Original Research Article

PM2.5 prediction of innovation priority discrete nonlinear gray model based on gray wolf optimization algorithm

ABSTRACT:

PM2.5 is one of the main factors of air pollution, so the prediction of PM2.5 is of great significance. For this reason, the innovation priority discrete nonlinear gray model based on gray wolf optimization algorithm is established, which is based on the innovation priority principle in the gray system principle. , Try to optimize the discrete nonlinear gray model, and use the gray wolf optimization algorithm to solve the innovation priority parameters. First, the basic theory of discrete nonlinear gray model is proposed. On this basis, the innovation priority principle is used to improve the cumulative generation sequence, and the cumulative generation with parameters is defined. Finally, using the minimum error criterion, the gray wolf optimization algorithm is used to solve the parameters. Take the monthly PM2.5 data and daily PM2.5 data of Mianyang City, Chengdu City, Zigong City and Panzhihua in Sichuan Province as examples. Apply the model to perform PM2.5 forecast analysis, and calculate the absolute average percentage between the predicted value and the observed value Error, and compare with the traditional gray model. The analysis shows that the established model has achieved good results, which verifies the practicability and reliability of the proposed model.

Key words: PM2.5;Discrete Nonlinear Grey Model;Innovation Priority Principle;Gray Wolf Optimization Algorithm

1. Introduction:

PM2.5 refers to the particulate matter in the ambient air with an aerodynamic equivalent diameter less than or equal to 2.5 microns. It can be suspended in the air for a longer time, and the higher its concentration in the air, the more serious the air pollution. Although PM2.5 is only a small component in the earth's atmosphere, it has an important impact on air quality and visibility. Compared with atmospheric particles with larger diameters, PM2.5 has a small particle size, large area, strong activity, easy to carry toxic and harmful substances, and has a long residence time in the atmosphere and a long transportation distance, which is harmful to human health and atmospheric environment. The impact of quality is greater.

With the rapid development of my country's economy in recent years, people's quality of life has improved significantly. But at the same time, factors such as dense population, industrial agglomeration, and traffic have led to extremely bad air pollution in many cities. Among them, the smog is concentrated in many places, which seriously harms people's lives and the environment, and PM2.5 has become one of the main factors of air pollution. With people’s attention and research on PM2.5 forecasting, a variety of forecasting methods have emerged at home and abroad, such as neural networks, support vector machines, ARMA time series, multiple linear regression, random forest algorithms, grey system models,etc

.Establishing a scientific and reasonable PM2.5 concentration prediction model can provide a guarantee for people's healthy life and provide a reference for environmental management departments.

Gray system theory has been widely used in economics, agriculture, military, ecology and other fields since the famous Chinese scholar Professor Deng Julong proposed in 1982[7-11] .With the continuous growth and development of grey system theory and its practicality in real life, more and more scholars are involved in it. On the basis of the classic grey prediction theory, a large number of scholars continuously optimize and improve the model to increase the scope of application of the model. Among them, Tan Guanjun first proposed the background value concept of the gray GM(1,1) model in 2000, revealing the root cause of the error source of the traditional gray model[12]. Since then, many scholars at home and abroad have successively proposed a series of high-precision background value construction formulas. Among them, the most representative result is the background value construction formula based on discrete inhomogeneous exponential law proposed by Wang Zhengxin et al. (2008), which can make the GM(1, 1) model and pure exponential sequence have complete coincidence[13].The background value transformation method has also been proved to be applicable to other univariate gray models, such as gray Verhulst model [14], non-homogeneous model (NGM) [15] and so on. However, the background value transformation method still has major limitations and is usually not applicable to multivariate models. Xie Naiming and Liu Sifeng first proposed the discrete gray GM(1,1) model in 2005, which can be regarded as an extended form of GM(1,1) and has complete coincidence with pure exponential sequences [16]. The modeling process of the discrete gray model is simpler and the accuracy is higher than that of the traditional model. Therefore, it is widely used in the transformation of various traditional models, such as the discrete NGM model [17], the gray Verhulst model [18], etc. Successfully used to establish a multivariate discrete gray model [19]. Scholars have improved the gray model in many aspects, further increasing the scope of application of the model, but there is still room for improvement.

