q
p	+50%	0.5863	1.8854	709.2	277.3
	+20%	0.4583	1.1712	543.1	286.2
	-20%	0.4355	1.1799	519.4	328.5
	-50%	0.3501	1.5598	511.1	348.0
α	+50%	0.8346	1.1444	611.5.	384.21
	+20%	0.1409	1.3076	618.3	251.5
	-20%	0.5512	1.8333	678.4	299.7
	-50%	0.6065	1.6664	775.2	363.52
β	+50%	0.7067	1.7045	621.8	229.24
	+20%	0.6160	1.1046	770.7	252.15
	-20%	0.0125	1.4254	678.2	363.65
	-50%	0.2745	1.9635	688.4	316.24
M	+50%	0.0255	1.0481	500.8	333.8
	+20%	0.3596	1.9908	528.7	335.69
	-20%	0.7630	1.8240	587.8	348.95
	-50%	0.9197	1.4880	691.1	397.52
R	+50%	0.7033	1.0360	760.2.3	298.14
	+20%	0.8095	1.4061	771.4	307.56
	-20%	0.9253	1.9785	624.8	338.74
	-50%	0.2535	1.8235	569.2	357.62

Observations

From the results presented in tables 1 and 2, the followings are observed:

1. An increment in selling price (p) the overall system total cost increases and it is highly sensitive with respect to the selling price for both case 1 and case 2.

2. When there is an increment in demand parameter (α), total cost of the system also increases for both case 1 and case 2.

3. It was observed that as parameter (β) increase, value of total cost decreases for both cases.

4. With increase and decrease in the value of M, there is corresponding decrease and increase in total cost and quantity for both cases.

5. The increment in inflation parameter (R) results to a corresponding increase in the total cost for both cases.

7. **Conclusion**

In this study, an inventory model for deteriorating items with three-parameter Weibull distribution considering quadratic demand and trade credit under inflation was developed. Shortages were allowed and partially backlogged. The result of the sensitivity analysis indicate that the parameter p, α, β, M, & R are more sensitive toward changes in t_2, T, Q and total cost. However, the proposed model can be extended in several ways; we may extend the model by considering non-zero lead time and demand as stochastic. Furthermore, the parameters of the model can be considered as fuzzy variables.

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APPENDIX

The results below were obtained with the help of a software **Mathematica 11** version 11.3.0.0

Holding Cost:

1. $HC = \int_0^{t_2} I(t)e^{-Rt}dt = \int_0^{t_1} I_1(t)e^{-Rt}dt + \int_{t_1}^{t_2} I_2(t)e^{-Rt}dt$

$$
= \int_0^{t_1} \left[\begin{aligned}
&(at_1 + \frac{bt_1^{\,2}}{2} + \frac{ct_1^{\,3}}{3}) + a\left((t_2 - t) + \frac{\alpha[(t_2-\gamma)^{\beta+1} - (t-\gamma)^{\beta+1}]}{\beta+1} \right) \\
&+b\left(\frac{(t_2^{\,2} - t^2)}{2} + \alpha\left(\frac{t_2[(t_2-\gamma)^{\beta+1} - (t-\gamma)^{\beta+1}]}{\beta+1} - \frac{[(t_2-\gamma)^{\beta+2} - (t-\gamma)^{\beta+2}]}{(\beta+1)(\beta+2)} \right)\right) \\
&+c\left(\frac{(t_2^{\,3} - t^3)}{3} + \alpha\left(\frac{t_2^{\,2}[(t_2-\gamma)^{\beta+1} - (t-\gamma)^{\beta+1}]}{\beta+1} \right.\right. \\
&\left.\left. - \frac{2}{\beta+1}\left(\frac{t_2[(t_2-\gamma)^{\beta+2} - (t-\gamma)^{\beta+2}]}{(\beta+2)} - \frac{[(t_2-\gamma)^{\beta+3} - (t-\gamma)^{\beta+3}]}{(\beta+2)(\beta+3)} \right)\right)\right) \\
&-\alpha\left(a(t_2 - t_1) + \frac{b(t_2^{\,2} - t_1^{\,2})}{2} + \frac{c(t_2^{\,3} - t_1^{\,3})}{3} \right)(t_1-\gamma)^{\beta} \\
&-(at + \frac{bt^2}{2} + \frac{ct^3}{3})
\end{aligned} \right] e^{-Rt}dt
$$

