

A Mathematical Model of Poverty and Cybercrime Dynamics in South-South, Nigeria

By

Abstract

Poverty and cybercrime are two topical concepts that have attracted a lot of interest in Nigeria and in the global community today. This study used the compartmental mathematical modelling approach to provide a comprehensive framework for analyzing the dynamics of poverty and its relationship with cybercrime in the South-South region of Nigeria. The model was derived from a five-compartment representation from which a set of five ordinary differential equations (ODEs) was developed, which captures its dynamism. The stability analysis and determination of the conditions for stability at steady state of the system were done using the Routh Array criterion. The ODEs were solved numerically using the fourth-order Runge Kutta method. Findings reveals that intervention programmes at different stages can help decrease the population in both impoverished and cybercrime compartments in the region.

Keyword: Compartmental Model, Poverty, Cybercrime, Stability Analysis, Ordinary Differential Equation, Runge-Kutta Method.

1. Introduction

The issue of poverty has always been a subject of interest for both developed and developing nations in the world. A great amount of efforts and resources are invested annually by government and non-governmental agencies to tackle the menace of its scourge. Instead of abating, it is rather expanding. Fields (1994) defines poverty as the inability, by individuals and communities, to command sufficient resources to satisfy basic needs and necessities of life. The United Nations (1998) explained the effects of poverty to include denial of decent choices and opportunities, violation of human worth and dignity, and lack of capacity to participate meaningfully and effectively in society. Cybercrime, on the other hand, is a category of criminal conduct that entails the use of computers and other network instruments to cheat or to hurt other people. They include phishing, identity theft, hacking, hijacking, stalking, viruses and malware, software piracy, advocating terrorism, and pornography. Most cybercrimes are committed by those who want to make money,

and this has proved to be attractive to youths as a means of acquiring wealth. Ogunleye and Oloyede (2020) identify poverty as a key factor for cybercrime using different socioeconomic factors in Nigeria. Their study's main point is that poverty is among the main factors that lead to an increased incidence of cybercrime. Those who have limited opportunities to make a living become involved in criminal activities for monetary reward. By doing a statistical analysis, the authors explain how the rates of unemployment, education level, and income determine the increase in cybercrimes. Nwagbara and Nwafor (2021) explore the socioeconomic factors contributing to cybercrime among youths in Lagos State, Nigeria. Their studies reveal certain economic conditions correlate with the incidence of cybercrimes among young people. The authors add that increased cases of unemployment and poverty ensure youths participate in cybercrime so as to generate an income. In turn, they employ both qualitative and quantitative approaches to show that another variable is the educational background, and the possibility of using a technical tool also defines the extent of youth engagement in cybercriminality acts, while in 2022 Adebayo and Adepoju examine the correlation between youth unemployment and cybercrime in Nigeria; policy imperatives arising from the lesson learned. The authors claim that the high rate of unemployment of the young people leads to the propensity of the young people to indulge in cybercrimes since they look for other ways of making their living. Using qualitative and quantitative methodologies integrated into the present research, one realizes that the lack of employment opportunities makes youths easily becoming fertile ground for recruiters to engage them into the business of cybercrimes as a way of survival. Adesina (2017) concluded that there was a strong relationship between poverty and cybercrime in Nigeria, while Alabi et al (2023) also identified poverty as well as peer pressure as major determinants of youth involvement in cybercrime in Nigeria. Ugwuanyi et al. (2020) identified common cybercrimes in Nigeria to include ATM fraud, phishing, and identity theft and observed that a sizeable proportion of internet users in Nigeria have encountered cybercrimes of varying degrees. The SIR class of epidemiological models has been used extensively to simulate the effect of intervention programmes on the dynamics of a disease. For instance, Aghanenu et al. (2022) simulated the effect of vaccines on the spread of COVID-19, while Urumese and Igabari (2023) investigated the effect of social distancing and community lockdown on the spread of the pandemic. Compartmentalized epidemiological models have been adapted by several researchers to explain the spread of social diseases such as poverty, and such mathematical models have been instrumental in analyzing, predicting, and suggesting strategies to minimize poverty and its associated social vices. Rao, Keshri, Mishra, and Panda in 2019 developed a differential e-epidemic model to analyze Distributed Denial of Service (DDoS) attacks on computer networks, particularly in sectors of greatest importance. Their model replicates the propagation process of DDoS attacks toward the distinguished resources and also illustrates the weaknesses inside the interconnected systems, which are crucial for the infrastructure. This paper contributes to our understanding of DDoS attack dynamics by conceptualizing them as viruses that spread through systems networks and using this analysis to help construct better defense measures for critical networked assets. Thus, this approach is more applicable when considering threat in the context of some system and is conducive to the notion of building effective defense mechanisms in the cyber sphere, and also in 2023, Rao, Rauta, Kund, Sethi, and Behera (2023) developed a mathematical model to investigate the behaviour of Distributed Denial of Service (DDoS) attacks within computer networks. This area has become increasingly important due to the ongoing threats posed by DDoS attacks to cybersecurity. This work relates to their model in revealing the features of the network traffic flow during an attack that could be used in presenting patterns of such attacks. Notably, the authors defined certain quantitative measures concerning network traffic, which it is believed might serve as early warning indicators of a DDoS attack. In a study on poverty and drug addiction in Bangladesh, Sakib, Islam, Shahrear, and Habiba (2017) developed a compartmental model to investigate the impact of governmental and non-governmental intervention programmes. Their model classified the population into five groups: non-impooverished, impooverished, drug-