Grey wolf optimization algorithm[20] is an intelligent optimization algorithm proposed by the scholar Mirjalili of Griffith University in Australia in 2014. The algorithm is inspired by the prey-predation activities of gray wolves and developed an optimized search method. It has the characteristics of strong convergence performance, few parameters, and easy implementation. In recent years, it has received extensive attention from scholars, and it has been successfully applied to the fields of workshop scheduling, parameter optimization, image fusion, etc[21-23].

In summary, regarding the forecast and analysis of PM2.5 at home and abroad in recent years, the focus of scholars is mainly on classic models such as BP neural network, support vector machine and ARMA time series. For the discrete gray model, PM2.5 There is still a lot of digging space in the forecasting research of. At the same time, there are many researches on the improvement of the discrete gray model, but there are still some shortcomings. Based on this situation, this paper proposes an innovation-first discrete nonlinear gray model based on gray wolf optimization algorithm. The improved model has the characteristics of more generalization and stronger adaptability, and can accurately describe and predict the system. This paper uses the innovation priority discrete nonlinear gray model based on the gray wolf optimization algorithm to predict PM2.5 in Mianyang, Chengdu, Zigong, and Panzhihua in Sichuan Province, and conduct a prediction accuracy test to obtain the PM2.5 prediction Practical method.

1. Discrete nonlinear grey model

Definition 1: Let the sequence $X^{(0)} = \left(X^{(0)}(1), X^{(0)}(2), \cdots, X^{(0)}(n)\right)$,$X^{(1)}$ is the 1-AGO sequence of $X^{(0)}$.

which is

$$X^{(1)} = \left(X^{(1)}(1), X^{(1)}(2), \cdots, X^{(1)}(n)\right) \tag{1}$$

Where

$$X^{(1)}(k)=\sum_{i=1}^{k}X^{(0)}(i),k=1,2,\cdots,n \tag{2}$$

Definition 2: Set $X^{(0)},X^{(1)}$ as shown in definition 1, then call

$$\frac{dX^{(1)}}{dt}+aX^{(1)}=bt+c \tag{5}$$

It is the whitening differential equation of the discrete nonlinear gray model.

Theorem 1: The time response of the whitening differential equation of the discrete nonlinear gray model is:

$$X^{(1)}(t+1)=e^{-a}x^{(1)]}(t)+\frac{b}{a}(1-e^{-a})t+(1-e^{-a})\left(\frac{c}{a}-\frac{b}{a^2}\right)+\frac{b}{a} \tag{6}$$

Let $\alpha=e^{-a};\beta=\frac{b}{a}(1-e^{-a});\gamma=(1-e^{-a})\left(\frac{c}{a}-\frac{b}{a^2}\right)+\frac{b}{a}$,then

$$X^{(l)}(t+1)=\alpha X^{(1)}(t)+\beta t+\gamma \tag{7}$$

Proof 1: Solve the albino differential equation using the constant variation method.

The homogeneous equation corresponding to formula (5) is:

$$\frac{dX^{(1)}(t)}{dt}+aX^{(1)}(t)=0\Rightarrow\frac{dX^{(1)}(t)}{dt}=-aX^{(1)}(t)$$

The general solution is:

$$X^{(1)}(t)=C_1e^{-at}$$

Using the constant variation method, the C_1 in the above formula can be replaced with $u(t)$, and it is easy to get:

$$u(t)=\frac{b}{a}te^{at}-\frac{b}{a^2}e^{at}+\frac{h}{a}e^{at}+C$$

Therefore

$$X^{(1)}(t)=\frac{b}{a}t-\frac{b}{a^2}+\frac{c}{a}+Ce^{-at}$$

When t=1,, $X^{(1)}(1)=X^{(0)}(1)$,therefore

$$C=\frac{x^{(1)}(1)-\frac{b}{a}+\frac{b}{a^2}-\frac{c}{a}}{e^{-a}}$$

Which is

$$X^{(1)}(t+1)=e^{-a}X^{(1)}(t)+\frac{b}{a}(1-e^{-a})t+(1-e^{-a})\left(\frac{c}{a}-\frac{b}{a^2}\right)+\frac{b}{a}$$

