IMPUTATION METHODS FOR MISSING VALUES IN ESTIMATING POPULATION PROPORTION UNDER DIAGONAL SYSTEMATIC SAMPLING SCHEME

Abstract

In this paper, we have proposed a regression-type and exponential type imputation methods which are free of unknown parametersforestimating missing values or non-responses while estimating population proportion under diagonal systematic sampling design. The estimators of the proposed imputation methods were derived. The properties (biases and MSEs) of the class of estimators of the proposed imputation methods were derived up to first order approximation. Results of numerical illustration using simulated data revealed that the proposed estimators are more efficient and practicable than exiting estimators considered in the study.

Keywords: Imputation methods, Proportion, Diagonal Systematic sampling, Non-response.

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1.0 Introduction

Population parameters like total, mean, variance, proportion can be estimated using efficient estimators in which the study attribute are associated with auxiliary information under the assumption that, full sample information is available in sample survey is one of the research interests. Several authors like Khan and Shabbir (2017), have worked extensively in this direction. However, surveys like medical and social science surveys often face the problem of non-response due to involvement of human in data collection. These lost values in turn create complications in data handling and analysis. Over time, many methods have been developed to address the problem of estimating unknown parameters in the presence of missing values. Imputation is a common technique used to handle situations where data is missing. Missing values can be completed with specific substitutes and data can be analyzed using standard methods. Information about unit of characteristic of interest observed and auxiliary attribute help improve the accuracy of demographic parameter estimates. Hansen and Hurwitz (1946) were the first to consider the problem of nonresponse. Several authors also proposed imputation methods to deal with non-response or missing values. Recent among them include Singh and Horn (2000), Singh and Deo (2003), Wang and Wang (2006), Kadilar and Cingi (2008), Toutenburg et al. (2008), Singh (2009), Diana and Perri (2010), Al-Omari et al. (2013), Singh et al. (2014), Gira (2015), Singh et al. (2016), Bhushan and Pandey (2016), Prasad (2017). However, some of the estimators in above aforementioned literatures depend on unknown parameters of study attribute y which make them impracticable in real life application.

Recently, Azeem (2021) suggested a new approach called diagonal systematic sampling. The approach arrange the population of size $N = nk$ units, having n rows and k columns where $n \le k$, draw a random r number where $1 \le r \le k$. The conventional estimator for the procedure was proposed and it efficiency was compared to that of conventional estimators based on simple random sampling with replacement and linear systematic sampling. However, his proposed estimator did not consider the situation of presence of outliers and non-response. Therefore, in the current study, we have considered the modification of Azeem (2021) estimator of population proportion under theframework of diagonal systematic sampling in the presence of non-response and outliers using imputation approach.

1.1 Notations

Consider a finite population $U = U_{1,} U_{2,} U_{3.......} U_N$ of size N units numbered from 1 to N in some order. A sample of size n is taken at random from the first k units and every k^{th} subsequent unit; Then, N = nk where n and k are positive integers; thus, there will be k samples (clusters) each of size n and observe the study variate y and auxiliary variate x for each and every unit selected in the sample.

Let every population unit $(y_{i,} x_i)$ for i = 1, 2… N belongs to one of two mutually exclusive classes H and H^c where H is the class of units having the characteristics of interest. That is, let

$$y_i = \begin{cases} 1, & if\ i^{th} unit\ of\ population\ belongs\ to\ class\ H, \\ 0, & otherwise. \end{cases} \quad (1.1)$$

$$x_i = \begin{cases} 1, & if\ i^{th} unit\ of\ population\ belongs\ to\ class\ H, \\ 0, & otherwise. \end{cases} \quad (1.2)$$

Then, the systematic sample means are defined as $p_{a(sys)} = 1/n\sum_{i=1}^{n} y_i = \frac{a_{(sys)}}{n}$ and $p_{b(sys)} = 1/n\sum_{i=1}^{n} x_i = \frac{b_{(sys)}}{n}$ are unbiased estimators of the population means $P_A = \frac{1}{N}\sum_{i=1}^{N} Y_i = \frac{A}{N}$ and $P_B = 1/N\sum_{i=1}^{N} x_i = \frac{B}{N}$ for Y and X respectively.

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The usual sample proportion $p_{a(sys)}$ estimator of population proportion P_A is given in (1.3) as

$$p_{a(sys)} = \frac{a_{(sys)}}{n} \qquad (1.3)$$

The variance of sample proportion $V\left(p_{a(sys)}\right)$ estimator of population proportion P_A is given as in (1.4).

$$V\left(p_{a(sys)}\right) = \frac{p(1-p)}{n} p_A^* \qquad (1.4)$$

Azeem (2021) adopted the steps involved in the Subramani (2000) diagonal systematic sampling method which are as follows:

1. Arrange the population having $N = nk$ units in a table having n rows and k columns where $n \le k$.
2. Draw a random number r , where $1 \le r \le k$.
3. The units are drawn in such a way that the sampled units are the entries in the diagonal/broken diagonal of the table as shown in Table 1.

The selected units in diagonal systematic sampling scheme are as follows (Subramani, 2000).

$$S_r = \left\{ y_r, y_{(k+1)+r}, y_{2(k+1)+r}, ..., y_{(n-1)(k+1)+r} \right\}, \quad if\ r \le k-n+1, \qquad (1.5)$$

$$S_r = \left\{ y_r, y_{(k+1)+r}, y_{2(k+1)+r}, ..., y_{t(k+1)+r=(t+1)k}, y_{(t+1)k+1}, y_{(t+2)k+2}, ..., y_{(n-1)k+(n-t-1)} \right\}, \quad if\ r > k-n+1, \quad (1.6)$$

where $0 \le t \le n-1$

Table 1: Arrangement of the Population units

S/N	1	2	…	K
1	y_1	y_2	…	y_k
2	y_{k+1}	y_{k+2}	…	y_{2k}
3	y_{2k+1}	y_{2k+2}	…	y_{3k}

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$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
n	$y_{(n-1)k+1}$	$y_{(n-1)k+2}$	$\cdots$	y_{nk}

The first and second order inclusion probabilities under diagonal systematic sampling are given by:

$$\pi_i^* = \frac{1}{k} \tag{1.7}$$

$$\pi_{ij}^* = \begin{cases} \frac{1}{k}, & if\ i^{th} and\ j^{th} units\ are\ from\ the\ same\ diagonal\ or\ broken\ diagonal, \\ 0, & otherwise \end{cases} \tag{1.8}$$

1.2 Estimator Of Population Proportion In Diagonal Systematic Sapling And Its Properties

Azeem (2021) suggested the sample proportion based on the diagonal systematic sampling scheme given by:

$$p_{ad(sys)} = \frac{a_{d(sys)}}{n} \tag{1.9}$$

where $a_{d(sys)} = \begin{cases} \sum_{i=0}^{n-1} y_{i(k+1)+r}, & if\ r \le k-n+1, \\ \sum_{i=o}^{t} y_{i(k+1)+r} + \sum_{i=1}^{n-t-1} y_{(t+i)k+i}, & if\ r > k-n+1. \end{cases}$

The variance of $p_{ad(sys)}$ is given by:

$$Var\left(p_{ad(sys)}\right) = \frac{1}{k}\sum_{i=1}^{k}\left(p_{ad(sys)} - P\right)^2 \tag{1.10}$$

Theorem 1 Under diagonal systematic sampling scheme, the sample proportion can be written in the form of Horvitz–Thompson estimator pHT, suggested by Horvitz and Thompson (1952). Also, $p_{ad(sys)}$ is unbiased for population proportion P.