$$
+ \int_{t_1}^{t_2} \left[\begin{aligned}
&a\left((t_2 - t) + \frac{\alpha[(t_2-\gamma)^{\beta+1} - (t-\gamma)^{\beta+1}]}{\beta+1} \right) \\
&+b\left(\frac{(t_2^{\,2} - t^2)}{2} + \alpha\left(\frac{t_2[(t_2-\gamma)^{\beta+1} - (t-\gamma)^{\beta+1}]}{\beta+1} - \frac{[(t_2-\gamma)^{\beta+2} - (t-\gamma)^{\beta+2}]}{(\beta+1)(\beta+2)} \right)\right) \\
&+c\left(\frac{(t_2^{\,3} - t^3)}{3} + \alpha\left(\frac{t_2^{\,2}[(t_2-\gamma)^{\beta+1} - (t-\gamma)^{\beta+1}]}{\beta+1} \right.\right. \\
&\left.\left. - \frac{2}{\beta+1}\left(\frac{t_2[(t_2-\gamma)^{\beta+2} - (t-\gamma)^{\beta+2}]}{(\beta+2)} - \frac{[(t_2-\gamma)^{\beta+3} - (t-\gamma)^{\beta+3}]}{(\beta+2)(\beta+3)} \right)\right)\right) \\
&-\alpha\left(a(t_2 - t) + \frac{b(t_2^{\,2} - t^2)}{2} + \frac{c(t_2^{\,3} - t^3)}{3} \right)(t-\gamma)^{\beta}
\end{aligned} \right] e^{-Rt}dt
$$

$$
\frac{\beta^{4}}{(\beta+1)(\beta+2)(\beta+3)(\beta+4)(\beta+5)(\beta+6)}
\left[\left[
\begin{array}{l}
(t_2-t_1)(\mathrm{t}_1-\gamma)(\mathrm{R}\,t_1-1)\left(\frac{1}{3}ct_1+(\frac{1}{3}ct_1+\frac{1}{2}b)t_2+\frac{1}{3}ct_1+\frac{1}{2}t_1b+a\right)\beta^{5} \\
+\left(
\begin{array}{l}
-\frac{19}{3}\left(\frac{18}{19}t_1^2\gamma+\gamma t_1-\frac{20}{19}\right)ct_2^3 \\
-\frac{19}{2}\left(\frac{18}{19}t_1^2\gamma+\gamma t_1-\frac{20}{19}\right)bt_2^2 \\
-\frac{19}{2}\left(\frac{18}{19}t_1^2\gamma+\gamma t_1-\frac{20}{19}\right)at_2 \\
+17\left(
\begin{array}{l}
\frac{4}{17}Rct_1^4+\left(\left(\frac{13}{34}c+\frac{7}{17}b\right)R-\frac{13}{51}c\right)t_1^3 \\
+\left(\left(\frac{15}{34}b+\frac{16}{17}a\right)R-\frac{14}{51}c-\frac{15}{34}b\right)t_1^2 \\
+\left(Ra-\frac{8}{17}b-a\right)
\end{array}
\right)t_1-\frac{18}{17}a
\end{array}
\right)t_1
\end{array}
\right]\beta^{4}\right]
$$

$$
+\frac{\beta^{3}}{(\beta+1)(\beta+2)(\beta+3)(\beta+4)(\beta+5)(\beta+6)}
\left[
+\left(
\begin{array}{l}
-\frac{137}{3}\left(\frac{121}{137}t_1^2\gamma+\gamma t_1-\frac{155}{137}\right)ct_2^3 \\
-\frac{137}{2}\left(\frac{121}{137}t_1^2\gamma+\gamma t_1-\frac{155}{137}\right)bt_2^2 \\
-137\left(\frac{121}{137}t_1^2\gamma+\gamma t_1-\frac{155}{137}\right)at_2 \\
+104\left(
\begin{array}{l}
\frac{49}{312}Rct_1^4+\left(\left(\frac{7}{39}c+\frac{67}{208}b\right)R-\frac{7}{39}c\right)t_1^3 \\
+\left(\left(\frac{77}{208}b+\frac{7}{8}a\right)R-\frac{5}{24}c-\frac{77}{208}b\right)t_1^2 \\
+\left(-a+Ra-\frac{89}{208}b\right)t_1-\frac{119}{104}a
\end{array}
\right)t_1
\end{array}
\right)\right]
$$