Proved

Theorem 2: Set$X^{(0)},X^{(1)}$as shown in Definition 1, then the parameters of the discrete nonlinear gray model $\hat{a}=[\alpha,\beta,\gamma]^T$ satisfies:

$$\begin{aligned}
&1)\rightarrow\text{If }m=N+1,|B|\neq0,\text{then }\hat{a}=B^{-1}Y;\\
&2)\rightarrow\text{If }m>N+1,|B^TB|\neq0,\text{then }\hat{a}=(B^TB)^{-1}B^TY;\\
&3)\rightarrow\text{If }m<N+1,|B^TB|\neq0,\text{then }\hat{a}=B^T(B^TB)^{-1}Y;
\end{aligned} \tag{8}$$

Where

$$B=\begin{bmatrix}X^{(1)}(1) & 2 & 1\\ X^{(1)}(2) & 3 & 1\\ \vdots & \vdots & \vdots\\ X^{(1)}(n-1) & n & 1\end{bmatrix},Y=\begin{bmatrix}X^{(1)}(2)\\ X^{(1)}(3)\\ \vdots\\ X^{(1)}(n)\end{bmatrix} \tag{9}$$

Proof 2: When $k=2,3\ldots n$, from equation 7 there is:

$$\begin{cases} x^{(1)}(2) = \alpha x^{(1)}(1) + 2\beta + \gamma \\ x^{(1)}(3) = \alpha x^{(1)}(2) + 3\beta + \gamma \\ \vdots \\ x^{(1)}(n) = \alpha x^{(1)}(n-1) + n\beta + \gamma \end{cases}$$

In matrix form: $Y = B\hat{a}$

1) If m=N+1 and $|B| \neq 0$ then $\hat{a} = B^{-1}Y$;

2) If m>N+1 and $|B^T B| \neq 0$ then $\hat{a} = \left(B^T B\right)^{-1} B^T Y$;

3) When m<N+1, B is a column full rank matrix, and $|B^T B| \neq 0$, $\hat{a} = B^T \left(B^T B\right)^{-1} Y$.

According to the solution of the above formula α, β, γ, Therefore it can be concluded that $a = -\ln\alpha, b = \frac{\alpha\beta}{1-\alpha}, c = \frac{\alpha\gamma - b}{1-\alpha} + \frac{b}{a}$, the estimated parameter value is brought into formula (7) and a cumulative reduction is performed Simulation and prediction of raw value series.

2.Innovation priority discrete nonlinear gray model based on gray wolf optimization algorithm

In this paper, considering the structure of the model and the predictive effect, in order to meet the needs of complex system changes in real life, the gray action quantity in the traditional discrete gray model is changed from a constant term to a one-time term, and new information is defined in combination with the principle of new information priority. Priority accumulation is generated, and the gray wolf optimization algorithm is used to solve the innovation priority parameters.

Definition 3 Set $X^{(0)}$ as shown in Definition 1, let:

$$\begin{aligned} &X^{(1)}(1) = X^{(0)}(1) \\ &X^{(1)}(2) = \lambda X^{(0)}(1) + X^{(0)}(2) \\ &X^{(1)}(3) = \lambda^2 X^{(0)}(1) + \lambda X^{(0)}(2) + X^{(0)}(3) \\ &\cdots \\ &X^{(1)}(n) = \lambda^{n-1} X^{(0)}(1) + \lambda^{n-2} X^{(0)}(2) + \cdots + X^{(0)}(n) \end{aligned} \tag{10}$$

Which is

$$x^{(1)}(k) = \sum_{i=1}^{k} \lambda^{k-i} X^{(0)}(i) \tag{11}$$

Among them $\lambda \in (0,1)$, λ is called the new information priority parameter, which is used as a parameter to adjust the weight of the new and old information in the sequence generation process. The one-time innovation priority accumulation sequence (1-NIPAGO) called $X^{(1)}$ is called $X^{(0)}$.

Its cumulative reduction value is:

$$x^{(0)}(k+1) = x^{(1)}(k+1) - \lambda x^{(1)}(k), k = 1,2 \ldots n \tag{12}$$

Definition 4: Set $X^{(0)}$ and $Z^{(1)}$ as shown in Definition 1, and $X^{(1)}$ as shown in Definition 3. Then it is called

$$\frac{dX^{(1)}}{dt} + aX^{(1)} = bt + c \tag{13}$$

It is the whitening differential equation of the innovation-first discrete nonlinear gray model based on the gray wolf optimization algorithm, referred to as GWO_NIPDNGM(1,1).

