Research on Mathematical Intuitive Imagination Literacy in China

ABSTRACT

In the process of discovering, formulating, analyzing, and solving problems, intuitive imagination is always an essential part, which is the thinking basis for exploring and forming argumentative ideas, conducting mathematical reasoning, and constructing abstract structures. It plays a vital role in students' innovative consciousness and practical ability. Thus mathematical intuitive imagination literacy has become a focus in educational research. By its relevant research findings in China, the following conclusions can be drawn: (1) Previous research on mathematical intuitive imagination literacy mainly focused on five aspects: connotation, the academic profile of high school students, influencing factors, cultivation strategies, and evaluation methods. (2) Among them, connotation, the academic profile of high school students, and cultivation strategies of mathematical intuitive imagination literacy are the hot topics of current research. (3) In terms of research methods, scholars mainly used test questions, questionnaires, or interviews to investigate the academic profile of students, and most of them used theoretically critical thinking to analyze the influencing factors and cultivation strategies. (4) Previous research adopted a single research method and lacked empirical research; besides, the research on the influencing factors is not systematic and comprehensive enough. Meanwhile, the research conclusions on the cultivation strategies lack feasibility. Therefore, to find out more comprehensive influencing factors and more operable cultivation strategies, it is necessary to improve the research method further and conduct more in-depth and systematic research on the influencing factors of mathematical intuitive imagination literacy from an empirical perspective.

Keywords: High school students; Intuitive imagination literacy; Core literacy; Mathematics

1. INTRODUCTION

Intuitive imagination is the literacy of using geometric intuition and spatial imagination to perceive the forms and changes of things and using spatial forms, especially graphs, to understand and solve mathematical problems. The position relations and morphological changes of objects in our lives are inseparable from visual imagination, but also mathematical thinking needs to be developed by means of intuition and imagination (Zhu, 2020). For high school students, intuitive imagination can help make complex and abstract problems concise and visual, which is a favorable way to solve mathematical problems (Du, 2019). At the same time, intuitive imagination plays a vital role in promoting students' innovative spirit and practical ability, so the value of mathematics education cannot be underestimated. In 2018, the Ministry of Education of the People's Republic of China promulgated The Curriculum Standards for General

High School Mathematics (2017 Edition) (hereinafter referred to as the Curriculum Standards (2017 Edition)). It clearly stated that mathematical intuitive imagination literacy should be fully implemented at the high school level (Ministry of Education of the People's Republic of China, 2018). In recent years, scholars have paid attention to mathematical intuitive imagination literacy, but there is a lack of organization of relevant research results. Thus, this paper intends to review and organize the existing relevant literature and systematically analyze the current situation and shortcomings of the relevant research. This research will provide references for strategies to improve the mathematical intuitive imagination literacy of high school students and help researchers grasp the characteristics and current situation of current research to promote further research.

The research question of this paper is: What is the current research situation on mathematical intuitive imagination literacy? The following specific questions will be included: (1) What previous research has been done on mathematical intuitive imagination literacy? (2) Which aspects have been researched more by scholars? (3) What are the main research methods scholars have used to research mathematical intuitive imagination literacy? (4) What are the gaps in the current research on intuitive imagination?

2. LITERATURE SOURCES

2.1 Source of Materials

This paper adopts the literature analysis method, and all the literature comes from China National Knowledge Infrastructure (CNKI). China National Knowledge Infrastructure is the most authoritative literature search tool for national academic journals in China, containing all the contents of journals and dissertations in China. The selection of this database can ensure the persuasiveness and reliability of the research in this paper.

2.2 Data Collection

In the process of literature search, 139 articles were searched with the subject terms of "intuitive imagination literacy" and "high school students," and 1778 articles were searched with the subject term of "intuitive imagination literacy." Since this paper focuses on high school students' mathematical intuitive imagination literacy, 45 references were selected in consideration of the research questions and the number of citations in the literature.

3. RESULTS

Through the collation of all the literature, we found that scholars' research on mathematical intuitive imagination literacy mainly focused on five aspects:

connotation, the academic profile of high school students, influencing factors, cultivation strategies, and evaluation methods.

