Multiparametric rational solutions of order N to the KPI equation

Abstract

We present multiparametric rational solutions to the Kadomtsev-Petviashvili equation (KPI). These solutions of order N depend on $2N-2$ real parameters and can be expressed as a quotient of a polynomial of degree $2N(N+1)-2$ in x, y and t by a polynomial of degree $2N(N+1)$ in x, y and t, depending on $2N-2$ real parameters.
Explicit expressions of the solutions at order 3 are given. We study the patterns of their modulus in the (x,y) plane for different values of time t and parameters.

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1 Introduction

The Kadomtsev-Petviashvili equation (KPI) is a well-known nonlinear partial differential equation in two spatial and one temporal coordinates which can be written in the following form :

$$(4u_t - 6uu_x + u_{xxx})_x = 3u_{yy}, \tag{1}$$

with subscripts x, y and t denoting partial derivatives.
The KP equation first appeared in 1970, in a paper written by Kadomtsev and Petviashvili [11]. The discovery of the KP equation happened almost simultaneously with the development of the inverse scattering transform (IST) as it is explained in Manakov et al. [13]. In 1974 Dryuma showed how the KP equation could be written in Lax form [1], and Zakharov extended the IST to equations in

two spatial dimensions, including the KP equation, and obtained several exact solutions to the KP equation.
In 1981 Dubrovin constructed for the first time [3] the solutions to KPI given in terms of Riemann theta functions in the frame of algebraic geometry .
There is a lot of studies which deal with solutions to the KPI equation. We can cite in particular the works of Krichever [12], Satsuma [16], Matveev [2], Veselov [17], Freeman [4].

In this paper, we express rational solutions in terms of a quotient of a polynomial of degree $2N(N+1)-2$ in x, y and t by a polynomial of degree $2N(N+1)$ in x, y and t depending on $2N-2$ real parameters. This representation allows to obtain an infinite hierarchy of solutions to the KPI equation, depending on $2N-2$ real parameters .
That provides an effective method to construct an infinite hierarchy of rational solutions of order N depending on $2N-2$ real parameters. We present here only the rational solutions of order 3, depending on 4 real parameters, and the representations of their modulus in the plane of the coordinates (x,y) according to real parameters a_1, b_1, a_2, b_2 and time t.

2 Rational solutions of order N to the KPI equation depending on $2N-2$ real parameters

We consider the matrix M defined by :

$$m_{ij} = \sum_{k=0}^{i} c_{i-k} \left(\frac{\sqrt{p^2-4}}{3} \partial_p \right)^k \sum_{l=0}^{j} c_{j-l} \left(\frac{\sqrt{q^2-4}}{3} \partial_q \right)^l \times \left(\frac{1}{p+q} \exp\left(\frac{1}{2}(p+q)(-x+\frac{3}{4}t) - \frac{1}{4}(p^2-q^2)iy \right) \right)_{p=q=-1}. \quad (2)$$

The coefficients c_j are defined by :

$$c_{2j} = 0, \quad c_{2j+1} = a_j + ib_j \quad 1 \leq j \leq N-1, \quad (3)$$

where a_j and b_j are arbitrary real numbers.
Then we have the following result :

Theorem 2.1 *The function v defined by*

$$v(x,y,t) = -2\,\partial_x^2 \left(\ln \det (m_{2i-1,2j-1})_{1 \leq i,j \leq N} \right) \quad (4)$$

is a solution to the KPI equation (1), depending on $2N-2$ parameters a_k, b_k, $1 \leq k \leq N-1$.

Proof The ideas and arguments are the same as those set out in the article [19]. We do not reproduce them here and the reader will be able to reproduce the steps of the proof given in this paper.
□

3 Rational solutions of order 3 depending on 4 parameters

In the following, we explicitly construct rational solutions to the KPI equation of order 3 depending on 4 parameters.
Because of the length of the expression of the solution, we only give the expression without parameters and we present it in the appendix.

We give patterns of the modulus of the solutions in the plane (x, y) of coordinates in functions of parameters a_1, b_1, a_2, b_2 and time t.
In all the following figures, if the parameters are not quoted then there are equal to 0.

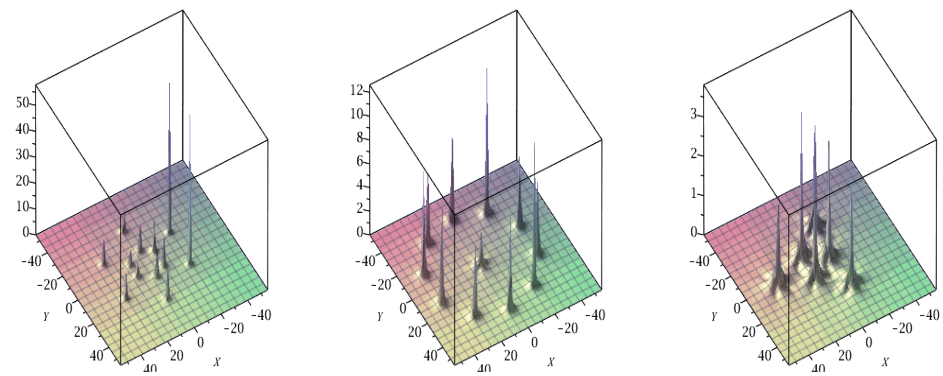

Figure 1. Solution of order 3 to KPI, on the left for $t = 0$, $a_1 = 10^4$; in the center for $t = 0$, $a_2 = 10^8$; on the right for $t = 0$, $b_1 = 10^4$.

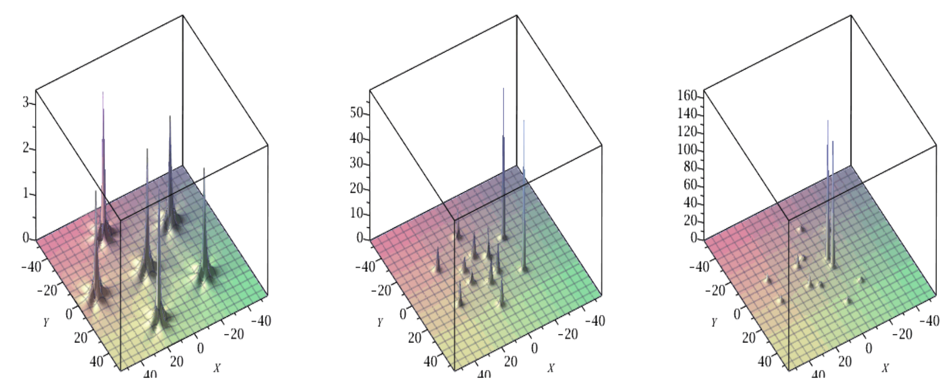

Figure 2. Solution of order 3 to KPI, on the left for $t = 0$, $b_2 = 10^8$; in the center for $t = 0$, $a_1 = 10^4$, $a_2 = 10^4$; on the right for $t = 0$, $a_1 = 10^4$, $a_2 = 10^4$, $b_1 = 10^4$, $b_2 = 10^8$.

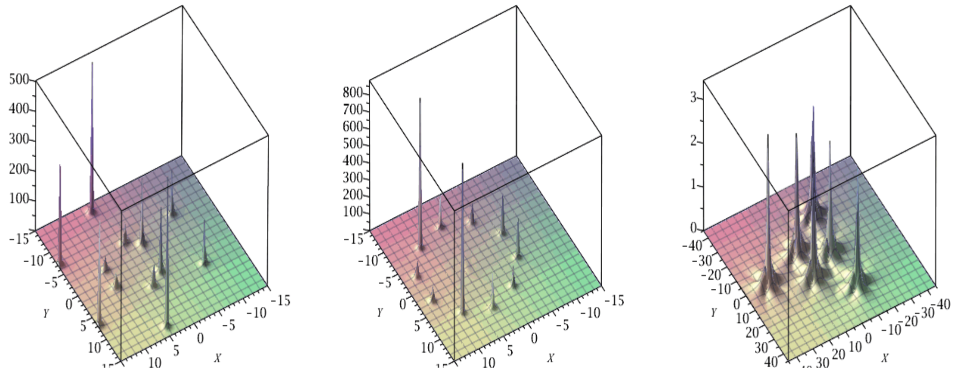

Figure 3. Solution of order 3 to KPI, on the left for $a_1 = 10^3$; in the center for $a_2 = 10^5$; on the right for $b_1 = 10^4$; here $t = 1$.