Theorem 3: The time response of the whitening differential equation of GWO_NIPDNGM(1,1) is:

$$X^{(1)}(t+1) = e^{-a}x^{(1)]}(t) + \frac{b}{a}(1-e^{-a})t + (1-e^{-a})\left(\frac{c}{a} - \frac{b}{a^2}\right) + \frac{b}{a} \tag{14}$$

Let $\alpha = e^{-a}; \beta = \frac{b}{a}(1-e^{-a}); \gamma = (1-e^{-a})\left(\frac{c}{a} - \frac{b}{a^2}\right) + \frac{b}{a}$,then

$$X^{(l)}(t+1) = \alpha X^{(1)}(t) + \beta t + \gamma \tag{15}$$

The proof process is the same as Theorem 1.

Theorem 4: Set $X^{(0)}$ as shown in Definition 1, and $X^{(1)}$as shown in Definition 3. Then the parameter $\hat{a} = [\alpha, \beta, \gamma]^T$ of GWO_NIPDNGM(1,1) satisfies:

$$\begin{aligned} &1) \rightarrow \text{If } m = N+1, |B| \neq 0, \text{then } \hat{a} = B^{-1}Y; \\ &2) \rightarrow \text{If } m > N+1, |B^T B| \neq 0, \text{then } \hat{a} = (B^T B)^{-1} B^T Y; \\ &3) \rightarrow \text{If } m < N+1, |B^T B| \neq 0, \text{then } \hat{a} = B^T (B^T B)^{-1} Y; \end{aligned} \tag{16}$$

among them:

$$B = \begin{bmatrix} X^{(1)}(1) & 2 & 1 \\ X^{(1)}(2) & 3 & 1 \\ \vdots & \vdots & \vdots \\ X^{(1)}(n-1) & n & 1 \end{bmatrix}, Y = \begin{bmatrix} X^{(1)}(2) \\ X^{(1)}(3) \\ \vdots \\ X^{(1)}(n) \end{bmatrix} \tag{17}$$

The proof process is the same as Theorem 2.

The gray wolf optimization algorithm is an optimization search method developed by the gray wolf that is inspired by the prey-predation activities of gray wolves. It has the characteristics of strong convergence performance, few parameters, and easy implementation. Gray wolves have a very strict social hierarchy, as shown in Figure 1.

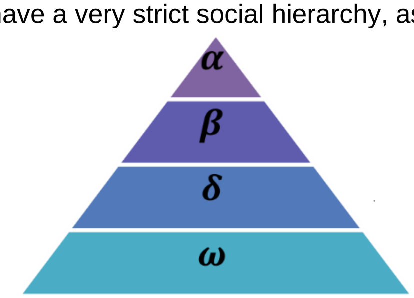


Figure 1 :The gray wolf optimization algorithm

Grey wolf hunting includes the following three main parts: tracking, chasing and approaching prey; hunting, surrounding and harassing prey until it stops moving; attacking prey.

According to the innovation priority discrete nonlinear model, one can get a result. In order to find the optimal, the average absolute percentage error of the model is calculated, and the innovation priority parameter is solved by the gray wolf optimization algorithm using the minimum error criterion.

$$MAPE = \frac{1}{n}\sum_{i=1}^{n}\left|\frac{true\ value - predicted\ value}{predicted\ value}\right| * 100\% \tag{18}$$

3.Case analysis

PM2.5 is derived from the residues emitted by combustion in the processes of daily power generation, industrial production, and automobile exhaust emissions. Mianyang is the only science and technology city in China approved by the Party Central Committee and the State Council. It is the second largest economy in Sichuan and Chengdu. The regional central city of Chongqing City Group, an important national defense scientific research and electronic industry production base; Chengdu is an important central city in western China approved by the State Council, an important national high-tech industrial base, a commercial logistics center and a comprehensive transportation hub; Panzhihua Iron and Steel is an important central city in western China. The largest and important iron and steel production base in western China, the largest vanadium product and railway steel production base in China, the largest titanium raw material production base in China, the only chlorinated titanium dioxide production base in China, and the second largest vanadium producer in the world. Zigong is one of the earliest provincial cities and industrial towns in Sichuan Province; as a well salt center in China, it has now developed into an industrial city with multiple pillar industries; Mianyang, Chengdu, Panzhihua, and Zigong are among the relatively large-scale cities in Sichuan Province. Large cities with rapid economic development. Therefore, collect PM2.5 data of the above cities, and apply the innovation-first discrete nonlinear gray model based on the gray wolf optimization algorithm to the urban PM2.5 prediction research, including 4 The data is used as the basic data to build a model, and the fifth to eighth data are used to calculate the innovation priority parameters, and the latter two data are used as predictions and compared with the traditional gray models GM(1,1) and DGM(1,1).