ProofBy definition

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$$p_{ad(sys)} = \frac{a_{d(sys)}}{n} = \frac{ka_{d(sys)}}{kn} = \frac{k\sum_{i \in S} y_i}{N}$$

where s denotes the sample drawn from population.

$$p_{ad(sys)} = \frac{1}{N}\left(\sum_{i \in S} \frac{y_i}{1/k}\right) = \frac{1}{N}\sum_{i \in S} \frac{y_i}{\pi_i} \tag{1.11}$$

where $\pi_i = 1/k$

Taking expectation on both sides of (1.9) yield

$$E\left(p_{ad(sys)}\right) = E\left(\frac{a_{d(sys)}}{n}\right) = E\left(\frac{ka_{d(sys)}}{kn}\right) = \frac{k}{N}\sum_{i=1}^{n} E(y_i) = \frac{k}{N}\sum_{i=1}^{n}\left[\sum_{i \in S} y_i \frac{1}{nk}\right] = \frac{\sum_{i \in S} y_i}{N} = \frac{A}{N} = P \tag{1.12}$$

where s denotes the sample drawn from population.

Remark 1 Although Horvitz–Thompson estimator is usually applied to estimate the finite population mean or population total, it can also be used to estimate population proportion by simply treating the variable as binary variable having possible values 0 and 1.

Remark 2 Using Sen–Yates–Grundy approach suggested by Sen (1953) and Yates and Grundy (1953).

The variance of $P_{ad(sys)}$ can be written as:

$$Var\left(P_{ad(sys)}\right) = \frac{1}{N^2}\left\{\frac{1}{2}\sum_{i=1}^{N}\sum_{\substack{j=1 \\ j \neq i}}^{N}\left(\pi_i^* \pi_j^* - \pi_{ij}^*\right)\left(\frac{y_i}{\pi_i^*} - \frac{y_j}{\pi_j^*}\right)^2\right\} \tag{1.13}$$

Also, the estimate of variance $Var\left(P_{ad(sys)}\right)$ denoted by $\widehat{Var}\left(P_{ad(sys)}\right)$ is given below:

$$\widehat{Var}\left(P_{ad(sys)}\right) = \frac{1}{2N^2}\sum_{i=1}^{n}\sum_{\substack{j=1 \\ j \neq i}}^{n}\left(\frac{\pi_i^* \pi_j^* - \pi_{ij}^*}{\pi_{ij}^*}\right)\left(\frac{y_i}{\pi_i^*} - \frac{y_j}{\pi_j^*}\right)^2 \tag{1.14}$$

The values of π_i^* and π_{ij}^* can be used from (1.7) and (1.8) in expression (1.13) and (1.14) to obtain the sampling variance of the sample proportion and its estimator under the diagonal systematic sampling scheme.

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2.0 Materials and Method

2.1 Proposed Imputation Schemes

Having study the estimator of Azeem (2021), the following imputation schemes were presented: Let consider $(y,x)\in\Re^{+}$ be pair of study and associated auxiliary attributes measured on the studied population. Also, let J denotes the set of responses with n_1 units, J^c denotes the set of non-responses having $n-n_1$ units or missing units (out of n) and S denotes the set of n units sampled without replacement from the N units in the population of interest. For each $i\in J$, the value of y_i is observed. However, for unit $i\in J^c$, y_i is missing due to non-response and obtained using different methods of imputation.

2.1.1 First Proposed Imputation

Motivated by Audu et al. (2023), the regression-type compromised imputation scheme for population proportion under diagonal systematic sampling design is proposed as in (2.1)

$$y_{.i}=\begin{cases} y_i & if \quad i\in J \\ \dfrac{p^{*}_{ad(sys)}+\hat{\beta}_{\phi}\left(P_B-p^{*}_{bd(sys)}\right)}{V_1 p^{*}_{bd(sys)}+V_2}(V_1P_B+V_2) & if \quad i\in J^{c} \end{cases} \tag{2.1}$$

where $p^{*}_{ad(sys)}=\dfrac{a^{*}_{d(sys)}}{r}$, $p^{*}_{bd(sys)}=\dfrac{b^{*}_{d(sys)}}{r}$, $\hat{\beta}_{\phi}=s_{(ab)r}/s^{2}_{(b)}$, $s_{(ab)r}=\left(r-1\right)^{-1}\sum_{i=1}^{r}(y_i-p^{*}_{a})(x_i-p^{*}_{b})$,

$s^{2}_{(b)}=\left(r-1\right)^{-1}\sum_{i=1}^{r}(x_i-p^{*}_{b})$, $a^{*}_{d(sys)}=\begin{cases}\sum_{i=0}^{n_1-1} y_{i(k+1)+r}, & if\ r\le k-n_1+1, \\ \sum_{i=o}^{t} y_{i(k+1)+r}+\sum_{i=1}^{n_1-t-1} y_{(t+i)k+i}, & if\ r>k-n_1+1\end{cases}$, $b^{*}_{d(sys)}=\begin{cases}\sum_{i=0}^{n_1-1} x_{i(k+1)+r}, & if\ r\le k-n_1+1, \\ \sum_{i=o}^{t} x_{i(k+1)+r}+\sum_{i=1}^{n_1-t-1} x_{(t+i)k+i}, & if\ r>k-n_1+1\end{cases}$,

V_1 and V_2 are either constants or known functions of auxiliary attributes like coefficients of skewness $\beta_{1(b)}$, kurtosis $\beta_{2(b)}$, variation C_B, standard deviation S_B etc, that are arbitrarily chosen to generate the members of proposed class of estimators