3.1 Connotation of Mathematical Intuitive Imagination Literacy

Based on the perspective of mathematical thinking, Wu et al. explained the connotation of intuitive imagination as the best prototype of life. They proposed perceiving the form and change of things through two ways: mathematical intuition and mathematical imagination. Mathematical image thinking should be used to produce new mathematical representations, which have gone through three stages: prototype intuition, representation intuition, and imagination intuition (Wu et al., 2018). Liu believed that intuitive imagination includes “intuition” and “imagination.” At the same time, mathematical intuitive imagination literacy is a long-formed ability, way, and habit of using intuitive imagination to observe, operate, think and express, which is a more stable character of thinking and value orientation integrated into body and mind (Liu, 2020). Shen and Wang believed that intuitive imagination is the fundamental way to know things. Unlike abstraction, intuitive imagination is simple, direct, easy to grasp and use, and it is a necessary foundation for further abstraction. In short, intuitive imagination is to look and think or reason (Shen and Wang, 2017). It is commonly recognized that intuitive imagination includes geometric intuition and spatial imagination. Sun believes that “intuitive imagination” as mathematical literacy is a juxtaposed and abbreviated phrase consisting of “geometric intuition” and “spatial imagination.” Geometric intuition is the perceptual understanding through geometric figures, and spatial imagination is the imagination of spatial forms. Spatial imagination provides the method for geometric intuition, and geometric intuition provides the cognitive basis for spatial imagination, constituting a nested circular interaction (Sun, 2017). Mi and Cui also divided intuitive imagination into geometric intuition and spatial imagination and divided spatial imagination into three levels: awareness of space, understanding imagination, and spatial imagination (Mi and Cui, 2018). Pan and Feng also hold the same view, and they believe that the carrier of intuitive imagination is “shape,” which includes not only plane and space geometric figures, but also diagrams, symbols, various function images, and their transformations, etc. (Pan and Feng, 2020). While Shen and Wang, from the perspective of PME, believe that intuitive imagination relies more on geometric intuition, including the intuition of plane and space geometry (Shen and Wang, 2018).

The Ministry of Education comprehensively generalized the above viewpoints and gave the most formal statement in the latest version of the Curriculum Standards (2017 Edition). The new curriculum standard points out that intuitive imagination refers to the literacy of using geometric intuition and spatial imagination to perceive the forms and changes of things and using spatial forms, especially graphs, to understand and solve mathematical problems. Mathematical intuitive imagination literacy mainly includes: recognizing the position relationship, morphological changes, and laws of motion of things with the help of spatial forms; using graphics to describe and analyze mathematical problems; establishing the connection between shapes and numbers, constructing intuitive models of mathematical problems, and exploring ways of solving problems. In addition, it expounds on the mathematical value and primary performance of the mathematical intuitive imagination literacy and requirements that students can reach through high school mathematics courses. Intuitive imagination is mainly manifested as establishing the connection between shapes and numbers, describing problems using geometric figures, understanding problems with the help of geometric intuition, and using spatial imagination to recognize things (Ministry of Education of the People's Republic of China, 2018).

3.2 High School Students' Academic Profile of Mathematical Intuitive Imagination Literacy

Before and after the promulgation of the Curriculum Standards (2017 Edition), scholars investigated the current situation of high school students' imagination literacy profile. In 2017, Weng found that the overall profile of mathematical intuitive imagination literacy of high school students was not high, and the overall profile was at Level 1. In addition, the number of students who could reach that level decreased as the level of mathematical intuitive imagination literacy increased level by level (Weng, 2017). Based on a "pyramidal" assessment framework constructed from three dimensions: content, structure, and process, Yin conducted a study on the assessment of high school students' mathematical intuitive imagination literacy and found that the level of geometric intuitive imagination was higher than the level of intuitive algebraic imagination (Yin, 2017).