3.1 Forecast data

The PM2.5 values of Mianyang City, Chengdu City, Zigong City and Panzhihua City of Sichuan Province for ten consecutive months and ten consecutive days were randomly selected for prediction research.

Table 1 Monthly content of PM2.5

Monthly content of PM2.5				
Time	Chengdu	Mianyang	Zigong	Panzhihua
2018.01	80	76	93	40
2018.02	64	46	87	33
2018.03	58	54	59	34
2018.04	45	41	50	29
2018.05	33	30	36	23
2018.06	25	23	26	21
2018.07	20	15	20	24
2018.08	31	21	29	28
2018.09	22	19	25	27
2018.10	42	37	46	28

Table 2 Daily content of PM2.5

Daily content of PM2.5				
Time	Chengdu	Mianyang	Zigong	Panzhihua
2021.02.11	106	99	137	37
2021.02.12	122	120	190	30
2021.02.13	52	36	76	38
2021.02.14	49	37	68	33

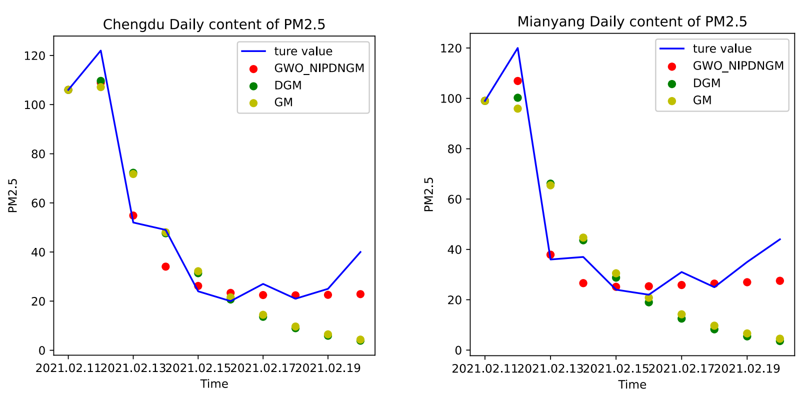
2021.02.15	24	24	41	39
2021.02.16	20	22	28	33
2021.02.17	27	31	44	32
2021.02.18	21	25	40	33
2021.02.19	25	35	44	36
2021.02.20	40	44	56	32

3.2 Forecast data

The PM2.5 values of Mianyang City, Chengdu City, Zigong City, and Panzhihua City of Sichuan Province for ten consecutive months were used as the independent variable sequence. The GWO_NIPDNGM(1,1) model is used to predict the settlement. For the PM2.5 in Chengdu and Panzhihua, the relative errors between the predicted values of the three models and the observed values are shown in Table 3. The data and observations of the three models are shown in Figure 2.

Table 3 Forecast results of PM2.5 monthly data

Time	**Chengdu**				**Panzhihua**			
	Relative value	**GWO-NIPDNGM (1,1)**	**DGM (1,1)**	**GM (1,1)**	**Relative value**	**GWO-NIPDNGM (1,1)**	**DGM (1,1)**	**GM (1,1)**
2018.01	99	0.0000	0.0000	0.0000	40	0.0000	0.0000	0.0000
2018.02	120	0.5209	1.3835	0.9278	33	-2.1528	-0.2168	-0.3888
2018.03	36	-8.0645	-4.2286	-4.4502	34	-6.0598	-3.2019	-3.3031
2018.04	37	-5.1302	-0.7784	-0.8342	29	-3.0189	-0.0668	-0.1050
2018.05	24	-0.0558	3.3679	3.4262	23	1.7701	4.1812	4.1989
2018.06	22	3.2004	4.9090	5.0428	21	3.1494	4.5352	4.6023
2018.07	31	4.9725	4.5971	4.7781	24	-0.0056	-0.0110	0.0995
2018.08	25	-8.2022	10.7713	-10.5640	28	-3.7929	-5.4636	-5.3152
2018.09	35	-0.6452	-5.3639	-5.1452	27	-2.2902	-5.8283	-5.6468
2018.10	44	-21.5798	-28.3185	-28.0988	28	-2.5586	-8.1103	-7.9002
MAPE		18.0906	23.8202	23.7234		11.0799	15.0238	14.9941