The estimator of the first proposed imputation can be obtained using the function defined in (2.2)

$$T=\frac{1}{n}\left(\sum_{i\in J}^{r} y_{.i}+\sum_{i\in J^{c}}^{n_1-r} y_{.i}\right) \tag{2.2}$$

$$t_{1(i)}=\frac{1}{n}\left(\sum_{i=1}^{r} y_i+\sum_{i=1}^{n-r}\frac{p^{*}_{ad(sys)}+\hat{\beta}_{\phi}(P_B-p^{*}_{bd(sys)})}{V_1p^{*}_{bd(sys)}+V_2}\left(V_1P_B+V_2\right)\right) \tag{2.3}$$

$$t_{1(i)}=\frac{1}{n}\left(rp^{*}_{ad(sys)}+\left(n-r\right)\frac{p^{*}_{ad(sys)}+\hat{\beta}_{\phi}(P_B-p^{*}_{bd(sys)})}{V_1p^{*}_{bd(sys)}+V_2}\left(V_1P_B+V_2\right)\right) \tag{2.4}$$

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$$t_{1(i)}=\frac{r}{n}p^{*}_{ad(sys)}+\left(1-\frac{r}{n}\right)\frac{p^{*}_{ad(sys)}+\hat{\beta}_{\phi}(P_B-p^{*}_{bd(sys)})}{V_1p^{*}_{bd(sys)}+V_2}\left(V_1P_B+V_2\right) \tag{2.5}$$

Using error terms defined in (2.6), (2.5) can be expressed as obtained in (2.7)

$$e_0=\frac{p^{*}_{ad(sys)}-P_A}{P_A},\ e_1=\frac{p^{*}_{bd(sys)}-P_B}{P_B}\text{ Such that } p^{*}_{ad(sys)}=\left(1+e_0\right)P_A,\ p^{*}_{bd(sys)}=\left(1+e_1\right)P_B \tag{2.6}$$

$$t_{1(i)}=\frac{r}{n}\left(1+e_0\right)P_A+\left(1-\frac{r}{n}\right)\left[\frac{\left(1+e_0\right)P_A+\hat{\beta}_{\phi}\left(P_B-\left(1+e_1\right)P_B\right)}{V_1\left(1+e_1\right)+V_2}\right]\left(V_1P_B+V_2\right) \tag{2.7}$$

$$t_{1(i)}=\frac{r}{n}\left(1+e_0\right)P_A+\left(1-\frac{r}{n}\right)\frac{P_A+P_Ae_0-\hat{\beta}_{\phi}P_Be_1}{\left(V_1P_B+V_2\right)\left[1+\frac{V_1P_Be_1}{V_1P_B+V_2}\right]}\left(V_1P_B+V_2\right) \tag{2.8}$$

$$t_{1(i)}=\frac{r}{n}\left(1+e_0\right)P_A+\left(1-\frac{r}{n}\right)\left(P_A+P_Ae_0-\hat{\beta}_{\phi}P_Be_1\right)(1+He_1)^{-1} \tag{2.9}$$

where $H=\frac{V_1P_B}{V_1P_B+V_2}$

$$t_{1(i)}=\frac{r}{n}\left(1+e_0\right)P_A+\left(1-\frac{r}{n}\right)\left(P_A+P_Ae_0-\hat{\beta}_{\phi}P_Be_1\right)(1-He_1+H^2e_1^2) \tag{2.10}$$

$$t_{1(i)}=\frac{r}{n}P_A+\frac{r}{n}P_Ae_0+\left(1-\frac{r}{n}\right)\left(P_A-HP_Ae_1+H^2P_Ae_1^2+P_Ae_0-HP_Ae_0e_1-\hat{\beta}_{\phi}P_Be_1+H\hat{\beta}_{\phi}P_Be_1^2\right) \tag{2.11}$$

$$t_{1(i)}=\frac{r}{n}P_A+\frac{r}{n}P_Ae_0+P_A-\frac{r}{n}P_A+P_Ae_0-\frac{r}{n}P_Ae_0+\left(1-\frac{r}{n}\right)\begin{pmatrix}-\left(HP_A+\hat{\beta}_{\phi}P_B\right)e_1\\+\left(H^2P_A+H\hat{\beta}_{\phi}P_B\right)e_1^2-HP_Ae_0e_1\end{pmatrix} \tag{2.12}$$

$$t_{1(i)}-P_A=P_Ae_0+\left(1-\frac{r}{n}\right)\left(-\left(HP_A+\hat{\beta}_{\phi}P_B\right)e_1+\left(H^2P_A+H\hat{\beta}_{\phi}P_B\right)e_1^2-HP_Ae_0e_1\right) \tag{2.13}$$

By taking the expectation of (2.13), the Bias of the first proposed estimator denoted by $t_{1(i)}$ is obtained as in (2.14)

$$Bias(t_{1(i)})=\left(1-\frac{r}{n}\right)\left(\left(H^2P_A+H\hat{\beta}_{\phi}P_B\right)E\left(e_1^2\right)-HP_AE\left(e_0e_1\right)\right) \tag{2.14}$$

$$Bias(t_3)=\left(1-\frac{r}{n}\right)\begin{pmatrix}\left(H^2P_A+H\hat{\beta}_{\phi}P_B\right)\frac{1}{N_1^2P_B}\left\{\frac{1}{2}\sum_{i=1}^{N_1}\sum_{\substack{j=1\\j\neq i}}^{N_1}\left(\pi_i\pi_j-\pi_{ij}\right)\left(\frac{x_i}{\pi_i}-\frac{x_j}{\pi_j}\right)^2\right\}\\-HP_A\frac{1}{N_1^2P_AP_B}\left\{\frac{1}{2}\sum_{i=1}^{N_1}\sum_{\substack{j=1\\j\neq i}}^{N_1}\left(\pi_i\pi_j-\pi_{ij}\right)\left(\frac{y_i}{\pi_i}-\frac{y_j}{\pi_j}\right)^2\left(\frac{x_i}{\pi_i}-\frac{x_j}{\pi_j}\right)^2\right\}\end{pmatrix} \tag{2.15}$$

Also, by taking expectation and squaring both sides of (2.13), the MSE of the first proposed estimator denoted by $t_{1(i)}$ is obtained as in (2.16)