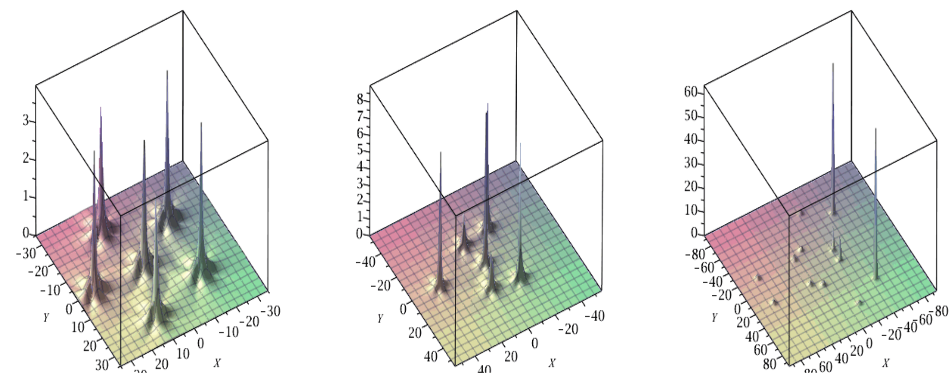

Figure 4. Solution of order 3 to KPI, on the left for $t = 1$, $b_2 = 10^7$; in the center for $t = 1$, $a_1 = 10^3$, $a_2 = 10^5$, $b_1 = 10^4$, $b_2 = 10^7$; on the right for $t = 1$, $a_1 = 10^5$, $a_2 = 10^5$, $b_1 = 10^5$, $b_2 = 10^5$.

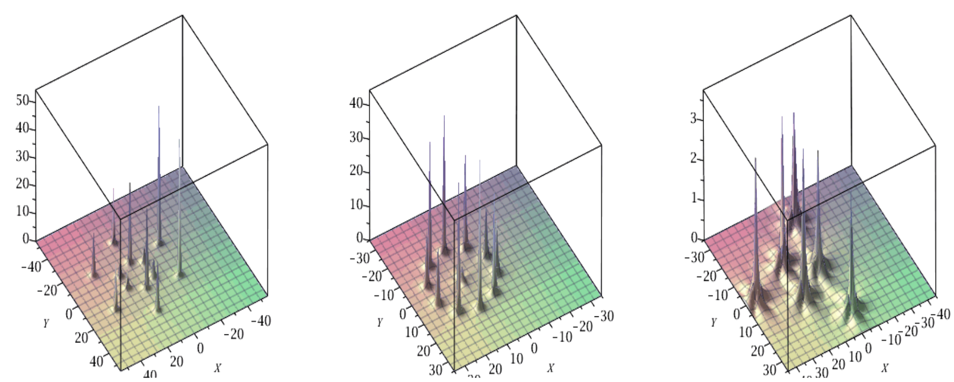

Figure 5. Solution of order 3 to KPI, on the left for $t = 10$, $a_1 = 10^4$; in the center for $t = 10$, $a_2 = 10^4$; on the right for $t = 10$, $b_1 = 10^4$.

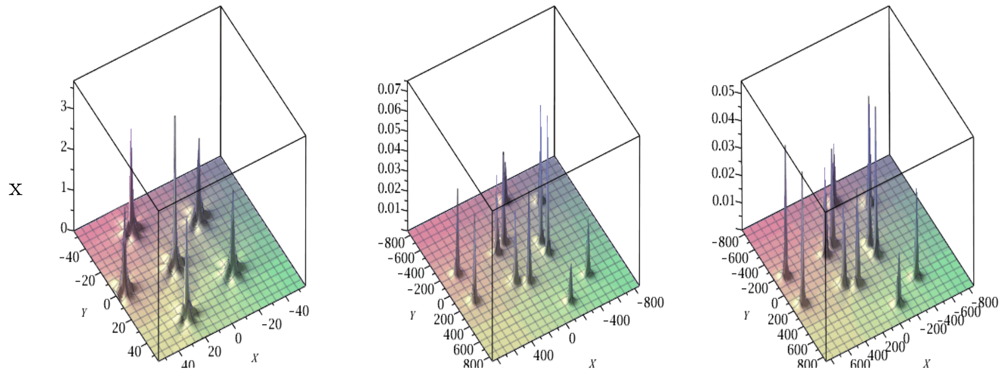

Figure 6. Solution of order 3 to KPI, on the left for $t = 10$, $b_1 = 10^8$; in the center for $t = 10$, $a_1 = 10^8$, $a_2 = 10^8$, $b_1 = 10^8$, $b_2 = 10^8$; on the right for $t = 10^2$, $a_1 = 10^8$, $a_2 = 10^8$, $b_1 = 10^8$, $b_2 = 10^8$.

The previous study shows the appearance of different types of configurations.
If $a_1 \neq 0$ and the other parameters equal to 0, we get 12 peaks on two concentric rings, 6 on the first and 6 on the second one.
For $a_2 \neq 0$ and the other parameters equal to 0, we get 10 peaks on a ring and a peak in the center of the ring.
If $b_1 \neq 0$ and the other parameters equal to 0, we get 6 peaks on a triangle.
For $b_2 \neq 0$ and the other parameters equal to 0, we get 5 peaks on a ring and one peak in the center of the ring.
In the case where two parameters a_1 and a_2 are not equal to 0, the other parameters being equal to 0, for the same values of parameters, we get 12 peaks on two concentric rings, which shows the predominance of the parameter a_1 over the parameter a_2 on the structure of the solutions.
In the case where two parameters b_1 and b_2 are not equal to 0, the other parameters being equal to 0, for the same values of parameters, we get 6 peaks on a ring with a peak in the center of the ring, which shows also the predominance of the parameter b_1 over the parameter b_2 on the structure of the solutions.
In the case where all parameters a_1, a_2, b_1, b_2, are not equal to 0, for the same values of parameters, we get 6 couples of 2 peaks on two rings.

4 Conclusion

In this article, rational solutions to the KPI equation have been built in terms of quotients of a polynomial of degree $2N(N+1)-2$ in x, y and t by a polynomial of degree $2N(N+1)$ in x, y and t, depending on $2N-2$ parameters.
Other approaches to build solutions of KPI equation had been realized, and we can mention those most significant. In 1990, Hirota and Ohta [9] built, the

solutions as particular case of a hierarchy of coupled bilinear equations given in terms of Pfaffians. In 1993, the Darboux transformations was used to obtain among others the solutions of the multicomponent KP hierarchy [15], but no explicit solutions were given. More recently, in 2013, wronskians identities of bilinear KP hierarchy were given [10]. In 2014, solutions of KPI equation were constructed [18]; an explicit solution at order 1 was built and only one asymptotic study has been carried out for order higher than 2.
In 2016, three types of representations of the solutions were given in [8], in terms of Fredholm determinants, wronskians and degenerate determinants.
The structures of the solutions given in this paper looks like to these given in [8] which were deduced from the solutions to the NLS equation [5, 6, 7].
But, to the best of my knowledge, some of the structures of these solutions had never been presented. It will be relevant to find explicit solutions for higher orders and try to describe the structure of these rational solutions.

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Appendix The general solution depending on 4 parameter being too large, we present only the solution without parameters. In can be written as :

$$v(x,y,t) = -2\frac{n(x,y,t)}{(d(x,y,t))^2}$$

with

$$n(x,y,t) = \sum_{i=0}^{2N(N+1)-2} n_i(y,t)x^i \text{ and } d(x,y,t) = \sum_{i=0}^{N(N+1)} d_i(y,t)x^i$$

$\mathbf{n_{22}} = -3377699720527872$, $\mathbf{n_{21}} = 55732045388709888\, t + 272467777455915008$, $\mathbf{n_{20}} = -4388898574360903 68\, t^2 - 30399297484750848\, y^2 - 4291367494930661376\, t - 10655516718358593536$, $\mathbf{n_{19}} = 21944492871804518 40\, t^3 + 321852562119799603 20\, t^2 + 2229281815548395520\, y^2 + (455989462271262720\, y^2 + 159832750775378903040)t + 268346983796871004160$, $\mathbf{n_{18}} = -7817725585580359680\, t^4 - 118219490218475520\, y^4 - 152879967006904811520\, t^3 + (-3248924918682746880\, y^2 - 113880834927457468 4160)t^2 - 790156554622153523 20\, y^2 + (-31767265871564636160\, y^2 - 3823944519105411809280)t - 4877083144468331233280$, $\mathbf{n_{17}} = 21107859081066971136\, t^5 + 515969888648303738880\, t^4 + 780248635441938432 0\, y^4 + (146201621340723609 60\, y^2 + 5124637571735586078720)t^3 + (214429044633061294080\, y^2 + 25811625503961529712640)t^2 +$