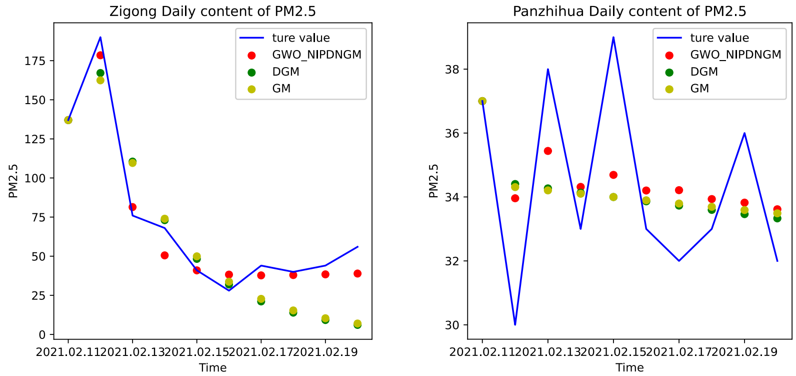


Figure 2 PM2.5 monthly data

Table 4 respectively predicts the monthly PM2.5 content of Chengdu and Panzhi. It is found that the MAPE value of the GWO_NIPDNGM(1,1) model is the smallest, which is 18.090571 for Chengdu and 11.079855 for Panzhihua; as can be seen from Figure 2, GWO_NIPDNGM (1,1) The predicted value obtained by the model is closer to the observed value curve, and the error is smaller. However, the predicted values of the GM(1,1) model and the DGM(1,1) model are far from the observed values, and the errors are large. To sum up, the GWO_NIPDNGM(1,1) model has a relatively small relative error between the predicted value and the observed value. The prediction effect is better than GM(1,1) model and DGM(1,1) model.

The PM2.5 values of Mianyang City, Chengdu City, Zigong City, and Panzhihua City of Sichuan Province for ten consecutive days were used as the independent variable sequence. The GWO_NIPDNGM(1,1) model is used to predict the settlement. For PM2.5 in Mianyang City and Zigong City, the relative errors between the predicted values of the three models and the observed values are shown in Table 4. The data and observations of the three models are shown in Figure 3.

Table 4 Forecast results of PM2.5 monthly data

Time	**Mianyang**				**Zigong**			
	Relative value	**GWO-NIPDNGM (1,1)**	**DGM (1,1)**	**GM (1,1)**	**Relative value**	**GWO-NIPDNGM (1,1)**	**DGM (1,1)**	**GM (1,1)**
2021.02.11	76	0.0000	0.0000	0.0000	137	0.0000	0.0000	0.0000
2021.02.12	46	-13.8946	-19.7704	-24.0893	190	-1.1359	-22.8901	-27.5856
2021.02.13	54	1.7428	30.1258	29.4642	76	5.4916	34.4858	33.6136
2021.02.14	41	-10.4798	6.6260	7.6829	68	-1.7417	5.0484	5.9783
2021.02.15	30	1.0778	4.7819	6.4985	41	-1.5289	7.2965	8.9280
2021.02.16	23	3.3094	-3.0113	-1.1832	28	1.0279	3.9315	5.6964
2021.02.17	15	-5.1724	-18.4724	-16.7914	44	-6.2293	-22.8883	-21.2583
2021.02.18	21	1.3947	-16.7350	-15.3018	40	-2.0256	-26.0418	-24.6516
2021.02.19	19	-8.0296	-29.5472	-28.3805	44	-5.5948	-34.7715	-33.6413
2021.02.20	37	-16.4525	-40.4026	-39.4818	56	-1.7089	-49.8985	-49.0089
MAPE		18.3594	56.8064	55.1663		17.2488	47.7424	47.9469