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$$MSE(t_{1(i)}) = E\left[P_A e_0 + \left(1-\frac{r}{n}\right)\left(-\left(HP_A+\hat{\beta}_\phi P_B\right)e_1 + \left(H^2 P_A + H\hat{\beta}_\phi P_B\right)e_1^2 - HP_A e_0 e_1\right)\right]^2 \quad (2.16)$$

$$MSE(t_{1(i)}) = P_A^2 E\left(e_0^2\right) + \left(1-\frac{r}{n}\right)^2 \left(HP_A+\hat{\beta}_\phi P_B\right)^2 E\left(e_1^2\right) - 2\left(1-\frac{r}{n}\right)P_A\left(HP_A+\hat{\beta}_\phi P_B\right)E\left(e_0 e_1\right) \quad (2.17)$$

$$MSE(t_{1(i)}) = P_A^2 \frac{1}{N_1^2 P_A}\left\{\frac{1}{2}\sum_{i=1}^{N_1}\sum_{\substack{j=1\\ j\neq i}}^{N_1}\left(\pi_i\pi_j - \pi_{ij}\right)\left(\frac{y_i}{\pi_i}-\frac{y_j}{\pi_j}\right)^2\right\}$$

$$+\left(1-\frac{r}{n}\right)^2\left(HP_A+\hat{\beta}_\phi P_B\right)^2 \frac{1}{N_1^2 P_B}\left\{\frac{1}{2}\sum_{i=1}^{N_1}\sum_{\substack{j=1\\ j\neq i}}^{N_1}\left(\pi_i\pi_j - \pi_{ij}\right)\left(\frac{x_i}{\pi_i}-\frac{x_j}{\pi_j}\right)^2\right\} \quad (2.18)$$

$$-2\left(1-\frac{r}{n}\right)P_A\left(HP_A+\hat{\beta}_\phi P_B\right)\frac{1}{N_1^2 P_A P_B}\left\{\frac{1}{2}\sum_{i=1}^{N_1}\sum_{\substack{j=1\\ j\neq i}}^{N_1}\left(\pi_i\pi_j - \pi_{ij}\right)\left(\frac{y_i}{\pi_i}-\frac{y_j}{\pi_j}\right)^2\left(\frac{x_i}{\pi_i}-\frac{x_j}{\pi_j}\right)^2\right\}$$

2.1.2 Second Proposed Imputation

Motivated by Audu and Singh (2021), the exponential-type regression compromised imputation scheme for population proportion under diagonal systematic sampling design is proposed as in (2.19)

$$y_{.i} = \begin{cases} y_i & if \quad i \in J \\ \dfrac{p^*_{ad(sys)} + \hat{\beta}_\phi\left(P_B - p^*_{bd(sys)}\right)}{V_1 p^*_{bd(sys)} + V_2}(V_1 P_B + V_2)\exp\left(\dfrac{P_B - p^*_{bd(sys)}}{P_B + p^*_{bd(sys)}}\right) & if \quad i \in J^c \end{cases} \quad (2.19)$$

Using (2.2), the estimator of the second proposed imputation can be obtained as follows:

$$t_{2(i)} = \frac{1}{n}\left(\sum_{i=1}^{r} y_i + \sum_{i=1}^{n-r}\frac{p^*_{ad(sys)} + \hat{\beta}_\phi\left(P_B - p^*_{bd(sys)}\right)}{V_1 p^*_{bd(sys)} + V_2}(V_1 P_B + V_2)\exp\left(\frac{P_B - p^*_{bd(sys)}}{P_B + p^*_{bd(sys)}}\right)\right) \quad (2.20)$$

$$t_{2(i)} = \frac{1}{n}\left(r p^*_{ad(sys)} + (n-r)\frac{p^*_{ad(sys)} + \hat{\beta}_\phi\left(P_B - p^*_{bd(sys)}\right)}{V_1 p^*_{bd(sys)} + V_2}(V_1 P_B + V_2)\exp\left(\frac{P_B - p^*_{bd(sys)}}{P_B + p^*_{bd(sys)}}\right)\right) \quad (2.21)$$

$$t_{2(i)} = \frac{r}{n}p^*_{ad(sys)} + \left(1-\frac{r}{n}\right)\frac{p^*_{ad(sys)} + \hat{\beta}_\phi\left(P_B - p^*_{bd(sys)}\right)}{V_1 p^*_{bd(sys)} + V_2}(V_1 P_B + V_2)\exp\left(\frac{P_B - p^*_{bd(sys)}}{P_B + p^*_{bd(sys)}}\right) \quad (2.22)$$

Using error terms defined in (2.6), (2.11) can be expressed as obtained in (2.23)

$$t_{2(i)} = \frac{r}{n}\left(1+e_0\right)P_A + \left(1-\frac{r}{n}\right)\frac{P_A + P_A e_0 - \hat{\beta}_\phi P_B e_1}{V_1\left(1+e_1\right)P_B + V_2}(V_1 P_B + V_2)\exp\left(\frac{P_B - \left(1+e_1\right)P_B}{P_B + \left(1+e_1\right)P_B}\right) \quad (2.23)$$

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$$t_{2(i)}=\frac{r}{n}(1+e_0)P_A+\left(1-\frac{r}{n}\right)\frac{P_A+P_Ae_0-\hat{\beta}_\phi P_Be_1}{(V_1P_B+V_2)\left[1+\frac{V_1P_Be_1}{V_1P_B+V_2}\right]}(V_1P_B+V_2)\exp\left(\frac{-e_1}{2+e_1}\right) \qquad (2.24)$$

$$t_{2(i)}=\frac{r}{n}(1+e_0)P_A+\left(1-\frac{r}{n}\right)\left(P_A+P_Ae_0-\hat{\beta}_\phi P_Be_1\right)\left(1+\frac{V_1P_Be_1}{V_1P_B+V_2}\right)^{-1}\exp\left(\frac{-e_1}{2}\left(1+\frac{e_1}{2}\right)^{-1}\right) \qquad (2.25)$$

$$t_{2(i)}=\frac{r}{n}(1+e_0)P_A+\left(1-\frac{r}{n}\right)\left(P_A+P_Ae_0-\hat{\beta}\phi P_Be_1\right)(1+He_1)^{-1}\exp\left(\frac{-e_1}{2}+\frac{e_1^2}{4}\right) \qquad (2.26)$$

$$t_{2(i)}=\frac{r}{n}P_A+\frac{r}{n}P_Ae_0+\left(1-\frac{r}{n}\right)\left(P_A+P_Ae_0-\hat{\beta}_\phi P_Be_1\right)(1-He_1+H^2e_1^2)\left(1-\frac{e_1}{2}+\frac{e_1^2}{8}\right) \qquad (2.27)$$

$$t_{2(i)}=\frac{r}{n}P_A+\frac{r}{n}P_Ae_0+\left(1-\frac{r}{n}\right)\begin{pmatrix}P_A+P_Ae_0-\left(\frac{P_A}{2}+HP_A+\hat{\beta}_\phi P_B\right)e_1-P_Ae_0e_1\left(\frac{1}{2}+H\right)\\+\left(\frac{P_A}{8}+\frac{HP_A}{2}+H^2P_A+\frac{\hat{\beta}_\phi P_B}{2}+H\hat{\beta}\phi P_B\right)e_1^2\end{pmatrix} \qquad (2.28)$$

$$t_{2(i)}-P_A=P_Ae_0+\left(1-\frac{r}{n}\right)\begin{pmatrix}-\left(\frac{P_A}{2}+HP_A+\hat{\beta}_\phi P_B\right)e_1-P_Ae_0e_1\left(\frac{1}{2}+H\right)\\+\left(\frac{P_A}{8}+\frac{HP_A}{2}+H^2P_A+\frac{\hat{\beta}_\phi P_B}{2}+H\hat{\beta}\phi P_B\right)e_1^2\end{pmatrix} \qquad (2.29)$$