$$
+\frac{\beta^{2}}{(\beta+1)(\beta+2)(\beta+3)(\beta+4)(\beta+5)(\beta+6)}
\left[+\left(\begin{array}{l}
-\frac{461}{3}\left(\frac{372}{461}t_1^2\gamma+\gamma t_1-\frac{580}{461}\right)ct_2^3\\
-\frac{461}{2}\left(\frac{372}{461}t_1^2\gamma+\gamma t_1-\frac{580}{461}\right)bt_2^2\\
-461\left(\frac{372}{461}t_1^2\gamma+\gamma t_1-\frac{580}{461}\right)at_2\\
+268\left(\begin{array}{l}
\frac{13}{134}Rct_1^4+\left(\left(\frac{63}{268}c+\frac{23}{201}b\right)R-\frac{23}{201}c\right)t_1^3\\
+\left(\left(\frac{153}{536}b+\frac{54}{67}a\right)R-\frac{28}{201}c-\frac{153}{536}b\right)t_1^2\\
+\left(-\frac{97}{268}b-a+Ra\right)t_1-\frac{171}{134}a
\end{array}\right)t_1
\end{array}\right)\right]
$$

$$
+\frac{\alpha t_1^{\beta+2}}{(\beta+1)(\beta+2)(\beta+3)(\beta+4)(\beta+5)(\beta+6)}
\left[+\left(\begin{array}{l}
-234\left(\frac{254}{351}t_1^2\gamma+\gamma t_1-\frac{58}{39}\right)ct_2^3\\
-351\left(\frac{254}{351}t_1^2\gamma+\gamma t_1-\frac{58}{39}\right)bt_2^2\\
-702\left(\frac{254}{351}t_1^2\gamma+\gamma t_1-\frac{58}{39}\right)at_2\\
+240\left(\begin{array}{l}
\frac{1}{18}Rct_1^4+\left(\left(\frac{3}{20}b+\frac{1}{15}c\right)R-\frac{1}{15}c\right)t_1^3\\
+\left(\left(\frac{3}{16}b+\frac{3}{4}a\right)R-\frac{1}{12}c-\frac{3}{16}b\right)t_1^2\\
+\left(-a+Ra-\frac{1}{4}b\right)t_1-\frac{3}{2}a
\end{array}\right)t_1\\
-360\left(\frac{2}{3}t_1^2R+Rt_1\right)t_2\left(\frac{1}{2}bt_2+\frac{1}{3}ct_2^2+a\right)
\end{array}\right)\right]
$$

$$
+\frac{2\alpha t_2^{\beta+2}}{(\beta+1)(\beta+2)(\beta+3)(\beta+4)(\beta+5)(\beta+6)}
$$

$$
\left(t_2+\gamma\right)\left(bt_2+ct_2^2+a\right)\left(t_2R-1\right)\beta^4
$$

$$
\left[+\left(\begin{array}{l}
+\left(\begin{array}{l}
12t_2^4Rc+\left(\left(\frac{27}{2}c+\frac{27}{2}b\right)R-\frac{27}{2}c\right)t_2^3 \\
+\left(\left(15a+15b\right)R-15b-15c\right)t_2^2 \\
+\left(\frac{-33}{2}a-\frac{33}{2}b+\frac{33}{2}Ra\right)t_2-18a
\end{array}\right)\beta^3 \\
+\left(\begin{array}{l}
49t_2^4Rc+\left(\left(\frac{123}{2}b+\frac{123}{2}c\right)R-\frac{123}{2}c\right)t_2^3 \\
+\left(\left(77b+78a\right)R-77b-78c\right)t_2^2 \\
+\left(\frac{193}{2}Ra-\frac{193}{2}a+193b\right)t_2-119a
\end{array}\right)\beta^2 \\
+\left(\begin{array}{l}
78t_2^4Rc+\left(\left(109c+109b\right)R-109c\right)t_2^3 \\
+\left(\left(164a+153b\right)R-164c-153b\right)t_2^2 \\
+\left(-231Rb-231Ra-231a\right)t_2-342a
\end{array}\right)\beta \\
+180\left(\frac{1}{2}bt_2+\frac{1}{3}ct_2^2+a\right)\left(\frac{2}{3}t_2^2+t_2\right)R
\end{array}\right)\right]
$$