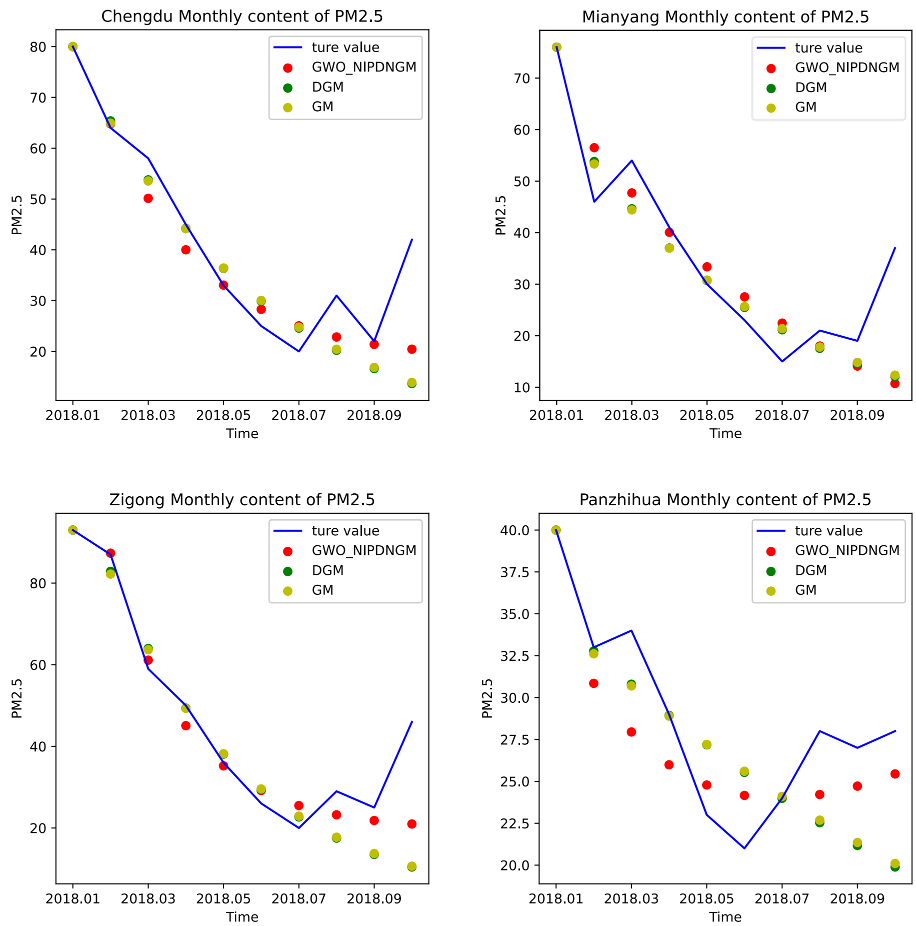


Figure 3 PM2.5 daily data

Table 3 predicts the daily PM2.5 content of the two cities in Zigong and Mianyang. It is found that the MAPE value of the GWO_NIPDNGM(1,1) model is also the smallest, 18.359481 in Mianyang and 17.248784 in Zigong; as can be seen from Figure 3, GWO_NIPDNGM(1,1) The predicted value obtained by the model is closer to the observed value curve, and the error is smaller. Compared with GM(1,1) model and DGM(1,1) model, the relative error is smaller.

In summary, combining the prediction errors of the monthly PM2.5 content and the daily PM2.5 content of the four cities respectively, it is concluded that the predicted value of the GWO-NIPDNGM(1,1) model is closer to the true value, and the prediction effect is excellent. In DGM(1,1), GM(1,1) models.

4. Conclusion

This article is carried out in-depth from theoretical research to practical application, and carries out detailed planning and design. Among them, theoretical research includes theoretical research on discrete nonlinear gray model, new information priority accumulation generation and gray wolf optimization algorithm. It mainly discusses the optimization method of discrete nonlinear gray model and the application of gray wolf optimization algorithm, focusing on innovation Prioritize the structure and attributes of the discrete nonlinear gray model; apply the innovation priority discrete nonlinear gray model based on the gray wolf optimization algorithm to predict PM2.5 in Mianyang, Chengdu, Zigong, and Panzhihua in Sichuan Province, and Comparing with GM(1,1) model and DGM(1,1) model, it can be seen that the established model has better prediction effect and higher reliability, which provides an effective method for future prediction analysis.