By taking the expectation of (2.29), the Bias of the second proposed estimator denoted by $t_{2(i)}$ is obtained as in (2.30)

$$Bias\left(t_{2(i)}\right)=\left(1-\frac{r}{n}\right)\left(\left(\frac{P_A}{8}+\frac{HP_A}{2}+H^2P_A+\frac{\hat{\beta}_\phi P_B}{2}+H\hat{\beta}\phi P_B\right)E\left(e_1^2\right)-P_A\left(\frac{1}{2}+H\right)E\left(e_0e_1\right)\right) \qquad (2.30)$$

$$Bias\left(t_{2(i)}\right)=\left(1-\frac{r}{n}\right)\begin{pmatrix}\left(\frac{P_A}{8}+\frac{HP_A}{2}+H^2P_A+\frac{\hat{\beta}_\phi P_B}{2}+H\hat{\beta}\phi P_B\right)\\ \frac{1}{N_1^2P_B}\left\{\frac{1}{2}\sum_{i=1}^{N_1}\sum_{\substack{j=1\\j\neq i}}^{N_1}\left(\pi_i\pi_j-\pi_{ij}\right)\left(\frac{x_i}{\pi_i}-\frac{x_j}{\pi_j}\right)^2\right\}\\ -P_A\left(\frac{1}{2}+H\right)\frac{1}{N_1^2P_AP_B}\left\{\frac{1}{2}\sum_{i=1}^{N_1}\sum_{\substack{j=1\\j\neq i}}^{N_1}\left(\pi_i\pi_j-\pi_{ij}\right)\left(\frac{y_i}{\pi_i}-\frac{y_j}{\pi_j}\right)^2\left(\frac{x_i}{\pi_i}-\frac{x_j}{\pi_j}\right)^2\right\}\end{pmatrix} \qquad (2.31)$$

Also, by taking expectation and squaring both sides of (2.29), the MSE of the second proposed estimator denoted by $t_{2(i)}$ is obtained as in (2.32)

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$$MSE(t_{2(i)}) = E\left[P_A e_0 + \left(1-\frac{r}{n}\right)\left(\begin{array}{l} -\left(\frac{P_A}{2} + HP_A + \hat{\beta}_\phi P_B\right)e_3 - P_A e_0 e_1\left(\frac{1}{2}+H\right) \\ +\left(\frac{P_A}{8} + \frac{HP_A}{2} + H^2 P_A + \frac{\hat{\beta}_\phi P_B}{2} + H\hat{\beta}_\phi P_B\right)e_1^2 \end{array} \right)\right]^2 \tag{2.32}$$

$$MSE(t_{2(i)}) = P_A^2 E\left(e_0^2\right) + \left(1-\frac{r}{n}\right)^2\left(\frac{P_A}{2} + HP_A + \hat{\beta}_\phi P_B\right)^2 E\left(e_1^2\right) - 2\left(1-\frac{r}{n}\right)P_A \left(\frac{P_A}{2} + HP_A + \hat{\beta}_\phi P_B\right)E\left(e_0 e_1\right) \tag{2.33}$$

$$MSE(t_{2(i)}) = P_A^2 \frac{1}{N_1^2 P_A}\left\{\frac{1}{2}\sum_{i=1}^{N_1}\sum_{\substack{j=1\\ j\neq i}}^{N_1}\left(\pi_i\pi_j - \pi_{ij}\right)\left(\frac{y_i}{\pi_i} - \frac{y_j}{\pi_j}\right)^2\right\} + \left(1-\frac{r}{n}\right)^2\left(\frac{P_A}{2} + HP_A + \hat{\beta}_\phi P_B\right)^2 \frac{1}{N_1^2 P_B}\left\{\frac{1}{2}\sum_{i=1}^{N_1}\sum_{\substack{j=1\\ j\neq i}}^{N_1}\left(\pi_i\pi_j - \pi_{ij}\right)\left(\frac{x_i}{\pi_i} - \frac{x_j}{\pi_j}\right)^2\right\} - 2\left(1-\frac{r}{n}\right)P_A\left(\frac{P_A}{2} + HP_A + \hat{\beta}_\phi P_B\right) \frac{1}{N_1^2 P_A P_B}\left\{\frac{1}{2}\sum_{i=1}^{N_1}\sum_{\substack{j=1\\ j\neq i}}^{N_1}\left(\pi_i\pi_j - \pi_{ij}\right)\left(\frac{y_i}{\pi_i} - \frac{y_j}{\pi_j}\right)\left(\frac{x_i}{\pi_i} - \frac{x_j}{\pi_j}\right)\right\} \tag{2.34}$$

3.0 Empirical Study

In this section, simulation studies to assess the performance of the proposed estimators $t_{1(i)}$ and $t_{2(i)}$ with respect to Azeem (2021) estimator under the effect of non-response and outliers were conducted. Data of size 500 units were generated for the study population using binomial distribution. A sample of size 100 was selected by the method of diagonal systematic sampling 500 times. The Biases, MSEs and PREs of the considered estimators were computed using (3.1), (3.2) and (3.3)

$$Bias(T) = \frac{1}{500}\sum_{j=1}^{500}(p_{ad} - P_A) \tag{3.1}$$

$$MSE(T) = \frac{1}{500}\sum_{j=1}^{500}(p_{ad} - P_A)^2 \tag{3.2}$$

$$PREs(T) = \frac{MSE\left(t_0\right)}{MSE(T)}\times 100 \tag{3.3}$$

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4.0 Results and Discussion

Table 2: Biases, MSEs and PREs of Estimators $p_{ad(sys)}$, $t_{1(i)}$ and $t_{2(i)}$ when P=0.5