$1796373301367406592000\, y^2+(1595963117949419520\, y^4+1066711348739907256320\, y^2+6584062245032247164928 0)t+$
$679589918660478002790 40, \quad \mathbf{n_{16}} = -448542005472673136 64\, t^6-253327479039590400\, y^6-1315723216053174534144\, t^5+$
$(-466017668023556505 60\, y^2-16334782259907180625920)t^4-247945677484882657280\, y^4+(-91132343969051049984 0\, y^2-$
$109699408391836501278720)t^3+(-101742648769275494 40\, y^4-680028484821690875904 0\, y^2-41973396812080575676416 0)t^2-$
$2932622480153727991808 0\, y^2+(-99481701018847150080\, y^4-2290375959243443404800 0\, y^2-866477146292109453557760)t-$
$753040926423114370252800, \quad \mathbf{n_{15}} = 768929152238868234 24\, t^7+263144643210634906828 8\, t^6+148618787703226368 00\, y^6+$
$(1118442403256535613 44\, y^2+392034774237772335022 08)t^5+(2733970319071531499520\, y^2+329098225175509503836160)t^4+$
$502754840801877950464 0\, y^4+(406970595077101977 60\, y^4+272011393928676350361 60\, y^2+167893587248322302705664 0)t^3+$
$(596890206113082900480\, y^4+1374225575546066042880 00\, y^2+519886287775265672134656 0)t^2+364818141054811661926400\, y^2+$
$(303992974847508480 0\, y^6+297534812981859188736 0\, y^4+351914697618447359016960\, y^2+903649111707737244303360 0)t+$
$679316770839478354837504 0, \quad \mathbf{n_{14}} = -1081306620335908454 40\, t^8-303992974847508480\, y^8-42291103373137752883 20\, t^7+$
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$7642313671944131491921920 0)t-506653800654076565782528 00, \quad \mathbf{n_{13}} = 1261524390391893196 80\, t^9+555070731772433006592 0\, t^8+$
$156049727088387686 40\, y^8+(314561925915900641280\, y^2+110259780254373469224960)t^7+(10765008131344155279360\, y^2+$
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$-122998628063209586688\, t^{10}-141863388262170624\, y^{10}-601326626086802423808 0\, t^9+(-383372347210003906560\, y^2-$
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$3427419460414042275840\, y^8+841737902450293667266560\, y^6+61260522427881646718976000\, y^4+1651249939280407411713638400\, y^2+$
$14859263768878687989635481600)t+7277822304196982485286912000, \quad \mathbf{n_{10}} = -691867282855553925 12\, t^{12}+141863388262170624\, y^{12}-$
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$38073328893624470921124249600)t^3+(-5266678289233084416\, y^{10}-14138105274207924387840\, y^8-3472168847607461377474560\, y^6-$
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$(-5149640993916793 6512\, y^{10}-48013900196312084643840\, y^8-7248552179610103452794880\, y^6-383707225981761767237222400\, y^4-$
$8049855140833019158541107200\, y^2-60042034009625105503617024000)t-27031856507283943793151180800, \quad \mathbf{n_9} =$
$399154201647434956 80\, t^{13}+2536846703803697725440\, t^{12}-520165756961292288 0\, y^{12}+(215646945305627197440\, y^2+$
$75588247791572436910080)t^{11}+(11597013503102618173440\, y^2+1395975857742160613867520)t^{10}+1596120743936377487360\, y^{10}+$
$(431574133555121356800\, y^4+288455930408556350668800\, y^2+17804364817175145558835200)t^9+(1898926187642533969920 0\, y^4+$
$4371914476744102379520000\, y^2+165394854253259645150822400)t^8+477089389475446915072000\, y^8+(386845161730080768000\, y^6+$
$378626851183103941017600\, y^4+44782777688757668256153600\, y^2+1149935411965289117515776000)t^7+(1323870109031831961600 0\, y^6+$
$4478451992474531187916800\, y^4+324973605046126293024768000\, y^2+6051234715184591069301964800)t^6+3611335660556747643289600 0\, y^6+$
$(144422193712563486720\, y^8+198425396220787226050560\, y^6+34551779582599188656947200\, y^4+1701683274877619044928716800\, y^2+$
$24070310631347567595828019200)t^5+(353032029075155189760 0\, y^8+1682144128841739455692800\, y^6+179965692735684706566144000\, y^4+$
$6416451779741092996448256000\, y^2 + 71387491675545882977107968000)t^4 + 11519991561441624299929600 00\, y^4 +$
$(13166695723082711040\, y^{10}+35345263185519810969600\, y^8+8680422119018653443686400\, y^6+631749137537529481789440000\, y^4+$
$17028514998829201433296896000\, y^2+153236157616561469893115904000)t^3+(193111537271879761920\, y^{10}+180052125736170317414400\, y^8+$
$27182070673537887947980800\, y^6+1438902097431606627139584000\, y^4+30186956778123821844529152000\, y^2+225157627536094145638563840000)t^2+$
$15289389136008170537222144000\, y^2+(-1063975411966279680\, y^{12}+9613608944566429286 40\, y^{10}+463177882816490137190400\, y^8+$
$47701500802400126396006400\, y^6+1926109125211116395298816000\, y^4+32018026421258240246415360000\, y^2+202738923804629578448633856000)t+$
$843924918332368161802813440 00, \quad \mathbf{n_8} = -192449347222870425 60\, t^{14}+303992974847508480\, y^{14}-1317208865436535357440\, t^{13}+$
$(-121301406734415298560\, y^2-42518389382759495761920)t^{12}+9110782046170513408 0\, y^{12}+(-711634919508569751552 0\, y^2-$