addicted, rehabilitated, and recovered. Their findings suggested that intervention programs could cut poverty and drug addiction incidences. Based on the above study, Islam, Sakib, Shahrear, and Rahman (2017) added the aspect of snatching into their modelling and analyzed the impact of intervention programmes on the rate of poverty, drug addiction, and snatching. From their simulation results, they found out that defensive intervention programmes from the government and non-government organizations, religious organizations, and individuals could curb these problems. In Nigeria, Oduwole and Shehu (2013) created a compartmental model for poverty and prostitution. Their study considered a population divided into five groups: non-prostituted, impoverished, prostituted, disease-infected, and rehabilitated. The outcomes of the studies established that intervention programs could help reduce both prostitution and poverty levels. The same population classification was used by Akinpelu and Ojo (2016) but with different flow diagrams; according to them, government intervention is highly important in mitigating the occurrence of poverty. In West Malaysia, Roslan, Zakaria, Alias, and Malik (2018) fostered a three-compartment mathematical model of impoverished, poor, and criminal. According to their simulation, the results showed that a high level of government interference decreased both crime and poverty. Having analyzed the existing bibliography and bearing in mind the direct correlation between poverty and cybercrime, the purpose of the current study is to construct a compartmental model that will identify the connections between poverty and cybercrime and explore strategies for their massive reduction.

2. Model Formulation

Putting a total end to cybercrime is almost impossible but, intensifying interventions in order to control poverty rate in a bid to lower the rate of cybercrime is possible. This model considered five classes of people in the South South region of Nigeria. The classes considered includes: non-impoverished class $N(t)$, impoverished class $P(t)$, cybercrime class $C(t)$, jailed class $J(t)$, and rehabilitation class $R(t)$. The selected classes are dynamic in nature, i.e., they are meant to change with time. Based on this fact, sets of first order ordinary differential equations were developed from Figure 1, which are shown in equations 1 to 5. Aside those five model variables listed above, there are also model parameters in Figure 1. The detailed description of the model variables and parameters are as shown in Tables 1 and 2, respectively.

In regard to the dynamics between the different classes, the following dynamics have been assumed in the model. First, people will transit from the non-impoverished class $N(t)$ to impoverished class $P(t)$ at the rate of β , due to unemployment or low paying jobs and with some policy influences from the government. Secondly there is a high rate of transition of individuals into the cybercrime class $C(t)$ due to poverty at the rate of γ . Furthermore, with a probability rate of α , some of the people in the non-impoverished class $N(t)$ may also join the cybercrime class $C(t)$ a situation that is however rare. In addition, members of the cybercrime class $C(t)$ risk arrests and incarceration with a probability of ϕ . Finally, those who have been rehabilitated may go back into perpetrating cybercrime at the rate of ω , due to interactions with members of the cybercrime class.