$$
+\frac{1}{2(\beta+1)(\beta+2)(\beta+3)}
$$

$$
\left[+\left(\begin{array}{l}
-(\beta+2)(\beta+3)\alpha t_1 a\left(\frac{2}{3}t_1^2\gamma+\gamma t_1\right)t_1^{\beta+2}-\frac{1}{2}(\beta+3)\,\mathrm{t}_1\,\alpha b\left(\frac{2}{3}t_1^2\gamma+\gamma t_1\right)\beta(\beta+1)t_1^{\beta+2} \\
-\frac{1}{3}(\beta+2)\alpha t_1 c\left(\frac{2}{3}t_1^2\gamma+\gamma t_1\right)\beta(\beta+1)t_1^{\beta+3}-(\beta+3)(\beta+2)\,\mathrm{t}_1\,\alpha a\left(\frac{2}{3}t_1^2\gamma+\gamma t_1\right)t_2^{\beta+1} \\
-(\beta+3)\,\mathrm{t}_1\,\alpha b\left(\frac{2}{3}t_1^2\gamma+\gamma t_1\right)(\beta+1)t_2^{\beta+2}-(\beta+2)\,\mathrm{t}_1\,\alpha c\left(\frac{2}{3}t_1^2\gamma+\gamma t_1\right)(\beta+1)t_2^{\beta+3}
\end{array}\right)\right]
$$

$$
+\frac{t^2}{2(\beta+1)(\beta+2)(\beta+3)}
\left[\left(\begin{array}{l}
\left(\frac{2}{3}t_2^2\gamma+\left(\frac{2}{3}t_2^2\gamma+\gamma t_1\right)t_2+\frac{2}{3}t_1^2\gamma++\gamma t_1\right)+ \\
\left(\left(bt_2+ct_2^2+a\right)\beta^2+\left(3ct_2^2+4bt_2+5a\right)\beta+2ct_2^2+3bt_2+6a\right)\left(t_1-t_2\right)\alpha\gamma t_2^{\beta} \\
+(\beta+3)(\beta+2) \\
+\left(\begin{array}{l}
\left(\frac{2}{3}t_1^2\gamma R+\gamma R t_1\right)\left(\frac{1}{2}bt_2+\frac{1}{3}ct_2^2+a\right)\alpha t_1^{\beta} \\
-\frac{1}{3}t_2\left(\begin{array}{l}
\frac{1}{3}t_2^4 Rc+\left(\left(\frac{2}{5}b+\frac{3}{5}c\right)R-\frac{3}{5c}\right)t_2^3 \\
+\left(\left(\frac{4}{3}b+\frac{1}{2}a\right)R-\frac{3}{2}c-\frac{3}{2}b\right)t_2^2 \\
+\left(2b+Ra-a\right)t_2-3a
\end{array}\right)\left(\beta+1\right)
\end{array}\right)
\end{array}\right)\right]
$$

2. $IP_1 = c\int_M^{t_2} I\left(t\right)e^{-Rt}dt$

$$
=c\int_M^{t_2}\left[\begin{array}{l}
a\left((t_2-t)+\frac{\alpha[(t_2-\gamma)^{\beta+1}-(t-\gamma)^{\beta+1}]}{\beta+1}\right) \\
+b\left(\frac{(t_2{}^2-t^2)}{2}+\alpha\left(\frac{t_2[(t_2-\gamma)^{\beta+1}-(t-\gamma)^{\beta+1}]}{\beta+1}-\frac{[(t_2-\gamma)^{\beta+2}-(t-\gamma)^{\beta+2}]}{(\beta+1)(\beta+2)}\right)\right) \\
+c\left(\frac{(t_2{}^3-t^3)}{3}+\alpha\left(\frac{t_2{}^2[(t_2-\gamma)^{\beta+1}-(t-\gamma)^{\beta+1}]}{\beta+1}\right.\right. \\
\left.\left.-\frac{2}{\beta+1}\left(\frac{t_2[(t_2-\gamma)^{\beta+2}-(t-\gamma)^{\beta+2}]}{(\beta+2)}-\frac{[(t_2-\gamma)^{\beta+3}-(t-\gamma)^{\beta+3}]}{(\beta+2)(\beta+3)}\right)\right)\right) \\
-\alpha\left(a(t_2-t)+\frac{b(t_2{}^2-t^2)}{2}+\frac{c(t_2{}^3-t^3)}{3}\right)(t-\gamma)^{\beta}
\end{array}\right]e^{-Rt}dt
$$