$856621549069053103964160)t^{11}+(-291312540149706915840\,y^4-1947077530257755367014440\,y^2-120179462515932232522213760)t^{10}-$
$12971177574759955169280\,y^{10}+(-1424194640731900477440\,y^4-3278935857558076784640000\,y^2-124046140689944733863116800)t^9+$
$(-3264006052097556480000\,y^6-31946640568574395023360\,y^4-3778546867488928259112960\,y^2-97025800384571269290393600)t^8-$
$273570641450663631912960\,y^8+(-12765890337092665344000\,y^6-431850727845758364549120\,y^4-31336740486590749684531200\,y^2-$
$583511918964228424539832320)t^7+(-16247496792663392256\,y^8-2232285707483856293068800\,y^6-38870752030424087239065600\,y^4-$
$191439368423732142554480640\,y^2-2707909946026601354530652160)t^6-1622891878510923880595456\,y^6+(-476593239251459506176\,y^8-$
$227089457393634826518528\,y^6-242953685193174353864294400\,y^4-866220990265047554520514560\,y^2-963731137619869420190957568)t^5+$
$(-2221879903270207488\,y^{10}-5964513162556468101120\,y^8-146482123258439776862208\,y^6-10660766695945810005196800\,y^4-$
$287356190605242774186885120\,y^2-2585860159779474804446330880)t^4-41957476331255592166359040\,y^4+$
$(-4345009588617294643\,2\,y^{10}-40511728290638321418240\,y^8-6115965901546024788295680\,y^6-32375297192211149110640640\,y^4-$
$679206527507785991501905920\,y^2-5066046619562118276867686400)t^3+(359091701538619392\,y^{12}-32445930187911698841\,6\,y^{10}-$
$156322535450565421301760\,y^8-160992565208100426586521600\,y^6-6500618297587517834133504000\,y^4-10806083917174656083165184000\,y^2-$
$68424386784062482726413926400)t^2-4429107875789391162192691200\,y^2+(351111885948872294\,4\,y^{12}-10773815021570548039680\,y^{10}-$
$322035337895926667673600\,y^8-243765157087580465922048000\,y^6-7775994303973096402452480000\,y^4-10320337666805515112624947200\,y^2-$
$56964931987434850921689907200)t-22025623726373501393489100800,\quad \mathbf{n_7}=7697973888914817024\,t^{15}+564518085187086581760\,t^{14}-$
$891712726219358208\,y^{14}+(5598526464665321472\,y^2+196238720228120749670\,4\,0)t^{13}+(355817459754284875776\,y^2+$
$42831077453452655198208\,0)t^{12}-10119588362701504512\,y^{12}+(158897749172567408640\,y^4+10620422892315029274624\,0\,y^2+$
$655524340995993995575296\,0)t^{11}+(854516784439140286464\,0\,y^4+196736151453484607078400\,0\,y^2+744276844139668403178700\,80)t^{10}+$
$67342955732018858557440\,y^{10}+(2176004034731704320\,00\,y^6+212977603790495966822400\,y^4+25190312449926188394086400\,y^2+$
$646838669230475128602624000)t^9+(957441775281949900800\,0\,y^6+32388804588431877341184\,00\,y^4+2350255536494306226339840\,00\,y^2+$
$4376339392231713184048742400)t^8+1162516693828937459957760\,0\,y^8+(139264258222829076480\,y^8+19133877492718768226304\,0\,y^6+$
$333177874546492176334848\,00\,y^4+1640908872203418364752691200\,y^2+23210656680228011610262732800)t^7+(476593239251459506176\,0\,y^8+$
$227089457393634826518528\,0\,y^6+242953685193174353864294400\,y^4+866220990265047554520514560\,0\,y^2+963731137619869420190957568\,00)t^6+$
$5786886560174859081482240\,00\,y^6+(26662558839242489856\,y^{10}+7157415795067761721344\,0\,y^8+1757785479101277322346496\,0\,y^6+$
$127929200351349720062361600\,0\,y^4+3448274287262913290242621440\,0\,y^2+31030321917353697653355970560\,0)t^5+$
$(651751438292594196480\,y^{10}+60767592435957482127360\,0\,y^8+917394885231903718244352\,00\,y^6+48562945788316723665960960\,00\,y^4+$
$1018809791261678987252858880\,00\,y^2+7599069929343177415301529600\,00)t^4+1232615214147659882299392000\,0\,y^4+$
$(-7181834030772387840\,y^{12}+648918603758233976832\,0\,y^{10}+31264507090113084260352\,00\,y^8+3219851304162008531730432\,00\,y^6+$
$13001236595175035668267008000\,y^4+2161216783434931216633036800\,00\,y^2+1368487735681249654528278528000)t^3+$
$(-10533356578466168832\,0\,y^{12}+32321445064711644119040\,y^{10}+96610601368778000302080\,00\,y^8+73129547126274139776614400\,0\,y^6+$
$23327982911919289207357440000\,y^4+309610130004165453378748416000\,y^2+1708947959623045527650697216000)t^2+$
$1014835582904979367363870720\,00\,y^2+(-1823957849085050880\,y^{14}-546646922770230804480\,y^{12}+77827065448559731015680\,y^{10}+$
$1641423848703981791477760\,0\,y^8+97373512710655432835727360\,0\,y^6+2517448579875335529981542400\,0\,y^4+2657464725473634697315614720\,00\,y^2+$
$132153742358241008360934604800\,0)t+476127648098879353374375936000,\quad \mathbf{n_6}=-2525897682300174336\,t^{16}+$
$253327479039590400\,y^{16}-197581329815480303616\,t^{15}+(-209944742424949555 20\,y^2-7358952008554528112640)t^{14}+$
$121687261931550801920\,y^{14}+(-1436955125930765844480\,y^2-172971658946635722915840)t^{13}+(-69517765262998241280\,y^4-$
$4646435015387825307648\,0\,y^2-286791899185747373064192\,0)t^{12}+8034061447258775224320\,y^{12}+(-40783755620958968217 60\,y^4-$
$93896799557344926105600\,0\,y^2-3552230392484781015171072\,0)t^{11}+(-1142402118234144768\,00\,y^6-1118132419900103825817 60\,y^4-$
$132249140362112489068953 60\,y^2-339590301345999442516377600)t^{10}-17771834633551809085440\,0\,y^{10}+(-55850770224780410880\,00\,y^6-$
$1889346934325192844902400\,y^4-137098239628834529869824000\,y^2-2552864645468499357361766400)t^9+(-913921694587315814 40\,y^8-$
$1255660710459669164851 20\,y^6-21864798017113549071974400\,y^4-1076846447383493301868953600\,y^2-15231993446399632619234918400)t^8-$
$355719775432813808975872 00\,y^8+(-357444929438594629632\,0\,y^8-170317093045226119888896\,0\,y^6-182215263894880765398220800\,y^4-$
$6496657426987856658903859200\,y^2-72279835321490206514321817600)t^7+(-23329738984337178624\,y^{10}-6262738820684291506176\,0\,y^8-$
$1538062294213617657053184\,0\,y^6-111938050307431005054566400\,0\,y^4-3017240001355049128962293760\,0\,y^2-271515316776844854466864742400)t^6-$
$1633054530068082320421683200\,y^6+(-684339010207223906304\,y^{10}-638059720577553562337280\,y^8-9632646294934989041565696\,0\,y^6-$
$509910930777325598492590080\,0\,y^4-1069750280824762936615501824\,00\,y^2-797902342581033628606660608000)t^5+$
$(94261571653887590 40\,y^{12}-851705667432682094592\,0\,y^{10}-4103466555577342309171200\,y^8-42260548367126361978961920\,0\,y^6-$
$17064123031167234314600448000\,y^4-283659702825834722183086080000\,y^2-1796140153081640171568365568000)t^4-$
$2880192398576512393019392000\,0\,y^4+(184333740123157954560\,y^{12}-5656252886324537720832\,0\,y^{10}-16906855239536150052864000\,y^8-$
$1279767074709797446090752000\,y^6-4082397009585875611287552000\,0\,y^4-5418177275072895434128097280\,00\,y^2-$
$2990658929340329673388720128000)t^3+(4787889353848258560\,y^{14}+1434948172271855861760\,y^{12}-2042960468024692939161 60\,y^{10}-$
$430873760284795220262912 00\,y^8-255605470865470511193784320\,0\,y^6-66083025221727557662015488000\,y^4-697584490436829108045348864000\,y^2-$
$3469035736903826469474533376000)t^2-17627232345147456075333632000\,0\,y^2+(46814918126516305920\,y^{14}+531278389041828986880\,0\,y^{12}-$
$3535505175930990074265 60\,y^{10}-61032126426019216647782400\,y^8-3038115444091801017778176000\,y^6-64712298742752143820718080000\,y^4-$
$532788681025114167866032128000\,y^2-2499670152519116605215473664000)t-83994715891417285180941926400\,0,\quad \mathbf{n_5}=$
$668619974726516736\,t^{17}+55569749010603835392\,t^{16}-557320453887098880\,0\,y^{16}+(6298342272748486656\,y^2+2207685602566358433792)t^{15}+$
$(461878433334889021440\,y^2+55598033232847196651520)t^{14}-10063743727322110361 60\,y^{14}+(24063841821807083520\,y^4+$
$16083813514804010680320\,y^2+992741189489125522145280)t^{13}+(1529390835785961308160\,y^4+352112998340043472896000\,y^2+$
$13320863971817928806891520)t^{12}-48511649286128140288000\,y^{12}+(4673463210957864960\,0\,y^6+457417808140951565107 20\,y^4+$
$5410192105722783643729920\,y^2+138923305096090681029427200)t^{11}+(2513284660115118489600\,y^6+85020612044633678020608\,0\,y^4+$
$6169420783297553844142080\,0\,y^2+1148789090460824710812794880)t^{10}-25994128530836161560576\,0\,y^{10}+(45696084729365790720\,y^8+$
$62783035522983458242560\,y^6+109323990085567745359872 00\,y^4+5384232236917466509344768 00\,y^2+7615996723199816309617459200)t^9+$
$(2010627728092094791680\,y^8+958033648379396924375040\,y^6+10249608594087043053649920\,0\,y^4+3654369802680669370633420800\,y^2+$
$40657407368338241164306022400)t^8+71499471492947202434662400\,y^8+(14997689347073900544\,y^{10}+40260463847256159682560\,y^8+$
$988754331994468493819904\,0\,y^6+71960175197634217535078400\,0\,y^4+1939654286585388725761474560\,0\,y^2+17454556078511454930012733440\,0)t^7+$
$(5132542576554179297 28\,y^{10}+47854479043316517175296\,0\,y^8+7224484721201241781174272\,0\,y^6+382433198082994198869442560\,0\,y^4+$
$8023127106185722024616263680\,0\,y^2+59842675693577522145499545600\,0)t^6+3620477123375379618201600000\,y^6+$
$(-848354144884988313 6\,y^{12}+766535100689413885132 8\,y^{10}+36931199000196080782540 80\,y^8+38034493530413725781065728\,0\,y^6+$
$1535771072805051088314040320\,0\,y^4+2552937325432512499647774720\,00\,y^2+161652613777347615441152901120\,0)t^5+$
$(-20737545763855269888\,0\,y^{12}+636328449711510493593 60\,y^{10}+190202121444781688094720\,00\,y^8+1439737959048522126852096000\,y^6+$
$4592696635784110062698496000\,0\,y^4+609544943445700736339410944000\,y^2+3364491295507870882562310144000)t^4+$
$5222879904880158986521804800\,0\,y^4+(-7181834030772387840\,y^{14}-215242225840778379264\,0\,y^{12}+30644407020370394087424\,0\,y^{10}+$
$6463106404271928303943680\,0\,y^8+38340820629820576679067648\,00\,y^6+9912453783259133649302323200\,0\,y^4+1046376735655243662068023296000\,y^2+$