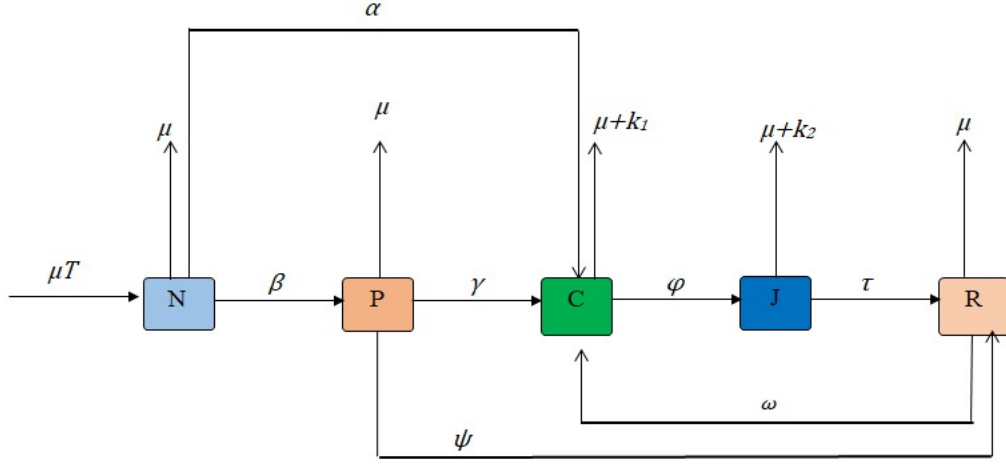


Figure 1: Flow diagram for the dynamic behaviour of the system

The variables and parameters used in the model are defined in Tables 1 and 2 respectively

Table 1: Description of the Model Variables

Variables	Description
$N(t)$	Non-impovertised class
$P(t)$	Impovertised class
$C(t)$	Cybercrime class
$J(t)$	Jailed class
$R(t)$	Rehabilitation class
$T(t)$	Total Population

Table 2: Description of the Model Parameters

Parameter	Description
β	Rate of movement from non-impovertised class $N(t)$ to impovertised class $P(t)$
α	Rate at which individuals in the non-impovertised class $N(t)$ engage cybercrime $C(t)$
γ	Rate at which individuals in the impovertised class $P(t)$ get involved in cybercrime $C(t)$
φ	Rate at which people in the cypercrime class $C(t)$ get caught by law enforcement agencies and moved to jailed class $J(t)$
τ	The rate at which individuals in the Jailed class $J(t)$ moved to rehabilitation class $R(t)$
ω	Transition rate from rehabilitation class $R(t)$ to cybercrime class $C(t)$
ψ	The rate of transition from impovertised class $P(t)$ to rehabilitation class $R(t)$
k_1	Death rate due to unnatural death in the cybercrime class $C(t)$
k_2	death rate due to gun duel in the jailed class $J(t)$
μ	Birth/Death rate

$$\frac{dN}{dt} = \mu T - \frac{\alpha NC}{T} - (\beta + \mu)N \quad (1)$$

$$\frac{dP}{dt} = \beta N - \frac{\gamma PC}{T} - (\psi + \mu)P \quad (2)$$

$$\frac{dC}{dt} = \frac{\gamma PC}{T} + \frac{\alpha NC}{T} + \frac{\omega RC(\alpha + \gamma)}{T} - (\varphi + \mu + k_1)C \quad (3)$$

$$\frac{dJ}{dt} = \varphi C - (\tau + \mu + k_2)J \quad (4)$$

$$\frac{dR}{dt} = \tau J - \frac{\omega RC(\alpha + \gamma)}{T} + \psi P - \mu R \quad (5)$$

3. Mathematical analysis of the Model at Steady State

A steady state in Mathematical modelling refers to a condition where the variables of a system remain constant over time, despite ongoing processes that strive to change them. This means that the system has reached a point of equilibrium where the rates of input and output are balanced. This simply means that $\frac{dA}{dt} = 0$, $A = N, P, C, J, \text{ and } R$. Subjecting equations 1 to 5 to definition of steady state, gives:

$$0 = \mu T - \frac{\alpha NC}{T} - (\beta + \mu)N = f_1 \quad (6)$$

$$0 = \beta N - \frac{\gamma PC}{T} - (\psi + \mu)P = f_2 \quad (7)$$

$$0 = \frac{\gamma PC}{T} + \frac{\alpha NC}{T} + \frac{\omega RC(\alpha + \gamma)}{T} - (\varphi + \mu + k_1)C = f_3 \quad (8)$$

$$0 = \varphi C - (\tau + \mu + k_2)J = f_4 \quad (9)$$

$$0 = \tau J - \frac{\omega RC(\alpha + \gamma)}{T} + \psi P - \mu R = f_5 \quad (10)$$