This situation did not improve well after the promulgation of the new standard. In 2018, Chen found that the overall profile of high school students' mathematical intuitive imagination literacy was low by testing students, and students' scores on the three dimensions of spatial imagination, number-shape combination, and geometric intuition declined in order (Chen, 2018). In 2020, Zheng et al. also obtained similar findings to Chen. Meanwhile, they found significant differences in mathematical intuitive imagination literacy between boys and girls and varying levels of mathematical intuitive imagination literacy among students from different geographical regions (Zheng et al., 2020). In 2021, Li further obtained through her study that there are three primary deficiencies in the development of mathematical intuitive imagination literacy of high school students: they are unable to make precise identification of mathematical concepts related to mathematical intuitive imagination literacy; they are unable to show solid basic skills of graphing in describing practical problems with graphs; and they are unable to solve problems comprehensively with refined mathematical language according to problem characteristics (Li, 2021).

3.3 Influencing Factors of Mathematical Intuitive Imagination Literacy

Based on the importance of mathematical intuitive imagination literacy in high school students' mathematics learning, more and more experts and scholars are devoted to improving students' mathematical intuitive imagination literacy level. Thus the research on its influencing factors becomes essential. The research on the influencing factors of intuitive imagination by Chinese scholars mainly focused on two aspects: teachers and students.

3.3.1 Teachers

Du's study on the factors that influence students' intuitive imagination found that the following behaviors of teachers will have positive impacts on students: (1) training students to observe real-world objects, models, and pictures; (2) allowing students to do more hands-on work and make models of spatial figures; (3) guiding students to abstract the observed objects and to imagine the shapes of objects from spatial figures (Du, 2019). Lin studied the link between GeoGebra software and mathematical intuitive imagination literacy development. Then she concluded that the reasonable use of software that can demonstrate the generation and dynamic changes of mathematical elements in mathematics teaching is beneficial to developing students' mathematical intuitive imagination literacy (Lin, 2021). From interviews with secondary school teachers, Fan obtained that many factors have an impact on the development of students' mathematical intuitive imagination literacy, such as the teachers' cognitive degree of mathematical intuitive imagination literacy, teaching methods, instructional design, teaching style, and teaching process used in the teachers' mathematics classroom, etc. (Fan, 2021). Pei found that the teacher's level of lecture, patience, and criticism of students significantly influenced several aspects of students' mathematical intuitive imagination literacy; that is to say, teacher quality is an essential factor in the development of students' mathematical intuitive imagination literacy, especially the positive qualities (Pei, 2020). Yang found that teachers' professionalism significantly impacted students' intuitive ability. In solving conventional intuitive problems, the difficulty of the question, the presentation, and whether students use schematics affect students' intuitive ability (Yang, 2012).

3.3.2 Students

3.3.2.1 Background Factors

Zheng et al. used a self-administered mathematical intuitive imagination literacy test paper to measure students. He found significant differences between male and female students' levels of mathematical intuitive imagination literacy and varying levels of mathematical intuitive imagination literacy among students from different geographical regions (Zheng et al., 2020). Shen found differences in arts and sciences and age differences in the mathematical intuitive imagination literacy levels of high school students. He also found that science students' mathematical intuitive imagination literacy levels are higher than those of liberal arts students overall, and there is little difference in students' literacy levels of similar age (Shen, 2018). Zhu analyzed the differences in the background information of high school students and found that different regions, types of schools, grades, and genders are factors that influence core mathematical literacy (Zhu, 2018). Chen pointed out that students' life experiences in childhood would play an indispensable role in intuitive imagination, such as the Rubik's Cube that boys used to play with as children, and the spatial sense of such toys would facilitate students' initial development of intuitive imagination ability (Chen, 2018).