$5203553605355739704211800064000)t^3+(-105333565784661688320\,y^{14}-11953763753441152204800\,y^{12}+795488664584472766709760\,y^{10}+$
$137322284458543237457510400\,y^8+6835759749206552290000896000\,y^6+145602672171192323596615680000\,y^4+1198774532306506877698572288000\,y^2+$
$5624257843168012361734815744000)t^2+210787132661036441791365120000\,y^2+(-1139973655678156800\,y^{16}-547592678691978608640\,y^{14}-$
$36153276512664488509440\,y^{12}+799732558509831408844800\,y^{10}+160073898944766214039142400\,y^8+7348745385306370441897574400\,y^6+$
$129608657935943057685872640000\,y^4+793225455531635523390013440000\,y^2+3779762215113777833142386688000)t+$
$1181260533177816038284197888000,\quad \mathbf{n_4} = -139295828068024320\,t^{18}+118219490218475520\,y^{18}-12258032869986140160\,t^{17}+$
$(-1476173970175426560\,y^2-517426313101490257920)t^{16}+55439311412930805760\,y^{16}+(-115469608333722255360\,y^2-$
$13899508308211799162880)t^{15}+(-6445671916555468800\,y^4-4308164334322502860800\,y^2-265912818613158622003200)t^{14}+$
$5532221782261917286400\,y^{14}+(-441170433399796531200\,y^4-101571057213474078720000\,y^2-3842556914947479463526400)t^{13}+$
$(-14604572534243328000\,y^6-14294306504404736409600\,y^4-1690685033038369888665600\,y^2-43413532842528337821696000)t^{12}+$
$227177327803263614976000\,y^{12}+(-856801588675608576000\,y^6-289842995606705720524800\,y^4-21032116306696206286848000\,y^2-$
$391632644475281151413452800)t^{11}+(-17136031773512171520\,y^8-23543638321118796840960\,y^6-4099649628208790450995200\,y^4-$
$201908708884404994100428800\,y^2-2855998771199931116106547200)t^{10}+4669515840522526969036800\,y^{10}+(-837761553371706163200\,y^8-$
$399180686824748718489600\,y^6-42706702475362679390208000\,y^4-1522654084450278904430592000\,y^2-16940586403474267151794176000)t^9+$
$(-70301668814408908800\,y^{10}-18872092428401324851200\,y^8-4634785931224071064780800\,y^6-337313321238910394695680000\,y^4-$
$9092129468369009652006912000\,y^2-81818231618022444984434688000)t^8+61764914608623050555392000\,y^8+(-274957638029688176640\,y^{10}-$
$256363280589195627724800\,y^8-38702596720720938113433600\,y^6-2048749275444611779657728000\,y^4-42981038068852082274299840000\,y^2-$
$320585762644165297208033280000)t^7+(5302213405531176960\,y^{12}-4790844379308836782080\,y^{10}-2308199937512255048908800\,y^8-$
$237715584565085786131660800\,y^6 - 9598569205031569301962752000\,y^4 - 159558582839532031227985920000\,y^2 -$
$1010328836108422596507205632000)t^6-6221605755593231572140032000\,y^6+(155531593228914524160\,y^{12}-47724633728363287019520\,y^{10}-$
$14265159108358626607104000\,y^8-1079803469286391595139072000\,y^6-34445224768380825470238720000\,y^4-457158707584275552254558208000\,y^2-$
$2523368471630903161921732608000)t^5+(6732969403849113600\,y^{14}+2017895867257297305600\,y^{12}-287291315815972444569600\,y^{10}-$
$60591622540049327849472000\,y^8-3594451934045679063662592000\,y^6-92929254218054377962209280000\,y^4-980978189676790933188771840000\,y^2-$
$4878331505021005972698562560000)t^4-70466594337127841503641600000\,y^4+(131666957230827110400\,y^{14}+14942204691801440256000\,y^{12}-$
$994360830730590958387200\,y^{10}-171652855573179046821888000\,y^8-8544699686508190362501120000\,y^6-182003340213990404495769600000\,y^4-$
$1498468165383133597123215360000\,y^2-7030322303960015452168519680000)t^3+(2137450604396544000\,y^{16}+1026736272547459891200\,y^{14}+$
$67787393461245915955200\,y^{12}-1499498547205933891584000\,y^{10}-300138560521436651323392000\,y^8-1377889759744944578557952000\,y^6-$
$24301623362989323316101120000\,y^4-1487297729121816606356275200000\,y^2-7087054153338333437141975040000)t^2-$
$12027970523768056497307648000\,y^2+(20899517020766208000\,y^{16}+377390389774579138560\,y^{14}+181918684822980526080000\,y^{12}+$
$9747798199063560585216000\,y^{10}-2681230180985520091299840000\,y^8-135767892126576735682560000000\,y^6-1958579964330059619945676800000\,y^4-$
$7904517474788866567176192000\,y^2-4429726999416810143565742080000)t-12755512195764738639003648000000,\quad \mathbf{n_3} =$
$21994078116003840\,t^{19}+2043005478331023360\,t^{18}-1733885856537640960\,y^{18}+(260501288854487040\,y^2+913105258414394572800)t^{17}+$
$(216505515625729228800\,y^2+26061578077897123430400)t^{16}-31549967189544009728\,y^{16}+(1289134383311093760\,y^4+$
$861632866864500572160\,y^2+53182563722631724400640)t^{15}+(9453652144281354240\,y^4+21765226545744445440000\,y^2+$
$823405053203031313612800)t^{14}-2082869791662580695040\,y^{14}+(3370285969440768000\,y^6+3298686116401093017600\,y^4+$
$390158084547316128153600\,y^2+10018507579045001035776000)t^{13}+(21420039716890214400\,y^6+72460748901676430131200\,y^4+$
$5258029076674051571712000\,y^2+97908161118820287853363200)t^{12}-803049459635389779148800\,y^{12}+(4673463210957864960\,y^8+$
$6420992269396035502080\,y^6+11180862622387610320896000\,y^4+55066011513928634754662400\,y^2+778908755781799395301785600)t^{11}+$
$(25132846601151184896\,y^8+11975420604742461554688\,y^6+128120107426088038170624000\,y^4+456796225335083671329177600\,y^2+$
$5082175921042280145538252800)t^{10}-216584149910947083845632000\,y^{10}+(234338896048029696\,y^{10}+62906974761337749504000\,y^8+$
$15449286437413570215936000\,y^6+1124377737463034648985600000\,y^4+30307098227896698840023040000\,y^2+272727438726741483281448960000)t^9+$
$(10310911426113306624\,y^{10}+9613623022094836039680\,y^8+14513473770270351792537600\,y^6+768280978291729417371648000\,y^4+$
$16117889275819530853023744000\,y^2 + 120219660991561986453012480000)t^8 - 1196567363524572416573440000\,y^8 +$
$(-2272377173799075840\,y^{12}+2053219019703787192320\,y^{10}+989228544648109306675200\,y^8+101878107670751051199283200\,y^6+$
$4113672516442101129412608000\,y^4 + 68382249788370870526279680000\,y^2 + 432998072617895398503088128000)t^7 +$
$(-77765796614457262080\,y^{12}+238623168641816435097600\,y^{10}+71325795541793133035520000\,y^8+539901734643195797569536000\,y^6+$
$1722261238419041273511936000\,y^4+22857935379213777612727910400\,y^2+126168423581545158096086630400)t^6+$
$8072038641088128117374976000\,y^6+(-4039781642309468160\,y^{14}-121073752035437838336\,y^{12}+172374789489583466741760\,y^{10}+$
$36354973524029596709683200\,y^8+2156671160427407438197555200\,y^6+55757552530832626777325568000\,y^4+588586913806074559913263104000\,y^2+$
$2926998903012603583619137536000)t^5+(-98750217923120332800\,y^{14}-11206653518851080192000\,y^{12}+745770623047943218790400\,y^{10}+$
$128739641679884285116416000\,y^8+6408524764881142771875840000\,y^6+136502505160492803371827200000\,y^4+1123851124037350197842411520000\,y^2+$
$5272741727970011589126389760000)t^4+65464304592799150794342400000\,y^4+(-2137450604396544000\,y^{16}-1026736272547459891200\,y^{14}-$
$67787393461245915955200\,y^{12}+1499498547205933891584000\,y^{10}+300138560521436651323392000\,y^8+1377889759744944578557952000\,y^6+$
$24301623362989323316101120000\,y^4+1487297729121816606356275200000\,y^2+7087054153338333437141975040000)t^3+$
$(-3134927553114931200\,y^{16}-566085584661868707840\,y^{14}-27287802723447078912000\,y^{12}-1462169729859534087782400\,y^{10}+$
$402184527147828013694976000\,y^8 + 20365183818986510352384000000\,y^6 + 293786994649508942991851520000\,y^4 +$
$1185677621218329985076428800000\,y^2+6644590499125215215348613120000)t^2-99037004287605522605015040000\,y^2+$
$(-354658470655426560\,y^{18}-166317934238792417280\,y^{16}-16596665346785751859200\,y^{14}-681531983409790844928000\,y^{12}-$
$14008547521567580907110400\,y^{10}+1852947438258691516661760000\,y^8+186648172667796947164200960000\,y^6+2113997830113835245109248000000\,y^4+$
$3608391157130416949192294400000\,y^2+38266536587294215917010944000000)t+99281850930100420567105536000000,\quad \mathbf{n_2} =$
$-2474333788050432\,t^{20}+303992974847508480\,y^{20}-241934859276042240\,t^{19}+(-3256266110681088\,y^2-1141381573017993216000)t^{18}+$
$108761931000997478400\,y^{18}+(-2865514177399357440\,y^2-344932651030991339520)t^{17}+(-181284522653122560\,y^4-$
$1211671219028203929600\,y^2-74787980234950862438400)t^{16}+1082260026453403893760\,y^{16}+(-1418047821642203136\,y^4-$
$3264783981861666816000\,y^2-1235107579804546970419200)t^{15}+(-5416531022315520000\,y^6-5301459829930328064000\,y^4-$
$627039778736758063104000\,y^2-16101172894893751664640000)t^{14}+533217188681411985408000\,y^{14}+(-3707314566384844800\,y^6-$
$125412834637516898304000\,y^4-9100434940397396951040000\,y^2-169456432705650498207744000)t^{13}+(-876274352054599680\,y^8-$
$1203936050511756656640\,y^6-20964117416976769351680000\,y^4-1032487715886161901649920000\,y^2-14604539170908738661908480000)t^{12}+$
$200750880729797650022400\,y^{12}+(-5140809532053651456\,y^8-2449517850970048954368\,y^6-262063856098816441712640000\,y^4-$
$9343559154581256913551360000\,y^2-103953598384955730249646080000)t^{11}+(-5272625161080668160\,y^{10}-1415406932130099363840\,y^8-$
$347608944841805329858560\,y^6-252984990929182796021760000\,y^4-6819097101276757239005184000\,y^2-61363673713516833738326016000)t^{10}+$
$56688844024154013931929600\,y^{10}+(-257772785652832665600\,y^{10}-240340575552370900992000\,y^8-36283684425675879481344000\,y^6-$
$1920702445729323543429120000\,y^4-40294723189548827132559360000\,y^2-300549152478904966132531200000)t^9+(639106080130990080\,y^{12}-$
$577467849291690147840\,y^{10}-2782205281822807425024000\,y^8-286532177823987331497984000\,y^6-11569703952493409426472960000\,y^4-$