To determine if the system is stable under steady state condition, firstly Jacobian matrix have to be defined:

$$JM = \begin{bmatrix} \frac{\partial f_1}{\partial N} & \frac{\partial f_1}{\partial P} & \frac{\partial f_1}{\partial C} & \frac{\partial f_1}{\partial J} & \frac{\partial f_1}{\partial R} \\ \frac{\partial f_2}{\partial N} & \frac{\partial f_2}{\partial P} & \frac{\partial f_2}{\partial C} & \frac{\partial f_2}{\partial J} & \frac{\partial f_2}{\partial R} \\ \frac{\partial f_3}{\partial N} & \frac{\partial f_3}{\partial P} & \frac{\partial f_3}{\partial C} & \frac{\partial f_3}{\partial J} & \frac{\partial f_3}{\partial R} \\ \frac{\partial f_4}{\partial N} & \frac{\partial f_4}{\partial P} & \frac{\partial f_4}{\partial C} & \frac{\partial f_4}{\partial J} & \frac{\partial f_4}{\partial R} \\ \frac{\partial f_5}{\partial N} & \frac{\partial f_5}{\partial P} & \frac{\partial f_5}{\partial C} & \frac{\partial f_5}{\partial J} & \frac{\partial f_5}{\partial R} \end{bmatrix} \quad (11)$$

Substituting equations 6 to 10 into equation 11 gives:

$$\begin{pmatrix} (-\frac{\alpha C}{T} - (\beta + \mu)) & 0 & -\frac{\alpha N}{T} & 0 & 0 \\ \beta & (-\frac{\gamma C}{T} - (\psi + \mu)) & -\frac{\gamma P}{T} & 0 & 0 \\ \frac{\alpha C}{T} & (\frac{\gamma C}{T}) & (\frac{\gamma P}{T} + \frac{\alpha N}{T} + \frac{\omega R(\alpha + \gamma)}{T} - (\varphi + \mu + k_1)) & 0 & \frac{\omega C(\alpha + \gamma)}{T} \\ 0 & 0 & \varphi & -(\tau + \mu + k_2) & 0 \\ 0 & \psi & (-\frac{\omega R(\alpha + \gamma)}{T}) & \tau & (-\frac{\omega C(\alpha + \gamma)}{T} - \mu) \end{pmatrix}$$

Secondly, the characteristics equation has to be defined. Following Islam et al., (2017) approach, the characteristics equation is given as:

$$\left[s - \left(-\frac{\alpha C}{T} - (\beta + \mu) \right) \right] \left[s - \left(-\frac{\gamma C}{T} - (\psi + \mu) \right) \right] \left[s - \left(\frac{\gamma P}{T} + \frac{\alpha N}{T} + \frac{\omega R(\alpha + \gamma)}{T} - (\varphi + \mu + k_1) \right) \right] \\ [s + (\tau + \mu + k_2)] \left[s - \left(-\frac{\omega C(\alpha + \gamma)}{T} - \mu \right) \right] = 0$$

$$\text{Let } \left(-\frac{\alpha C}{T} - (\beta + \mu)\right) = a, \left(-\frac{\gamma C}{T} - (\psi + \mu)\right) = b, \left(\frac{\gamma P}{T} + \frac{\alpha N}{T} + \frac{\omega R(\alpha + \gamma)}{T} - (\varphi + \mu + kS_1)\right) = c, \\ (\tau + \mu + k_2) = d, \left(-\frac{\omega C(\alpha + \gamma)}{T} - \mu\right) = e$$

$$(s - a)(s - b)(s - c)(s + d)(s - e) = 0 \quad (12)$$

Expansion of equation 12 gives:

$$s^5 + (-e + d - c - b - a)s^4 + (-cd - bd + bc - ad + ac + ab - de + ce + ae + be)s^3 + \\ (bcd + acd - abc + cde + bde - bce + ade - ace - abe + abd)s^2 + \\ (-bcde - acde - abde - abde + abce - abcd)s + abcde = 0 \quad (13)$$

Let $-e + d - c - b - a = a_1$, $-cd - bd + bc - ad + ac + ab - de + ce + ae + be = a_2$, $bcd + acd - abc + cde + bde - bce + ade - ace - abe + abd = a_3$, $-bcde - acde - abde - abde + abce - abcd = a_4$, $abcde = a_5$