ESTIMATOR	BIAS	MSE	PRE	ESTIMATOR	BIAS	MSE	PRE
$p_{ad(sys)}$ Azzem(2021)	-0.0009	0.0015	100	$p_{ad(sys)}$ Azzem(2021)	-0.0009	0.0015	100
Proposed Estimator $t_{1(i)}$				Proposed Estimator $t_{2(i)}$			
$t_{1(1)}$	0.0003	0.0013	125.1558	$t_{2(1)}$	-0.0022	0.0011	145.9438
$t_{1(2)}$	-0.0009	0.0015	108.1937	$t_{2(2)}$	-0.0036	0.0012	133.5281
$t_{1(3)}$	-0.0009	0.0015	105.095	$t_{2(3)}$	-0.0037	0.0012	130.5208
$t_{1(4)}$	-0.0006	0.0017	92.08923	$t_{2(4)}$	-0.0039	0.0013	116.735
$t_{1(5)}$	0.00032	0.0012	131.9072	$t_{2(5)}$	-0.0012	0.0011	147.008
$t_{1(6)}$	-0.0009	0.0015	105.095	$t_{2(6)}$	-0.0037	0.0012	130.5208
$t_{1(7)}$	-0.0006	0.0017	103.0892	$t_{2(7)}$	-0.0039	0.0013	116.735
$t_{1(8)}$	0.00032	0.0012	131.9072	$t_{2(8)}$	-0.0012	0.0011	147.008
$t_{1(9)}$	-0.0009	0.0014	112.0605	$t_{2(9)}$	-0.0034	0.0011	137.0513
$t_{1(10)}$	0.0005	0.0019	108.1483	$t_{2(10)}$	-0.0035	0.0015	102.0924
$t_{1(11)}$	-4.8208	0.0012	128.4142	$t_{2(11)}$	-0.0017	0.0010	146.9423
$t_{1(12)}$	-0.0650	0.0243	6.480046	$t_{2(12)}$	-0.0679	0.0245	6.420426
$t_{1(13)}$	0.0011	0.0021	74.64178	$t_{2(13)}$	-0.0031	0.0016	96.91431
$t_{1(14)}$	-0.0005	0.0013	122.6186	$t_{2(14)}$	-0.0025	0.0010	144.7304
$t_{1(15)}$	-0.0009	0.0016	111.5792	$t_{2(15)}$	-0.0039	0.0013	123.7993
$t_{1(16)}$	-0.0009	0.0016	102.2689	$t_{2(16)}$	-0.0039	0.0013	124.5315
$t_{1(17)}$	-0.0009	0.0016	100.6587	$t_{2(17)}$	-0.0039	0.0012	125.9936

Table 2 displays the outcomes of biases, mean squared errors (MSEs), and percentage relative efficiency (PREs) for the existing and proposed estimators when p = 0.5.The findings indicate that, with the exception of ($t_{1(4)}$,,$t_{1(12)}$, $t_{1(13)}$, $t_{2(12)}$ and $t_{2(13)}$), all the proposed estimators exhibit lower MSEsand higher PREs in comparison to the existing estimator considered in this investigation. Consequently, the proposed estimators in this context are more efficient than their competitor in the study and are likely to yield superiorestimates of the population proportion, particularly in scenarios involving non-response.

Table 3: Biases, MSEs and PREs of Estimators $p_{ad(sys)}$, $t_{1(i)}$ and $t_{2(i)}$ when P=0.6

ESTIMATOR	BIAS	MSE	PRE	ESTIMATOR	BIAS	MSE	PRE
$p_{ad(sys)}$ Azzem(2021)	-0.0009	0.0015	100	$p_{ad(sys)}$ Azzem(2021)	-0.0009	0.0015	100
Proposed Estimator $t_{1(i)}$				Proposed Estimator $t_{2(i)}$			
$t_{1(1)}$	0.0001	0.0018	108.9821	$t_{2(1)}$	0.0014	0.0017	112.8677
$t_{1(2)}$	-0.0003	0.0019	103.0852	$t_{2(2)}$	0.0003	0.0018	110.1613
$t_{1(3)}$	-0.0004	0.0019	101.8556	$t_{2(3)}$	0.0002	0.0018	109.3862
$t_{1(4)}$	-8.9443	0.0020	96.80879	$t_{2(4)}$	-8.2330	0.0019	105.8678
$t_{1(5)}$	0.0001	0.0018	108.8499	$t_{2(5)}$	0.0014	0.0018	112.8367
$t_{1(6)}$	-0.0004	0.0019	101.8556	$t_{2(6)}$	0.0002	0.0018	109.3862
$t_{1(7)}$	-8.9443	0.0020	96.80879	$t_{2(7)}$	-8.2330	0.0019	105.8678
$t_{1(8)}$	-0.0001	0.0018	108.8499	$t_{2(8)}$	0.0014	0.0018	112.8367
$t_{1(9)}$	-0.0004	0.0018	104.6066	$t_{2(9)}$	0.0005	0.0018	111.0486
$t_{1(10)}$	0.0010	0.0022	89.64492	$t_{2(10)}$	0.0005	0.0019	100.4655
$t_{1(11)}$	0.0001	0.0018	108.9158	$t_{2(11)}$	0.0014	0.0017	112.8524
$t_{1(12)}$	-0.0684	0.0060	32.39762	$t_{2(12)}$	-0.0683	0.0061	32.46881
$t_{1(13)}$	0.0009	0.0022	89.74117	$t_{2(13)}$	0.0005	0.0019	100.5394
$t_{1(14)}$	0.0002	0.0018	109.0494	$t_{2(14)}$	0.0015	0.0017	112.8826
$t_{1(15)}$	-0.0003	0.0019	100.0373	$t_{2(15)}$	4.6749	0.0018	108.1691
$t_{1(16)}$	-0.0003	0.0019	100.0186	$t_{2(16)}$	4.5844	0.0018	108.1562
$t_{1(17)}$	-0.0003	0.0019	99.98126	$t_{2(17)}$	4.4057	0.0018	108.1304

Table 3 displays the outcomes of biases, mean squared errors (MSEs), and percentage relative efficiency (PREs) for the existing and proposed estimators when p = 0.6.The findings indicate that, with the exception of $t_{1(4)}, t_{1(7)}, t_{1(10)}, t_{1(12)}, t_{1(13)}, t_{1(14)}, t_{1(17)}$ and $t_{2(12)}$, all the proposed estimators exhibit lower MSEsand higher PREs in comparison to the existing estimator considered in this investigation. Consequently, the proposed estimators in this context are more efficient than their competitor in the study and are likely to yield superiorestimates of the population proportion, particularly in scenarios involving non-response.