$$
\begin{aligned}
&192325077529793073355161600000\,y^2 - 1217807079237830808289935360000)t^8 + 503944783021321689759744000\,y^8 + \\
&(2499614891178983424 0\,y^{12} - 767003042062981398528 0\,y^{10} - 22926148567004935618560000\,y^8 - 1735398432781700777902080000\,y^6 - \\
&55358396949183469505740800000\,y^4 - 734719351474728566123397120000\,y^2 - 4055413615121094367374213120000)t^7 + \\
&(1514918115866050560\,y^{14} + 45402657013289189376 0\,y^{12} - 646405460585938000281600\,y^{10} - 13633115071511098766131200\,y^8 - \\
&80875168516027778932408320 0\,y^6 - 2090908219906223504149708800 0\,y^4 - 22072009267727795996747366400 0\,y^2 - \\
&109762458862972634385717657600 0)t^6 - 743652818116209934545715200 0\,y^6 + (44437598065404149760\,y^{14} + 5042994083482986086400\,y^{12} - \\
&335596780371574448455680\,y^{10} - 5793283875594792830238720 0\,y^8 - 288383614419651424734412800 0\,y^6 - 6142612732222176151732224000 0\,y^4 - \\
&50573300581680758902908518400 0\,y^2 - 237273377758650521510687539200 0)t^5 + (120231596497305600 0\,y^{16} + 577539153307946188800\,y^{14} + \\
&38130408821950827724800\,y^{12} - 843467932803337814016000\,y^{10} - 16882794029330811636940800 0\,y^8 - 775062989856531257543884800 0\,y^6 - \\
&1366966314168149436530688000 00\,y^4 - 836604972631021841075404800000\,y^2 - 3986467961252812558392360960000)t^4 - \\
&3512082537485187869048832000 0\,y^4 + (2351195664836198400 0\,y^{16} + 424564188496401530880 0\,y^{14} + 204658520425853091840000\,y^{12} + \\
&109662729739465056583680 0\,y^{10} - 301638395360871010271232000\,y^8 - 15273887864239882764288000000\,y^6 - 220340245987131707243888640000\,y^4 - \\
&8892582159137474888073216000 00\,y^2 - 4983442874343911411511459840000)t^3 + (398990779487354880\,y^{18} + 18710767601864146944 0\,y^{16} + \\
&186712485151339708416 00\,y^{14} + 766723481336014700544000\,y^{12} + 1575961596176352852049920 0\,y^{10} - 208456586804102795624448000\,y^8 - \\
&2099791942512715655597260800 0\,y^6 - 237824755887806465074790400000\,y^4 - 4059440051771719067841331200 00\,y^2 - \\
&4304985366070599290663731200000)t^2 + 2865255804638011716953702400 00\,y^2 + (390124317720969216 0\,y^{18} + 709874261764740218880\,y^{16} + \\
&468645703124080656384 00\,y^{14} + 1806861284179627003084800\,y^{12} + 4873143372996309386526720 0\,y^{10} + 269227656793028793729024000\,y^8 - \\
&1816208694244828826409369600 0\,y^6 - 147294685333798089287270400000\,y^4 + 2228332596471124258612838400 00\,y^2 - \\
&2233841645927259462759874560000)t - 4945985829663792497374003200 00, \quad \mathbf{n_1} = 176738127717888\,t^{21} + 18145114445703168\,t^{20} - \\
&222928181554839552\,y^{20} + (2570736403169280\,y^2 + 901090715540520960)t^{19} + (23879284811661312 0\,y^2 + 28744387585915944960)t^{18} - \\
&3334915524067852288 0\,y^{18} + (15995693175275520\,y^4 + 106912166384841523 20\,y^2 + 65989394324956643328 0)t^{17} + \\
&(13294198327895654 40\,y^4 + 3060734982995312640 00\,y^2 + 1157913356066762784768 0)t^{16} - 2089096018147420405760\,y^{16} + \\
&(54165310223155200\,y^6 + 5301459829930328064 0\,y^4 + 62703977873675806310 40\,y^2 + 16101172894893751664640 0)t^{15} + \\
&(397212274969804800 0\,y^6 + 1343708942544823910400\,y^4 + 97504660075686395904000\,y^2 + 181560463613196962365440 0)t^{14} - \\
&85546775641828045619200\,y^{14} + (101108579083223040\,y^8 + 13891569813597192192 0\,y^6 + 24189366250357810790400\,y^4 + \\
&1191331979868648348057600\,y^2 + 16851391351048544609894400)t^{13} + (642601191506706432 0\,y^8 + 3061897313712561192960\,y^6 + \\
&327579820123520552140800\,y^4 + 11679448943226571141939200\,y^2 + 129941997981194662812057600)t^{12} - 31510601225201164222464 00\,y^{12} + \\
&(71899434014736384\,y^{10} + 193010036199559004160\,y^8 + 47401219751155272253440\,y^6 + 3449795330852492673024000\,y^4 + \\
&92987687744683053259161600\,y^2 + 836777368820684096431718400)t^{11} + (386659178479248998 4\,y^{10} + 3605108633285563514880\,y^8 + \\
&544255266385138192220160\,y^6 + 28810536685939853151436800\,y^4 + 604420847843232406988390400\,y^2 + 4508237287183574491987968000)t^{10} - \\
&85529414940804476462694400\,y^{10} + (-106517680021831680\,y^{12} + 96244641548615024640\,y^{10} + 46370088030380123750400\,y^8 + \\
&4775536297066455524966400\,y^6 + 192828399208223490441216000\,y^4 + 3205417958829884555919360000\,y^2 + 20296784653963846804832256000)t^9 + \\
&(-46867779209605939 20\,y^{12} + 1438130703868090122240\,y^{10} + 429865285631342542848000\,y^8 + 32538720614656889585664000\,y^6 + \\
&1037969942797190053232640000\,y^4 + 13775987840151160614813696000\,y^2 + 76039005283520519388266496000)t^8 - \\
&7684121319119951253995520 00\,y^8 + (-324625310542725120\,y^{14} - 972914078856196915 20\,y^{12} + 13851545583984385720320\,y^{10} + \\
&2921381801038092592742400\,y^8 + 173303932534345240569446400\,y^6 + 4480517614084764651749376000\,y^4 + 47297162716559562850172928000\,y^2 + \\
&235205268992084216540823552000)t^7 + (-1110939951635103744 0\,y^{14} - 1260748520870746521600\,y^{12} + 838991950928936121139 20\,y^{10} + \\
&14483209688986982075596800\,y^8 + 720959036049128561836032000\,y^6 + 15356531830555440379330560000\,y^4 + 126433251454201897257271296000\,y^2 + \\
&593183444396626303776718848000)t^6 + 41338585069874134741155840 00\,y^6 + (-360694789491916800\,y^{16} - 173261745992383856640\,y^{14} - \\
&114391226465852483174 40\,y^{12} + 2530403798410013442048 00\,y^{10} + 50648382087992434910822400\,y^8 + 2325188969569593772631654400\,y^6 + \\
&41008989425044483095920640000\,y^4 + 250981491789306552322621440000\,y^2 + 1195940388375843767517708288000)t^5 + \\
&(-881698374313574400 0\,y^{16} - 1592115706861505740800\,y^{14} - 76746945159694909440000\,y^{12} - 4112352365229939621888 00\,y^{10} + \\
&1131143982603266288517120 00\,y^8 + 572770794908995603660800 0000\,y^6 + 82627592245174390216458240000\,y^4 + 333471830967655308302745600000\,y^2 + \\
&1868791077878966779316797440000)t^4 + 4589971349874067636224000000\,y^4 + (-199495389743677440\,y^{18} - 93553838009320734720\,y^{16} - \\
&9335624257566985420800\,y^{14} - 383361740668007350272000\,y^{12} - 7879807980881764260249600\,y^{10} + 104228293402051239781222 4000\,y^8 + \\
&10498959712563578277986304000\,y^6 + 118912377943903232537395200000\,y^4 + 202972002588585953392066560000\,y^2 + \\