The characteristics equation becomes:

$$s^5 + a_1s^4 + a_2s^3 + a_3s^2 + a_4s^1 + a_5 = 0 \quad (14)$$

Lastly, we apply the Routh Array method to obtain condition for stability in equation (14)

$$\begin{array}{cccc} s^5 & 1 & a_2 & a_4 \\ s^4 & a_1 & a_3 & a_5 \\ s^3 & b_1 & b_2 & 0 \\ s^2 & b_3 & b_4 & 0 \\ s^1 & b_5 & 0 & 0 \\ s^0 & b_6 & 0 & 0 \end{array}$$

$$b_1 = -\frac{1}{a_1} \begin{vmatrix} 1 & a_2 \\ a_1 & a_3 \end{vmatrix}, b_2 = -\frac{1}{a_1} \begin{vmatrix} 1 & a_4 \\ a_1 & a_5 \end{vmatrix}, b_3 = -\frac{1}{b_1} \begin{vmatrix} a_1 & a_3 \\ b_1 & b_2 \end{vmatrix}, b_4 = -\frac{1}{b_1} \begin{vmatrix} a_1 & a_5 \\ b_1 & 0 \end{vmatrix}, b_5 = -\frac{1}{b_3} \begin{vmatrix} b_1 & b_2 \\ b_3 & b_4 \end{vmatrix}, \\ b_6 = -\frac{1}{b_5} \begin{vmatrix} b_3 & b_4 \\ b_5 & 0 \end{vmatrix}$$

The system is stable if all values of b_i ($i = 1, 2, 3, 4, 5, 6$) are positive. it becomes unstable if any value of b_i is negative.

4. Model Simulation

Data used for the simulation are as shown in Tables 3 and 4. To simulate and analyse the model, data obtained from Nigeria Bureau of Statistics NBS (2019) and field survey for research purpose were used to derived initial value of the model variables (Table 3). Parameters value were assumed after careful comparison with what is obtainable in literature. Results of simulation are illustrated in Figures 2, 3, 4, and 5.

Table 3: Initial value of the model variables

Variables	T	N(0)	P(0)	C(0)	J(0)	R(0)
Values	33980000	25480000	8500000	260780	83084	151660

Source: NBS, (2019), Field Survey.

Table 4: Estimated value of the model parameters

Parameters	β	α	γ	φ	τ	ω	ψ	k_1	k_2
Values	0.078	0.31	0.32	0.22	0.35	0.44	0.09	0.02	0.0071

Source: Assumed.

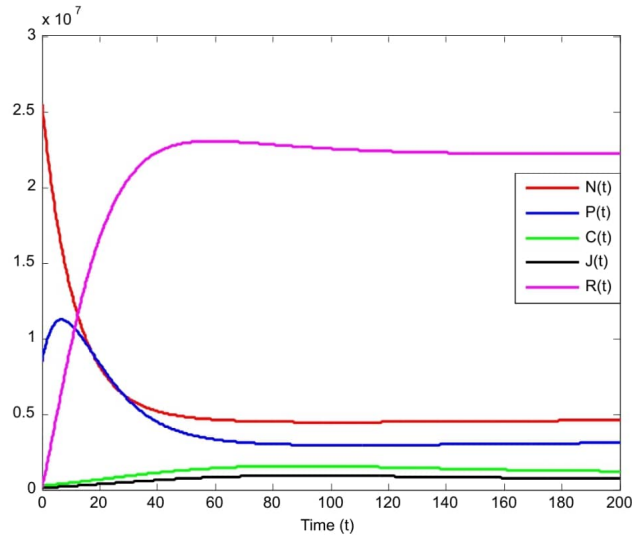


Figure 2: Simulation of different classes under intervention programmes.

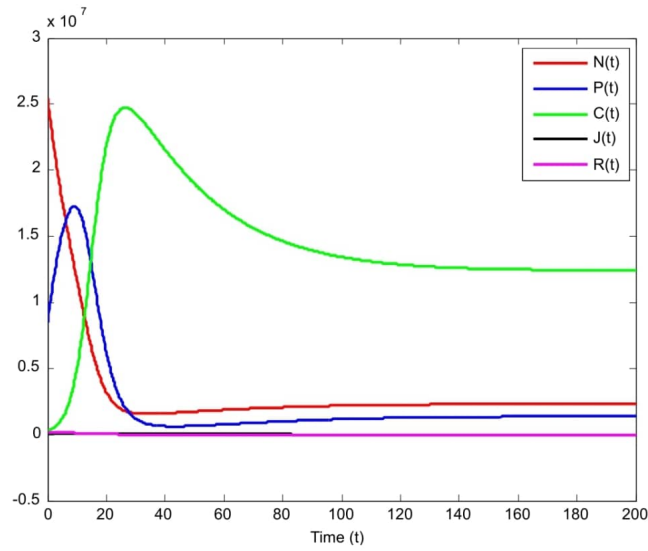


Figure 3: Simulation of different classes without an intervention programme.