References

[1]H Yuxiao, D Xianming. Research on PM_(2.5) diffusion and prediction based on Gaussian plume and multiple linear regression model[J]. Arid Land Resources and Environment, **29(06)**: 86-92.(2015)

[2]P Sijun, S Jiachao, Z Xue. PM_(2.5) prediction based on ARIMA model[J]. Safety and Environmental Engineering, 21(06): 125-128. (2014)

[3] D Xu, F Jingyu, L Shaoqing, S Wei. PM2.5 concentration prediction model based on random forest regression analysis [J]. Telecommunications Science, **33(07)**: 66-75.(2017)

[4]G Qijia, Y Yibin, Z Yongjiang. PM2.5 random forest prediction model fused with GNSS meteorological parameters[J/OL]. Science of Surveying and Mapping:,**1-7,**http://kns.cnki. net/kcms/detail/11.4415.P.20210126.1047.016.html.**,**(2021-03-22).

[5]F Yingying, Z Feng, D Zhenhong, L Renyi. Multi-step PM_(2.5) hour concentration prediction based on graph convolutional neural network and attention mechanism[J]. Journal of Zhejiang University (Science Edition), **48(01)**): 74-83.(2021)

[6]G Ming, Y Chunming. Shanghai PM2.5 concentration prediction based on modified grey Markov chain[J]. Journal of Natural Disasters, **25(05)**: 97-104.(2016)

[7] Z H Fei, Z Y Jiang, Z F He. Discrete GM(1,1) model and its application for forecasting of real estate prices[C]// International Conference on Management Science & Industrial Engineering. (2011).

[8] D Q Truong, K K Ahn . An accurate signal estimator using a novel smart adaptive grey model SAGM(1,1)[J]. Expert Systems with Applications, **39(9)**:7611-7620.(2012)

[9]LI, Junliang, LIAO, et al. Generalized accumulated GM(1,1) cosine model and its application[C]// IEEE International Conference on Grey Systems & Intelligent Services. (2011).

[10]Ou S L. Forecasting agricultural output with an improved grey forecasting model based on the genetic algorithm[J]. Computers & Electronics in Agriculture, **85(85)**:33-39.(2012)

[11]Hsu L C, Wang C H. Forecasting integrated circuit output using multivariate grey model and grey relational analysis[J]. Expert Systems with Applications, **36(2)**:1403-1409.(2009)

[12]T Guanjun. The background value construction method and application of GM (1, 1) model [J]. System Engineering Theory and Practice, **20(4)**: 99-103.(2000)

[13]W Zhengxin, D Yaoguo, L Sifeng. GM(1,1) model based on discrete exponential function optimization [J]. Systems Engineering Theory and Practice, **28(2)**: 61-67.(2008)

[14]X Pingping, D Yaoguo, Y Tianxiang, C Jie. Research on the modeling method of gray Verhulst model background value optimization[J]. China Management Science, **20(6)**: 154-159.(2012)

[15]T Mingyu, Z Xiaohua, Z Bo. Grey NGM(1,1,k) model background value optimization method[J]. Control and Decision, **32(03)**: 507-514.(,2017)

[16]X Naiming, L Sifeng. Discrete GM(1,1) model and grey prediction model modeling mechanism[J]. Systems Engineering Theory and Practice, **25(1)**:93-99.(2005)

[17]C Lizhi, L Sifeng, L Zhiping. Grey Discrete Verhulst Model[J]. Systems Engineering and Electronic Technology,**33(3)**: 590-593.(2011)

[18]X, N-Ming, et al. "On novel grey forecasting model based on non-homogeneous index sequence." Applied Mathematical Modelling 37.7 : 5059-5068.(2013)

[19] X Naiming, L Sifeng. Multivariate discrete grey model and its properties[J]. Systems Engineering Theory and Practice, **(06)**:143-150+165.(2008)

[20]S. Mirjalili, S. M. Mirjalili, A. Lewis, Grey Wolf Optimizer, Advances in Engineering Software, vol. 69, pp.46-61, (2014)

[21]J Tianhua. Hybrid gray wolf optimization algorithm for flexible job shop scheduling problem[J]. Control and Decision, **33(03):** 503-508.(2018)

[22]Z Jian, Z Chengwen, X Yi, L Yinhong, D Xianzhong. Optimal design of multi-machine power system stabilizer parameters based on gray wolf optimization algorithm[J]. Power System Technology, **41(09)**: 2987-2995.(2017)

[23]L Yilun, X Xihao. Infrared and visible image fusion based on improved gray wolf optimization[J]. Infrared Technology,**41(11)**: 1017-1024.(2019)

[24]W Zhengxin. Grey multivariable GM (1, N) power model and its application [J]. Systems engineering theory and practice,**34 (09)**: 2357-2363.(2014).