Table 4: Biases, MSEs and PREs of Estimators $p_{ad(sys)}$, $t_{1(i)}$ and $t_{2(i)}$ when P=0.7

ESTIMATOR	BIAS	MSE	PRE	ESTIMATOR	BIAS	MSE	PRE
$p_{ad(sys)}$ Azzem(2021)	-0.0006	0.0016	100	$p_{ad(sys)}$ Azzem(2021)	-0.0006	0.0016	100
Proposed Estimator $t_{1(i)}$				Proposed Estimator $t_{2(i)}$			
$t_{1(1)}$	-6.3700	0.0016	100.7015	$t_{2(1)}$	-0.0002	0.0016	98.5387
$t_{1(2)}$	-0.0005	0.0016	100.5071	$t_{2(2)}$	-0.0011	0.0016	100.5984
$t_{1(3)}$	-0.0005	0.0016	100.3393	$t_{2(3)}$	-0.0012	0.0016	100.8379
$t_{1(4)}$	-0.0004	0.0016	99.2362	$t_{2(4)}$	-0.0014	0.0016	101.3943
$t_{1(5)}$	3.8626	0.0016	100.6312	$t_{2(5)}$	-7.8749	0.0016	98.1435
$t_{1(6)}$	-0.0006	0.0016	100.3393	$t_{2(6)}$	-0.0012	0.0016	100.8379
$t_{1(7)}$	-0.0004	0.0016	99.2362	$t_{2(7)}$	-0.0014	0.0016	101.3943
$t_{1(8)}$	3.8626	0.0016	100.6312	$t_{2(8)}$	-7.8749	0.0016	98.1435
$t_{1(9)}$	-0.0005	0.0016	100.6604	$t_{2(9)}$	-0.0009	0.0016	100.2353
$t_{1(10)}$	0.0002	0.0017	96.95981	$t_{2(10)}$	-0.0012	0.0016	101.228
$t_{1(11)}$	-1.4861	0.0016	100.6703	$t_{2(11)}$	-0.0002	0.0016	98.3497
$t_{1(12)}$	0.0339	0.0914	1.7666	$t_{2(12)}$	0.0329	0.0915	1.7642
$t_{1(13)}$	0.0003	0.0017	96.5466	$t_{2(13)}$	-0.0011	0.0016	101.1309
$t_{1(14)}$	-0.0001	0.0016	100.7245	$t_{2(14)}$	-0.0003	0.0016	98.7024
$t_{1(15)}$	-0.0006	0.0016	99.9608	$t_{2(15)}$	-0.0014	0.0016	101.15
$t_{1(16)}$	-0.0006	0.0016	99.9803	$t_{2(16)}$	-0.0014	0.0016	101.1385
$t_{1(17)}$	-0.0006	0.0016	100.0186	$t_{2(17)}$	-0.0013	0.0016	101.1148

Table 4 displays the outcomes of biases, mean squared errors (MSEs), and percentage relative efficiency (PREs) for the existing and proposed estimators when p = 0.7.The findings indicate that, with the exception of $t_{1(4)}, t_{1(7)}, t_{1(10)}, t_{1(12)}, t_{1(13)}, t_{1(14)}, t_{1(15)}, t_{1(16)}, t_{2(1)}, t_{2(5)}, t_{2(8)}, t_{2(11)}, t_{2(12)}$ and $t_{2(14)}$, all the proposed estimators exhibit lower MSEsand higher PREs in comparison to the existing estimator considered in this investigation. Consequently, the proposed estimators in this context are more efficient than their competitor in the study and are likely to yield superiorestimates of the population proportion, particularly in scenarios involving non-response.

Table 5: Biases, MSEs and PREs of Estimators $p_{ad(sys)}$, $t_{1(i)}$ and $t_{2(i)}$ when P=0.8

ESTIMATOR	BIAS	MSE	PRE	ESTIMATOR	BIAS	MSE	PRE
$p_{ad(sys)}$ Azzem(2021)	-0.0035	0.0039	100	$p_{ad(sys)}$ Azzem(2021)	-0.0035	0.0039	100
Proposed Estimator $t_{1(i)}$				Proposed Estimator $t_{2(i)}$			
$t_{1(1)}$	-0.0036	0.0038	104.3494	$t_{2(1)}$	-0.0039	0.0037	106.4786
$t_{1(2)}$	-0.0039	0.0039	102.3417	$t_{2(2)}$	-0.0046	0.0037	105.6342
$t_{1(3)}$	-0.0039	0.0039	101.8392	$t_{2(3)}$	-0.0046	0.0037	105.3624
$t_{1(4)}$	-0.0037	0.0039	99.3582	$t_{2(4)}$	-0.0047	0.0038	103.8639
$t_{1(5)}$	-0.0036	0.0038	104.5048	$t_{2(5)}$	-0.0039	0.0037	106.5135
$t_{1(6)}$	-0.0039	0.0039	101.8392	$t_{2(6)}$	-0.0046	0.0037	105.3624
$t_{1(7)}$	-0.0037	0.0039	99.3582	$t_{2(7)}$	-0.0047	0.0038	103.8639
$t_{1(8)}$	-0.0036	0.0038	104.5048	$t_{2(8)}$	-0.0039	0.0037	106.5135
$t_{1(9)}$	-0.0039	0.0038	102.9154	$t_{2(9)}$	-0.0045	0.0037	105.9235
$t_{1(10)}$	-0.0029	0.0041	95.3261	$t_{2(10)}$	-0.0044	0.0039	101.1446
$t_{1(11)}$	-0.0036	0.0039	104.4269	$t_{2(11)}$	-0.0039	0.0037	106.497
$t_{1(12)}$	0.0139	0.0503	7.8559	$t_{2(12)}$	0.0129	0.0504	7.8408
$t_{1(13)}$	-0.0028	0.0042	94.7915	$t_{2(13)}$	-0.0043	0.0039	100.7688
$t_{1(14)}$	-0.0037	0.0038	104.2761	$t_{2(14)}$	-0.0040	0.0037	106.4596
$t_{1(15)}$	-0.0038	0.0039	100.9115	$t_{2(15)}$	-0.0047	0.0038	104.8271
$t_{1(16)}$	-0.0038	0.0039	100.9476	$t_{2(16)}$	-0.0047	0.0038	104.8486
$t_{1(17)}$	-0.0038	0.0039	101.0193	$t_{2(17)}$	-0.0047	0.0038	104.8912

Table 5 displays the outcomes of biases, mean squared errors (MSEs), and percentage relative efficiency (PREs) for the existing and proposed estimators when p = 0.8.The findings indicate that, with the exception of $t_{1(4)}$, $t_{1(7)}$, $t_{1(10)}$, $t_{1(12)}$, $t_{1(13)}$ and $t_{2(12)}$, all the proposed estimators exhibit lower MSEsand higher PREs in comparison to the existing estimator considered in this investigation. Consequently, the proposed estimators in this context are more efficient than their competitor in the study and are likely to yield superiorestimates of the population proportion, particularly in scenarios involving non-response.