3.3.2.2 Subjective Factors

(1) Students' Learning Situation

According to Li, the direct experience formed by students' hands-on and intuitive perception in the classroom is the necessary foundation for developing spatial imagination ability, and it is also powerful support for feeling and understanding geometric intuition. Meanwhile, dynamic software application is a favorable factor for developing students' intuitive imagination ability. Students can develop intuitive imagination ability through the observation and understanding of explicit and intuitive visualized mathematical materials (Li, 2019). In his research, Yin suggested that the students' understanding degree of basic knowledge, awareness of intuitive imagination, understanding of the process of intuitive imagination, and mastery of the format and basic methods of intuitive imagination all have an impact on students' mathematical intuitive imagination literacy (Yin, 2017). Duan et al. found that the experimental mathematics learning method of "doing mathematics" could significantly improve students' intuitive imagination, especially in chapters related to geometry (Duan et al., 2021). Shen suggested that the frequency of using intuitive imagination to solve problems, the number of making physical models, and the teaching style that students prefer have a specific influence on students' mathematical intuitive imagination literacy (Shen, 2018).

(2) Students' Learning Ability

Li and Luo believed that the assessment of mathematical core literacy was mainly based on the process and results of problem-solving. Problem-solving cannot be separated from individuals' extraction and organization of information from problem situations, which means that mathematical reading cannot be eliminated, so the mathematical reading ability is an essential influencing factor in assessing mathematical core literacy (Li and Luo, 2019). Tao and Wei found that all dimensions of logical reasoning ability positively correlated with all dimensions of intuitive imagination ability. At the same time, they found that logical reasoning ability and intuitive imagination ability were significantly and positively correlated with academic performance in mathematics (Tao, 2021). Zhou believed that the image and intuitive characteristics of graphic language could help students recognize and understand problems. The interpenetration and transformation of mathematical language can enable students to unify questions and diagrams better organically and graphically the known conditions, so the ability to use mathematical language for expression and communication is essential for developing students' mathematical intuitive imagination literacy (Zhou, 2014). Liu et al. believed that students could understand and solve problems more clearly by constantly transforming them with the help of geometric models, which helped them learn to see and think about problems with geometric intuition. Hence, the ability to understand and analyze problems from a transformed perspective is an excellent contribution to the development of mathematical intuitive imagination literacy (Liu et al., 2020).

(3) Students' Study Habits

Many researchers investigated students' mathematical intuitive imagination literacy and found that whether students develop good mathematical learning habits can impact their learning outcomes (Zhu and Hu, 2020). Lin concretized the learning habits that affect mathematical intuitive imagination literacy. Then she obtained that students' habits of thinking about mathematical problems using a combination of numbers and shapes, different ways of responding when encountering problems, reflective awareness, habits of using software to study problems in geometry, and relying on teachers for mathematical learning all affect the development of mathematical intuitive imagination literacy of high school students (Lin, 2021). Wu found that for the intuitive imagination, students are more accustomed to solving problems with the help of geometric intuition, followed by the combination of numbers and shapes, and finally by drawing and analyzing. These processes are inseparable from drawing, and it is essential to have good graphing habits and standardized graphing in this process (Wu, 2021).

(4) Non-Intellectual Factors

Hou and Cui pointed out that thinking in terms of students, education, and teaching should promote the development of students' mathematical learning strategies, and the development of good motivation, self-efficacy, and affective attitudes can effectively contribute to the improvement of academic and subject competence in mathematics (Hou and Cui, 2019). After an empirical study of middle school students' mathematical intuitive literacy, Xu found that at the student level, intuitive experiences and beliefs, visual representation style, and problem-solving perseverance all significantly and positively predicted students' mathematical intuitive literacy performance. In contrast, mathematical anxiety showed a negative predictive effect on students' mathematical intuitive literacy, and the effects of problem-solving perseverance and mathematical anxiety were more significant (Xu, 2019). Zhu and Hu also found that high school students' sense of belonging to the school, subjective attitudes toward learning mathematics, self-belief, persistence in learning mathematics, attributions of success or failure in learning mathematics, and mathematics teacher-student relationships similarly influence students' level of mathematical core literacy development (Zhu and

Hu, 2020).

(5) Students' learning in mathematics outside of class

After analyzing the data, Dong found that extracurricular learning was significantly correlated with each of the core mathematical literacies, and the correlation with intuitive imagination was high. Among them, five categories of extracurricular learning had a significant effect on mathematics core literacy. They included students doing homework and completing learning tasks, pre-reading and reviewing, reading outside of class, the number of mathematics competitions participated in, and summarizing mistakes and knowledge points, in addition to other influencing factors such as summarizing mistakes and knowledge points and extracurricular tutoring (Dong, 2018).