&2152492683035299645331865600000)t^3 + (-2925932382907269120\,y^{18} - 532405696323555164160\,y^{16} - 35148427734306049228800\,y^{14} - \\
&1355145963134720253136 00\,y^{12} - 36548575297472320398950400\,y^{10} - 2019207425947711595296768000\,y^8 + 13621565206836216198070272000\,y^6 + \\
&110471014000348566965452800000\,y^4 - 1671249447353343193959628800 00\,y^2 + 1675381234445444597069905920000)t^2 - \\
&2644544666141553199782297600 00\,y^2 + (-45598946227126272\,y^{20} - 16314289650149621760\,y^{18} - 1623390039680105840640\,y^{16} - \\
&79982578302211797811200\,y^{14} - 3011263210946964750336000\,y^{12} - 85033266036231020897894400\,y^{10} - 755917174531982534639616000\,y^8 + \\
&11154792271743149018185728000\,y^6 + 52681238062277818035732480000\,y^4 - 429788370695701757543055360000\,y^2 + \\
&7418978744495688746061004800 00)t + 1177731069679864357532467200 00, \quad \mathbf{n_0} = -6025163444928\,t^{22} - 96402615118848\,t^{20}y^2 - \\
&666487215636480\,t^{18}y^4 - 2538998916710400\,t^{16}y^6 - 5416531022315520\,t^{14}y^8 - 4493714625921024\,t^{12}y^{10} + 7988826001637376\,t^{10}y^{12} + \\
&30433622863380480\,t^8y^{14} + 45086848686489600\,t^6y^{16} + 37405385576939520\,t^4y^{18} + 17099604835172352\,t^2y^{20} + \\
&3377699720527872\,y^{22} - 648039801632256\,t^{21} - 9426033478287360\,t^{19}y^2 - 58650874976010240\,t^{17}y^4 - 198606137484902400\,t^{15}y^6 - \\
&370731456638484480\,t^{13}y^8 - 263631258054033408\,t^{11}y^{10} + 390564826746716160\,t^9y^{12} + 1190292805323325440\,t^7y^{14} + \\
&1322547561470361600\,t^5y^{16} + 731483095726817280\,t^3y^{18} + 167196136166129664\,ty^{20} - 33790901832769536\,t^{20} - \\
&445467359936839680\,t^{18}y^2 - 2485059295279841280\,t^{16}y^4 - 7441912400141352960\,t^{14}y^6 - 12063127262472437760\,t^{12}y^8 - \\
&7218348116146126848\,t^{10}y^{10} + 9121069489276846080\,t^8y^{12} + 21657718249047982080\,t^6y^{14} + 17541344626747637760\,t^4y^{16} + \\
&6117858618806108160\,t^2y^{18} + 580964351930793984\,y^{20} - 1134646878391418880\,t^{19} - 13503242572038144000\,t^{17}y^2 - \\
&67185447127241195520\,t^{15}y^4 - 176647921944955453440\,t^{13}y^6 - 245802861360379330560\,t^{11}y^8 - 119844225322340843520\,t^9y^{10} + \\
&135080198664722841600\,t^7y^{12} + 238817356029225861120\,t^5y^{14} + 133101424080888791040\,t^3y^{16} + 25011866430508892160\,ty^{18} - \\
&27495580968731934720\,t^{18} - 293924896282855342080\,t^{16}y^2 - 1295858906269168435200\,t^{14}y^4 - 2962576234447204515840\,t^{12}y^6 - \\
&3477756602278509281280\,t^{10}y^8 - 1298582398498536161280\,t^8y^{10} + 1429890330823156039680\,t^6y^{12} + 1750429548293809766400\,t^4y^{14} + \\
&608771264880039690240\,t^2y^{16} + 46071824188000174080\,y^{18} - 510844127676512993280\,t^{17} - 4875233003784319795200\,t^{15}y^2 - \\
&18898835776356954931200\,t^{13}y^4 - 37108313617168513105920\,t^{11}y^6 - 35822107135945211904000\,t^9y^8 - 8989199474238601297920\,t^7y^{10} + \\
&11512041773954236416000\,t^5y^{12} + 8787106933576512307200\,t^3y^{14} + 1566822013610565304320\,ty^{16} - 7547424794481446092800\,t^{16} - \\
&63821356064391875788800\,t^{14}y^2 - 215612208178280792064000\,t^{12}y^4 - 358165222279984164372480\,t^{10}y^6 - 273879543847321180569600\,t^8y^8 - \\
&31630047480125168025600\,t^6y^{10} + 71880326375251378176000\,t^4y^{12} + 29993466863329424179200\,t^2y^{14} + 1840193325741721518080\,y^{16} - \\
&90780231806598481182720\,t^{15} - 673814362109225258188800\,t^{13}y^2 - 1964354774041353623961600\,t^{11}y^4 - 2711560051221407465472000\,t^9y^6 - \\
&1551772466677176650956800\,t^7y^8 + 616852854784490943283 20\,t^5y^{10} + 338786490783680063078400\,t^3y^{12} + 64160081731371034214400\,ty^{14} - \\
&902753108091886318387200\,t^{14} - 5811730484042690828697600\,t^{12}y^2 - 14462129940616761783091200\,t^{10}y^4 - 16247243675094866303385600\,t^8y^6 - \\
&6331047760999054363852800\,t^6y^8 + 1477463996415330798796800\,t^4y^{10} + 11292237041051117813760 00\,t^2y^{12} + 6628127715586253783040 0\,y^{14} -
\end{aligned}
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$7496653729684307469926400\, t^{13} - 4121051235294766411284480 0\, t^{11} y^2 - 8649749523309917110272000 0\, t^9 y^4 - 7724561100526377448243200 0\, t^7 y^6 -$
$1696715973904899432775680 0\, t^5 y^8 + 9137143824368080099737600\, t^3 y^{10} + 2363295091890087316684800\, t y^{12} - 5229858555129275602698240 0\, t^{12} -$
$2404063469122413416939520 00\, t^{10} y^2 - 4200485263204466861015040 00\, t^8 y^4 - 2906486211961992215789568 00\, t^6 y^6 -$
$1954280501288463708979200 00\, t^4 y^8 + 3188747476358663283671040 0\, t^2 y^{10} + 2341367234189406398054400\, y^{12} - 3073798150352437153628160 00\, t^{11} -$
$1147998986679263384567808 000\, t^9 y^2 - 1645342696130940040642560 000\, t^7 y^4 - 8591561923634934054912000 00\, t^5 y^6 +$
$5048018564869289882419200 0\, t^3 y^8 + 6414706120560335734702080 0\, t y^{10} - 1522258849047288510362419 200\, t^{10} - 4434109004677459017203712 000\, t^8 y^2 -$
$5126123678130560386990080 000\, t^6 y^4 - 1968554946105670927122432 000\, t^4 y^6 + 2834689404494934504898560 00\, t^2 y^8 +$
$5909335605983379602800640 0\, y^{10} - 6336583773626709949022208 000\, t^9 - 1354641979866448899185049 6000\, t^7 y^2 -$
$1239413883677615853246873 6000\, t^5 y^4 - 3405391301709054049517568 000\, t^3 y^6 + 5763090989339963440496640 00\, t y^8 -$
$2205049396800789530070220 8000\, t^8 - 3137268647366331904032768 0000\, t^6 y^2 - 2229607086448185610076160 0000\, t^4 y^4 -$
$4183047101903680881819648 000\, t^2 y^6 + 4935367267175881297100800 00\, y^8 - 6355536904249567540464844 8000\, t^7 -$
$5002077464514829624541184 0000\, t^5 y^2 - 2761775350008714174136320 0000\, t^3 y^4 - 3100393880240560105586688 000\, t y^6 -$
$1494925485469804709397135 36000\, t^6 - 3805725048535986626101248 0000\, t^4 y^2 - 1975546427335418176339968 0000\, t^2 y^4 -$
$8008008843990377284239360 00\, y^6 - 2803186616818450168975196 16000\, t^5 + 4178123618383357984899072 0000\, t^3 y^2 -$
$3442478512405550727168000 000\, t y^4 - 4035923780691186834997248 00000\, t^4 + 1611706390108881590786457 60000\, t^2 y^2 +$
$3896811218593394060165120 000\, y^4 - 4188453086113611492674764 80000\, t^3 + 1983408499606164899836723 20000\, t y^2 -$
$2782117029185883279772876 80000\, t^2 + 9717492982337975460298752 0000\, y^2 - 8832983022598982681493504 0000\, t,$