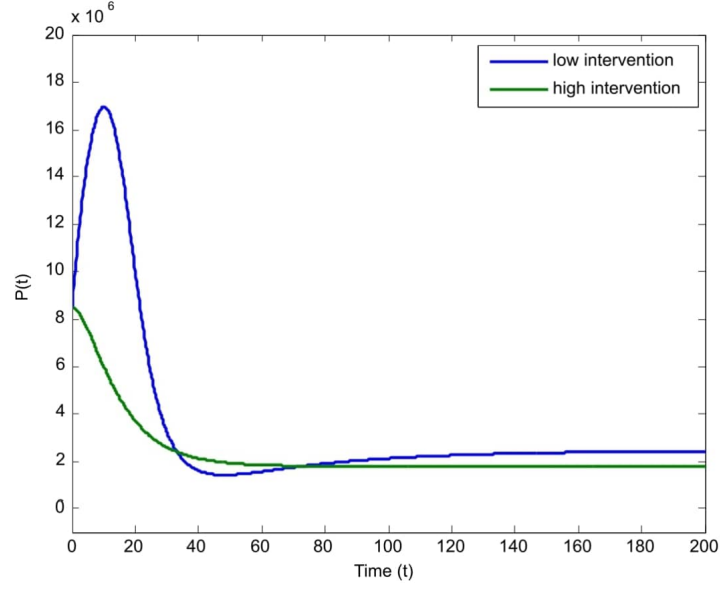


Figure 4: Simulation of $P(t)$ class under low and high intervention programmes.

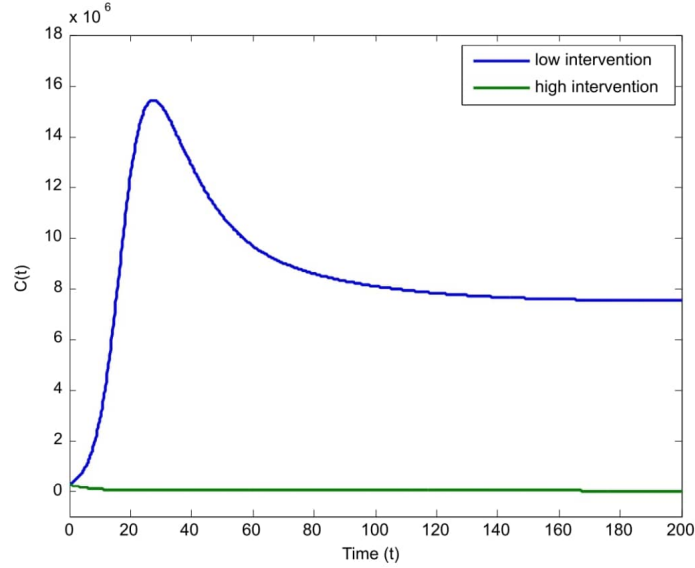


Figure 5: Simulation of $C(t)$ class under low and high intervention programmes.

5. Discussion

The simulation result under an intervention programme is as shown in Figure 2. The three intervention parameters of the model are φ , τ , and ψ and their numeric values are shown in Table 4. The following observations were drawn from Figure 2:

1. The $N(t)$ class reduces over time to a certain point, after which it becomes constant.
2. The $P(t)$ class increases slightly from the initial value, then continuously decreases over time and stabilizes.

3. The $C(t)$ and $J(t)$ classes have almost the same progression. Both classes increase over time for a period and then become stable.
4. The $R(t)$ class increases over time for a period and then becomes stable.