3.4 Cultivation Strategies of Mathematical Intuitive Imagination Literacy

At the macro level, some scholars have made many suggestions on strategies for developing mathematical intuitive imagination literacy of high school students by integrating teachers, students, and textbooks. Zhang and Han suggested guiding students to observe spatial geometry, make models of spatial figures, accumulate representations of spatial figures, and develop students' spatial intuitive imagination literacy through hands-on practice and operational confirmation (Zhang and Han, 2017). Cheng gave four steps to cultivate students' mathematical intuitive imagination literacy based on the solution of quadratic equations: drawing operation, graphical understanding, practical experience, and numerical integration (Cheng, 2020). Chang summed up the triple realm of the penetration of mathematical intuitive imagination literacy in mathematics teaching: for teachers, they should incorporate their teaching thinking based on careful study of curriculum standards; for students, they should establish an intuitive model to obtain the improvement of students' thinking level through step-by-step teaching; for teaching: elementary mathematics should focus on the mastery of geometry, problem-solving should pay attention to the charm of the combination of numbers and shapes, and analogical learning should seek the breakthrough of thinking innovation (Chang, 2020). From the PME perspective, Shen and Wang proposed that in the teaching process, teachers should pay attention to the teaching of mathematics in the critical period, provide appropriate prior organizers, stimulate students' metacognitive monitoring and regulation, and design appropriate inquiry-based problems (Shen and Wang, 2017). After conducting a study on the level of students' intuitive imagination ability, Li suggested that the application of modern information technology should be emphasized in cultivating students' intuitive imagination ability (Li, 2019). Yu believes that teachers should focus on two levels of teaching: numerical integration and graphical sense, train students to use graphs to describe and analyze concrete mathematical problems, organically combine mathematical language and intuitive graphs, and enhance students' ability to turn abstraction into intuition (Yu, 2017).

At the micro level, scholars propose cultivation strategies through geometric figures, background materials, and other specific mediums. Jin suggested that teachers must carefully study the tangents of background materials and the relationship with mathematical models in their teaching, use background materials rationally, and guide students to discover mathematical laws and models to solve problems (Jin, 2016). Cai proposed to use images to teach functions, to study geometric spatial position relationships using rectangles or squares, and to understand the relationship between vectors and geometry (Cai, 2019). By analyzing the main aspects of three-view teaching, Gao pointed out that three-view learning can improve students' spatial thinking ability. It is also an effective path to improve students’ mathematical intuitive imagination literacy (Gao, 2015). Gao made two suggestions for cultivating students’ intuitive imagination: the first is to use graphs to describe mathematical problems and develop the “graphical language” ability; the second is to use graphs to understand mathematical problems and expand students' spatial imagination ability (Gao, 2017). Yu believed that intuitive information could be dug from algebra and that algebraic structural equations could be transformed into geometric problems by constructing graphs while simplifying operations (Yu, 2018).

3.5 Evaluation Methods of Mathematical Intuitive Imagination Literacy

The Curriculum Standards (2017 Edition) emphasized that the evaluation of intuitive imagination literacy should be based on the curriculum objectives and academic quality standards, paying attention to the stage, continuity, and holistic nature. Meanwhile, it should be centered on the primary performance of the four aspects of intuitive imagination (Ministry of Education of the People’s Republic of China, 2018). Regarding Bloom's learning evaluation model, PISA learning evaluation model, and SOLO learning evaluation model, Yu proposed the theoretical concept of classifying mathematical literacy assessment into three forms: knowledge understanding, knowledge transfer, and knowledge innovation (Yu, 2017). Zhu constructed a three-dimensional assessment framework of “subject content × literacy components × observation indicators” and assigned weight values to the dimensions of mathematical core literacy and observation indicators for high school students. Then he obtained the expressions between mathematical core literacy and the three assessment dimensions of mathematical knowledge, problem-solving, and mathematical thinking (Zhu, 2020). Based on identifying concepts and theories related to mathematical intuitive imagination literacy, Zheng et al. used the Delphi method to construct an assessment index system of mathematical intuitive imagination literacy consisting of three primary and nine secondary indicators. Then they used a combination of quantitative and qualitative research methods to construct a mathematical intuitive imagination literacy assessment model (Zheng et al., 2021).