and

$\mathbf{d_{12}} = 16777216, \quad \mathbf{d_{11}} = -150994944\, t - 738197504, \quad \mathbf{d_{10}} = 622854144\, t^2 + 100663296\, y^2 + 6090129408\, t +$
$15435038720, \quad \mathbf{d_9} = -1557135360\, t^3 - 22837985280\, t^2 - 3690987520\, y^2 + (-754974720\, y^2 - 115762790400)t -$
$200991047680, \quad \mathbf{d_8} = 2627665920\, t^4 + 251658240\, y^4 + 51385466880\, t^3 + (2548039680\, y^2 + 390699417600)t^2 +$
$64760053760\, y^2 + (24914165760\, y^2 + 1356689571840)t + 1799188643840, \quad \mathbf{d_7} = -3153199104\, t^5 - 77078200320\, t^4 -$
$7381975040\, y^4 + (-5096079360\, y^2 - 781398835200)t^3 + (-74742497280\, y^2 - 4070068715520)t^2 - 708669603840\, y^2 +$
$(-1509949440\, y^4 - 388560322560\, y^2 - 10795131863040)t - 11563863900160, \quad \mathbf{d_6} = 2759049216\, t^6 + 335544320\, y^6 +$
$80932110336\, t^5 + (6688604160\, y^2 + 1025585971200)t^4 + 103347650560\, y^4 + (130799370240\, y^2 + 7122620252160)t^3 +$
$(3963617280\, y^4 + 1019970846720\, y^2 + 28337221140480)t^2 + 5308311142400\, y^2 + (38755368960\, y^4 + 3720515420160\, y^2 +$
$60710285475840)t + 54293755330560, \quad \mathbf{d_5} = -1773674496\, t^7 - 60699082752\, t^6 - 7381975040\, y^6 + (-6019743744\, y^2 -$
$923027374080)t^5 + (-147149291520\, y^2 - 8012947783680)t^4 - 890534625280\, y^4 + (-5945425920\, y^4 - 1529956270080\, y^2 -$
$42505831710720)t^3 + (-87199580160\, y^4 - 8371159695360\, y^2 - 136598142320640)t^2 - 28245114224640\, y^2 + (-1509949440\, y^6 -$
$465064427520\, y^4 - 23887400140800\, y^2 - 244321898987520)t - 186336485048320, \quad \mathbf{d_4} = 831409920\, t^8 + 251658240\, y^8 +$
$32517365760\, t^7 + (3762339840\, y^2 + 576892108800)t^6 + 75833016320\, y^6 + (110361968640\, y^2 + 6009710837760)t^5 +$
$(5573836800\, y^4 + 1434334003200\, y^2 + 39849217228800)t^4 + 5106984550400\, y^4 + (108999475200\, y^4 + 10463949619200\, y^2 +$
$170747677900800)t^3 + (2831155200\, y^6 + 871995801600\, y^4 + 44788875264000\, y^2 + 458103560601600)t^2 + 107786901913600\, y^2 +$
$(27682406400\, y^6 + 3339504844800\, y^4 + 105919178342400\, y^2 + 698761818931200)t + 460685574144000, \quad \mathbf{d_3} = -277136640\, t^9 -$
$12194012160\, t^8 - 3690987520\, y^8 + (-1612431360\, y^2 - 247239475200)t^7 + (-55180984320\, y^2 - 3004855418880)t^6 -$
$459024629760\, y^6 + (-3344302080\, y^4 - 860600401920\, y^2 - 23909530337280)t^5 + (-81749606400\, y^4 - 7847962214400\, y^2 -$
$128060758425600)t^4 - 19704840192000\, y^4 + (-2831155200\, y^6 - 871995801600\, y^4 - 44788875264000\, y^2 - 458103560601600)t^3 +$
$(-41523609600\, y^6 - 5009257267200\, y^4 - 158878767513600\, y^2 - 1048142728396800)t^2 - 292272524492800\, y^2 + (-754974720\, y^8 -$
$227499048960\, y^6 - 15320953651200\, y^4 - 323360705740800\, y^2 - 1382056722432000)t - 792542262067200, \quad \mathbf{d_2} =$
$62355744\, t^{10} + 100663296\, y^{10} + 3048503040\, t^9 + (453496320\, y^2 + 69536102400)t^8 + 23488102400\, y^8 + (17736744960\, y^2 +$
$965846384640)t^7 + (1254113280\, y^4 + 322725150720\, y^2 + 8966073876480)t^6 + 1727382159360\, y^6 + (36787322880\, y^4 +$
$3531582996480\, y^2 + 57627341291520)t^5 + (1592524800\, y^6 + 490497638400\, y^4 + 25193742336000\, y^2 + 257683252838400)t^4 +$
$49143821107200\, y^4 + (31142707200\, y^6 + 3756942950400\, y^4 + 119159075635200\, y^2 + 786107046297600)t^3 + (849346560\, y^8 +$
$255936430080\, y^6 + 17236072857600\, y^4 + 363780793958400\, y^2 + 1554813812736000)t^2 + 544467635404800\, y^2 + (8304721920\, y^8 +$
$1032805416960\, y^6 + 44335890432000\, y^4 + 657613180108800\, y^2 + 1783220089651200)t + 891610044825600, \quad \mathbf{d_1} =$
$-8503056\, t^{11} - 457275456\, t^{10} - 738197504\, y^{10} + (-75582720\, y^2 - 11589350400)t^9 + (-3325639680\, y^2 - 181096197120)t^8 -$
$76168560640\, y^8 + (-268738560\, y^4 - 69155389440\, y^2 - 1921301544960)t^7 + (-9196830720\, y^4 - 882895749120\, y^2 -$
$14406835322880)t^6 - 3871845908480\, y^6 + (-477757440\, y^6 - 147149291520\, y^4 - 7558122700800\, y^2 - 77304975851520)t^5 +$
$(-11678515200\, y^6 - 1408853606400\, y^4 - 44684653363200\, y^2 - 294790142361600)t^4 - 71259547238400\, y^4 + (-424673280\, y^8 -$
$127968215040\, y^6 - 8618036428800\, y^4 - 181890396979200\, y^2 - 777406906368000)t^3 + (-6228541440\, y^8 - 774604062720\, y^6 -$
$33251917824000\, y^4 - 493209885081600\, y^2 - 1337415067238400)t^2 - 634836431667200\, y^2 + (-150994944\, y^{10} - 35232153600\, y^8 -$
$2591073239040\, y^6 - 73715731660800\, y^4 - 816701453107200\, y^2 - 1337415067238400)t - 594406696550400, \quad \mathbf{d_0} =$
$, 531441\, t^{12} + 5668704\, t^{10} y^2 + 25194240\, t^8 y^4 + 59719680\, t^6 y^6 + 79626240\, t^4 y^8 + 56623104\, t^2 y^{10} + 16777216\, y^{12} +$
$31177872\, t^{11} + 277136640\, t^9 y^2 + 985374720\, t^7 y^4 + 1751777280\, t^5 y^6 + 1557135360\, t^3 y^8 + 553648128\, t y^{10} + 869201280\, t^{10} +$
$6483317760\, t^8 y^2 + 18393661440\, t^6 y^4 + 23994040320\, t^4 y^6 + 13212057600\, t^2 y^8 + 1677721600\, y^{10} + 15091349760\, t^9 +$
$94595973120\, t^7 y^2 + 211328040960\, t^5 y^4 + 193651015680\, t^3 y^6 + 57126420480\, t y^8 + 180122019840\, t^8 + 944765337600\, t^6 y^2 +$
$1615881830400\, t^4 y^4 + 971652464640\, t^2 y^6 + 108045271040\, y^8 + 1543589498880\, t^7 + 6702698004480\, t^5 y^2 + 8312979456000\, t^3 y^4 +$
$2903884431360\, t y^6 + 9663121981440\, t^6 + 34104449433600\, t^4 y^2 + 27643399372800\, t^2 y^4 + 4160078479360\, y^6 + 44218521354240\, t^5 +$
$123302471270400\, t^3 y^2 + 53444660428800\, t y^4 + 145763794944000\, t^4 + 306263044915200\, t^2 y^2 + 46939294924800\, y^4 +$
$334353766809600\, t^3 + 476127323750400\, t y^2 + 501530650214400\, t^2 + 356725555200000\, y^2 + 445805022412800\, t +$
198135565516800