Under non-intervention programme, φ , τ , and ψ were all equal to 0. Simulation result under non-intervention programme is as shown in Figure 3. The $J(t)$ and $R(t)$ classes are 0 in Figure 3 when there was no intervention programme. Comparing Figures 3 with 2, it was revealed that lack of intervention programmes increases the rates of those in impoverished and cybercrime classes. This experiment observed the reason why both governmental and non-governmental organisations in South-South region of Nigeria should invest in well meaning programmes that will mitigate against rise in poverty and cybercrime mostly among the youth. Having shed light on the need for intervention programmes, there is also need to numerically investigate the influence of low and high intervention in the control of poverty and cybercrime. For the low intervention the values of φ , τ , and ψ were assumed to be 0.05, 0.07 and 0.01 respectively while at high intervention the values were 0.42, 0.53 and 0.21 respectively. Figures 4 and 5 revealed that poverty and cybercrime rate can be reduced with the help of intervention programmes.

6. Conclusion

In this study, we developed a compartmental mathematical model that acts as a parameter in formulating an equation framework to analyze the flow and interaction between poverty and cybercrime in the south-south region of Nigeria. Our findings reveal that poverty and cybercrime can be massively reduced in this region provided there are concerted efforts by all stakeholders, including governmental and non-governmental organizations. Effective intervention programmes such as policies and initiatives targeted at raising people out of poverty, are a necessity. Education, skill acquisition, and job placement should deliberately form part of these programmes so that the practitioners reduce their economic incentives to engage in cybercrime. In addition, based on the estimates from our model, rehabilitation programmes for the offenders implicated in cybercrime are crucial in addressing poverty and reducing the incidence of cybercrime. As such, there is an effective process where offenders intervene with adequate means toward a successful reintegration process in society. Such programmes aim at ensuring once released offenders are upheld and they start earning for their livelihood, thus reducing rates of recidivism.

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Appendix

Numerical analysis of the Model

The system flow diagram in *Figure1* was simulated numerically using classical 4th order Runge-Kutta method in MATLAB environment. The algorithm for the simulation is given below:

Step 1: Initial value of $N(1), P(1), C(1), J(1), R(1)$.

Step 2: Choose a step size (h).

Step 3: Find the increment of each population class that make up the model.

$$k_1 = \mu \cdot T(i) - \frac{\alpha \cdot N(i) \cdot C(i)}{T(i)} - (\beta + \mu) \cdot N(i)$$

$$m_1 = \beta \cdot N(i) - \frac{\gamma \cdot P(i) \cdot C(i)}{T(i)} - (\psi + \mu) \cdot P(i)$$

$$p_1 = \frac{\gamma \cdot P(i) \cdot C(i)}{T(i)} + \frac{\alpha \cdot N(i) \cdot C(i)}{T(i)} + \frac{\omega \cdot R(i) \cdot C(i) \cdot (\alpha + \gamma)}{T(i)} - (\varphi + \mu + k_1) \cdot C(i)$$

$$j_1 = \varphi \cdot C(i) - (\tau + \mu + k_2) \cdot J(i)$$

$$r_1 = \tau \cdot J(i) - \frac{\omega \cdot R(i) \cdot C(i) \cdot (\alpha + \gamma)}{T(i)} - \psi \cdot P(i) - \mu \cdot R(i)$$

$$k_2 = \mu \cdot T(i) - \frac{\alpha \cdot (N(i) + \frac{k_1}{2}) \cdot (C(i) + \frac{p_1}{2})}{T(i)} - (\beta + \mu) \cdot (N(i) + \frac{k_1}{2})$$

$$m_2 = \beta \cdot (N(i) + \frac{k_1}{2}) - \frac{\gamma \cdot (P(i) + \frac{m_1}{2}) \cdot (C(i) + \frac{p_1}{2})}{T(i)} - (\psi + \mu) \cdot (P(i) + \frac{m_1}{2})$$

$$p_2 = \frac{\gamma \cdot (P(i) + \frac{m_1}{2}) \cdot (C(i) + \frac{p_1}{2})}{T(i)} + \frac{\alpha \cdot (N(i) + \frac{k_1}{2}) \cdot (C(i) + \frac{p_1}{2})}{T(i)} + \frac{\omega \cdot (R(i) + \frac{r_1}{2}) \cdot (C(i) + \frac{p_1}{2}) \cdot (\alpha + \gamma)}{T(i)} - (\varphi + \mu + k_1) \cdot (C(i) + \frac{p_1}{2})$$

$$j_2 = \varphi \cdot (C(i) + \frac{p_1}{2}) - (\tau + \mu + k_2) \cdot (J(i) + \frac{j_1}{2})$$