$$
=\frac{c_p I_p (M-t_2)\alpha M^{\gamma(\beta+1)}}{3(\beta+1)(\beta+2)(\beta+3)}
$$

$$
\left[\begin{array}{l}
(RM-1)(M-t_2)\left(ct_2^2+\left(Mc+\frac{3}{2}b\right)t_2+cM^2+\frac{3}{2}Mb3a\right)\beta^4 \\
+\left(\begin{array}{l}
(-13McR+14c)t_2^3+\left(-\frac{39}{2}MbR+21b\right)t_2^2 \\
+(-39MaR+42a)t_2+7M^4cR+\left(-8c+\frac{27}{2}bR\right)M^3 \\
+(-15b+33aR)M^2-36Ma
\end{array}\right)\beta^3 \\
+\left(\begin{array}{l}
(71c-59McR+14c)t_2^3+\left(\frac{213}{2}b-\frac{177}{2}MRb\right)t_2^2 \\
+(-177MaR+213a)t_2+14M^4cR+\left(\frac{69}{2}bR-17c\right)M^3 \\
+\left(114aR-\frac{87}{2}b\right)M^2-141Ma
\end{array}\right)\beta^2 \\
+\left(\begin{array}{l}
(-107McR+154c)t_2^3+\left(-\frac{321}{2}MRb+231b\right)t_2^2 \\
+(462a-321MaR)t_2+8M\left(M^3cR+\left(\frac{45}{16}bR-\frac{5}{4}c\right)M^2+\left(15aR-\frac{15}{4}b\right)M-\frac{45}{2}a\right) \\
-180(-2+RM)t_2\left(a+\frac{1}{2}bt_2+\frac{1}{3}ct_2^2\right)
\end{array}\right)\beta
\end{array}\right]
$$

$$
+\frac{2c_p I_p (M-t_2)\alpha t_2^{\gamma(\beta+1)}}{(\beta+1)(\beta+2)(\beta+3)}
$$

$$
\left[\begin{array}{l}
(bt_2+ct_2^2+a)(-1+Rt_2)\beta^3+\left(\frac{15}{2}ct_2^3R+(9bR-9c)t_2^2+\left(-\frac{21}{2}b+\frac{21}{2}Ra\right)t_2-12a\right)\beta^2 \\
+\left(\frac{33}{2}ct_2^3R+(-24c+23bR)t_2^2+\left(\frac{67}{2}aR-\frac{67}{2}b\right)t_2-47a\right)\beta+30(t_2R-2)\left(a+\frac{1}{2}bt_2+\frac{1}{3}ct_2^2\right)
\end{array}\right]
$$

$$+\frac{c_p I_p (M-t_2)}{15(\beta+1)(\beta+2)(\beta+3)}$$

$$\left[\begin{array}{l} -\frac{15}{2}\left(\left(bt_2+ct_2^2+a\right)\beta+\left(4bt_2+5a+3ct_2^2\right)\beta+3bt_2+6a+2ct_2^2\right)\alpha\gamma\left(t_2R+RM-2\right)t_2^{\beta+2} \\ +\left(\beta+1\right)\left(\beta+2\right)\left(\beta+3\right)(M-t_2) \\ \left(\frac{3}{2}ct_2^3R+\left(-\frac{15}{4}c+3McR\frac{15}{8}bR\right)t_2^2+2M^2cR+\left(-\frac{5}{2}c+\frac{15}{4}bR\right)M+\frac{5}{2}Ra-5b\right)t_2 \\ +M^3cR+\left(\frac{15}{8}bR-\frac{5}{4}c\right)M^2+\left(5aR-\frac{5}{2}b\right)M-\frac{15}{2}a \end{array}\right]$$