4. DISCUSSION

As can be seen from the above research, scholars mainly focus on five aspects: connotation, the academic profile of high school students, influencing factors, cultivation strategies, and evaluation methods. Among them, connotation, the academic profile of high school students, and cultivation strategies of mathematical intuitive imagination literacy are the hot topics of current research, which is consistent with the conclusions obtained by Zhang and Pei's review and outlook on the research of mathematical intuitive imagination literacy of primary and secondary school students (Zhang and Pei, 2020). However, the research on the influencing factors and evaluation methods of mathematical intuitive imagination literacy is relatively rare.

Specifically, in the research of the connotation of mathematical intuitive imagination literacy, researchers primarily define mathematical intuitive imagination literacy in terms of the definition of literacy and the structural elements. At the same time, some researchers link it to thinking or ability, which blurs the two concepts of mathematical intuitive imagination literacy and intuitive imagination ability. Currently, most people are accustomed to defining them as "geometric intuition" and "spatial imagination," but scholars also have some differences in the interpretation of these two concepts.

The findings of scholars on the current level of high school students' mathematical intuitive imagination literacy are relatively consistent, which generally point to the conclusion that "the level of high school students' mathematical intuitive imagination literacy is not high." However, the research method of the relevant studies is single, mainly using test questions, questionnaires, or interviews. In their research, Zheng et al. have also pointed out that the tests on students do not entirely involve only the examination of mathematical intuitive imagination literacy (Zheng et al., 2020), and other influences will likely lead to some errors in the results.

In the research on the influencing factors of mathematical intuitive imagination literacy, scholars mainly focus on the two subjects of teachers and students. The analysis of the influencing factors is not comprehensive, systematic, and profound. In terms of research methods, scholars mostly use theoretical thinking to analyze the influencing factors, which leads to the intense subjectivity of research results. Therefore, in the future, it is necessary to improve the research method and conduct a more objective and comprehensive analysis of the factors influencing high school students' mathematical intuitive imagination literacy from an empirical perspective.

In addition, scholars have mainly proposed cultivation strategies based on their own experiences or with reference to existing research. Most of the suggestions focus on improving teachers' behaviors and are not specific enough, which leads to the poor feasibility of these suggestions. Zhang et al. also concluded that the cultivation strategies proposed by scholars were too theoretical and lacked specificity and operability (Zhang et al., 2020). Therefore, subsequent research can be conducted from more perspectives to provide more feasible suggestions for cultivating mathematical intuitive imagination literacy.

Finally, there are few studies on the evaluation methods of mathematical intuitive imagination literacy, and there is no unified and valid opinion. Therefore, there is a need to dig deeper into the evaluation methods.

5. CONCLUSION

By summarizing the above five aspects of research, the following conclusions can be drawn: (1) Previous research on mathematical intuitive imagination literacy mainly focused on connotation, the academic profile of high school students, influencing factors, cultivation strategies, and evaluation methods. (2) Among them, connotation, the academic profile of high school students, and cultivation strategies of mathematical intuitive imagination literacy are the hot topics of current research. (3) In terms of research methods, scholars mainly used test questions, questionnaires, or interviews to investigate the academic profile of students, and most of them used theoretically critical thinking to analyze the influencing factors and cultivation strategies. (4) Previous research adopted a single research method and lacked empirical research; besides, the previous research on the influencing factors is not systematic and comprehensive enough. Meanwhile, the research conclusions on the cultivation strategies lack feasibility.

Therefore, it is necessary to improve further the research methods on the influencing factors and cultivation strategies in future research. It is suggested to conduct more deep and systematic research on the influencing factors of mathematical intuitive imagination literacy from an empirical perspective to find more comprehensive influencing factors and more operational cultivation strategies.

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