$$r_2 = \tau \cdot (J(i) + \frac{j_1}{2}) - \frac{\omega \cdot (R(i) + \frac{r_1}{2}) \cdot (C(i) + \frac{p_1}{2}) \cdot (\alpha + \gamma)}{T(i)} - \psi \cdot (P(i) + \frac{m_1}{2}) - \mu \cdot (R(i) + \frac{r_1}{2})$$

$$k_3 = \mu \cdot T(i) - \frac{\alpha \cdot (N(i) + \frac{k_2}{2}) \cdot (C(i) + \frac{p_2}{2})}{T(i)} - (\beta + \mu) \cdot (N(i) + \frac{k_2}{2})$$

$$m_3 = \beta \cdot (N(i) + \frac{k_2}{2}) - \frac{\gamma \cdot (P(i) + \frac{m_2}{2}) \cdot (C(i) + \frac{p_2}{2})}{T(i)} - (\psi + \mu) \cdot (P(i) + \frac{m_2}{2})$$

$$p_3 = \frac{\gamma \cdot (P(i) + \frac{m_2}{2}) \cdot (C(i) + \frac{p_2}{2})}{T(i)} + \frac{\alpha \cdot (N(i) + \frac{k_2}{2}) \cdot (C(i) + \frac{p_2}{2})}{T(i)} + \frac{\omega \cdot (R(i) + \frac{r_2}{2}) \cdot (C(i) + \frac{p_2}{2}) \cdot (\alpha + \gamma)}{T(i)} - (\varphi + \mu + K_1) \cdot (C(i) + \frac{p_2}{2})$$

$$j_3 = \varphi \cdot (C(i) + \frac{p_2}{2}) - (\tau + \mu + k_2) \cdot (J(i) + \frac{j_2}{2})$$

$$r_3 = \tau \cdot (J(i) + \frac{j_2}{2}) - \frac{\omega \cdot (R(i) + \frac{r_2}{2}) \cdot (C(i) + \frac{p_2}{2}) \cdot (\alpha + \gamma)}{T(i)} - \psi \cdot (P(i) + \frac{m_2}{2}) - \mu \cdot (R(i) + \frac{r_2}{2})$$

$$k_4 = \mu \cdot T(i) - \frac{\alpha \cdot (N(i) + k_3) \cdot (C(i) + p_3)}{T(i)} - (\beta + \mu) \cdot (N(i) + k_3)$$

$$m_4 = \beta \cdot (N(i) + k_3) - \frac{\gamma \cdot (P(i) + k_3) \cdot (C(i) + p_3)}{T(i)} - (\psi + \mu) \cdot (P(i) + m_3)$$

$$p_4 = \frac{\gamma \cdot (P(i) + m_3) \cdot (C(i) + p_3)}{T(i)} + \frac{\alpha \cdot (N(i) + k_3) \cdot (C(i) + p_3)}{T(i)} + \frac{\omega \cdot (R(i) + r_3) \cdot (C(i) + p_3) \cdot (\alpha + \gamma)}{T(i)} - (\varphi + \mu + K_1) \cdot (C(i) + p_3)$$

$$j_4 = \varphi \cdot (C(i) + p_3) - (\tau + \mu + k_2) \cdot (J(i) + j_3)$$

$$r_4 = \tau \cdot (J(i) + j_3) - \frac{\omega \cdot (R(i) + r_3) \cdot (C(i) + p_3) \cdot (\alpha + \gamma)}{T(i)} - \psi \cdot (P(i) + m_3) - \mu \cdot (R(i) + r_3)$$

Step 4: Calculate the next step of N, P, C, J and R .

$$N(i+1) = N(i) + \frac{h}{6} \cdot (k_1 + 2 \cdot k_2 + 2 \cdot k_3 + k_4)$$

$$P(i+1) = P(i) + \frac{h}{6} \cdot (m_1 + 2 \cdot m_2 + 2 \cdot m_3 + m_4)$$

$$C(i+1) = C(i) + \frac{h}{6} \cdot (p_1 + 2 \cdot p_2 + 2 \cdot p_3 + p_4)$$

$$J(i+1) = J(i) + \frac{h}{6} \cdot (j_1 + 2 \cdot j_2 + 2 \cdot j_3 + j_4)$$

$$R(i+1) = R(i) + \frac{h}{6} \cdot (r_1 + 2 \cdot r_2 + 2 \cdot r_3 + r_4)$$

$$t(i+1) = t(i) + h$$

Step 5: Repeat steps 3 and 4 until desired t is achieved.