Table 6: Biases, MSEs and PREs of Estimators $p_{ad(sys)}$, $t_{1(i)}$ and $t_{2(i)}$ when P=0.9

ESTIMATOR	BIAS	MSE	PRE	ESTIMATOR	BIAS	MSE	PRE
$p_{ad(sys)}$ Azzem(2021)	0.0016	0.0015	100	$p_{ad(sys)}$ Azzem(2021)	0.0016	0.0015	100
Proposed Estimator $t_{1(i)}$				Proposed Estimator $t_{2(i)}$			
$t_{1(1)}$	-0.0015	0.0013	109.5815	$t_{2(1)}$	-0.0004	0.0012	116.7586
$t_{1(2)}$	-0.0016	0.0014	103.3322	$t_{2(2)}$	-0.0008	0.0013	112.109
$t_{1(3)}$	-0.0016	0.0014	101.9937	$t_{2(3)}$	-0.0008	0.0013	111.0008
$t_{1(4)}$	-0.0015	0.0015	6.57065	$t_{2(4)}$	-0.0008	0.0014	106.2582
$t_{1(5)}$	-0.0015	0.0013	109.1763	$t_{2(5)}$	-0.0004	0.0013	116.4925
$t_{1(6)}$	-0.0016	0.0014	101.9937	$t_{2(6)}$	-0.0008	0.0013	111.0008
$t_{1(7)}$	-0.0015	0.0015	96.57065	$t_{2(7)}$	-0.0008	0.0014	106.2582
$t_{1(8)}$	-0.0015	0.0013	109.1763	$t_{2(8)}$	-0.0004	0.0013	116.4925
$t_{1(9)}$	-0.0016	0.0014	104.9822	$t_{2(9)}$	-0.0007	0.0013	113.4304
$t_{1(10)}$	-0.0011	0.0016	89.16261	$t_{2(10)}$	-0.0007	0.0015	99.34166
$t_{1(11)}$	-0.0015	0.0013	109.3771	$t_{2(11)}$	-0.0004	0.0013	116.6252
$t_{1(12)}$	-0.1184	0.0332	4.391399	$t_{2(12)}$	-0.1179	0.0331	4.399962
$t_{1(13)}$	-0.0012	0.0016	89.62094	$t_{2(13)}$	-0.0007	0.0015	99.77996
$t_{1(14)}$	-0.0015	0.0013	109.7969	$t_{2(14)}$	-0.0004	0.0012	116.8973
$t_{1(15)}$	-0.0016	0.0015	100.1201	$t_{2(15)}$	-0.0008	0.0013	109.4034
$t_{1(16)}$	-0.0016	0.0015	100.0597	$t_{2(16)}$	-0.0008	0.0013	109.3511
$t_{1(17)}$	-0.0016	0.0015	99.93893	$t_{2(17)}$	-0.0008	0.0013	109.2464

Table 6displays the outcomes of biases, mean squared errors (MSEs), and percentage relative efficiency (PREs) for the existing and proposed estimators when p = 0.9.The findings indicate that, with the exception of $t_{1(4)}, t_{1(7)}, t_{1(10)}, t_{1(12)}, t_{1(13)}, t_{1(17)}, t_{2(10)}, t_{2(12)}$ and $t_{2(13)}$, all the proposed estimators exhibit lower MSEsand higher PREs in comparison to the existing estimator considered in this investigation. Consequently, the proposed estimators in this context are more efficient than their competitor in the study and are likely to yield superiorestimates of the population proportion, particularly in scenarios involving non-response.

5.0 Conclusion

In Surveys involving medical fields, social surveys, household surveys it is common to experience the problem of non-response for a particular unit or units in the population. This non-sampling error may creep and lead to erroneous results due to missing values in the data set. In this study, we proposed two imputation class of estimators for estimating population means in situations when the study variables are characterized with non-responses or missing values. From the results of the empirical study, the proposed estimators performed more efficiently than other estimators considered in the study. Therefore, the proposed procedure for estimating non-responses or missing values in surveys or experiments, holding to its advantages over other considered procedures, is recommended for use for academic or practical purposes.

References

Al-Omari, A.I., Bouza, C.N.and Herrera, C. (2013) “Imputation Methods of Missing Data for Estimating the Populations Mean Using Simple Random Sampling with Known Correlation Coefficient” *Qual. Quant.* 47(1):353–365

Audu, A., Singh, R. and Supriya, K. (2021)“New Regression-Type Compromised Imputation Class of Estimators with Known Parameters of Auxiliary Variable”*Communications in Statistics -Simulation and Computation,* 52(10), 4789-4801.

Audu, A.& Singh, R. V. K. (2021)“Exponential-Type Regression Compromised Imputation Class of Estimators”*Journal of Statistics and Management Systems*, 24(6), 1253-1266.

Azeem, M. (2021) “On Estimation of Population Proportion in Diagonal Systematic Sampling” *Life Cycle Reliability and Safety Engineering*, 10, 249-254.

Bhushan, S. and Pandey, A.P. (2016) “Optimal imputation of missing data for estimation of Population mean.*” J. Stat. Manag. Syst.* 19(6):755–769

Diana, G. and Francesco Perri, P. (2010) “Improved Estimators of the Population Mean for Missing Data”*Communication in Statistics- Theory Methods* 39(18):3245–3251

Gira, A.A. (2015) Estimation of population mean with a New Imputation Methods. *Appl. Math. Sci.*9(34):1663–1672

Wang, H. and Wang, S. (2006) “Visualization of the Critical Patterns of Missing Values in

Classification data" *International Conference on Advances in Visual Information Systems, Springer*, pp. 267–274

Hansen, M.H. and Hurwitz, W.N. (1946) "The Problem of Non-Response in Sample Surveys." *Journal of the American Statistical Association*, 41, 517-529.

Kadilar, C. and Cingi, H. (2008) "Estimators for the Population Mean in the Case of Missing Data" *Communications in Statistics- Theory Methods* 37(14):2226–2236

Khan, Z., Shabbir, J. and Gupta, S. (2017)"New Sampling Design for Systematic Sampling" *Communications in Statistics – Theory and Methods*, 42(18), 3359-3370.

Prasad, S. (2017) "An exponential imputation in the case of missing data."*J. Stat. Manag Syst.* 20(6):1127–1140

Singh, G.N; Maurya, S; Khetan, M;Kadilar, C. (2016) "Some Imputation Methods for Missing Data in Sample Surveys" Hacet *J. Math. Stat.* 45(6):1865–1880

Singh, K., Singh, A., Priyanka and Singh, K.V. (2014) "Exponential-Type Compromised Imputation in Survey Sampling," *Journal of Statistics Application and probability:* Vol. 3:Iss. 2, Artcle 11.

Singh, S. (2009) "A New Method of Imputation in Survey Sampling"*Statistics* 43(5):499–511

Singh, S. and Horn, S. (2000) "Compromised Imputation in Survey Sampling"*Metrika*, 51, 266–276.

Singh, S. and Deo, B. (2003) "Imputation by Power Transformation"*Stat Pap* 44(4):555–579

Toutenburg, H. and Srivastava, V.K. (2008) "Amputation versus Imputation of Missing Values Through Ratio Method in Sample Surveys" *Stat. Pap.* 49(2):237–247

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