Effect of Prandtl Number on Deissler's Decay Law of MHD Turbulence at Four-point Correlations

Abstract

Deissler's decay law plays a great significance in Homogeneous and MHD turbulence flow. Fluid Dynamics are an interesting part of research work which focus on many branches of science, engineering science and also in meteorology. In turbulent flow, the fluid particles show an irregular movement and unpredictable behavior. The effect of Prandtl number on Deissler's energy decompose law of MHD turbulence at 4-point correlations has been described.

Introduction

In fluid dynamics, turbulent flow is a flow system characterized by disorganized and whose performance is actually irregular. In space and time, it shows small momentum circulation, high momentum convection and quick disparity of pressure and velocity. In this case, flow parameters are abruptly changed e.g., kinematics viscosity causes instability of the viscosity. The problem of turbulence is very difficult to solve for the case of nonlinearity. Turbulent flow problems are always treated statistically for its irregular conditions. Turbulent flow is always disorganized but not all disorganized flows are turbulent. In fact, turbulence is an inter-active movement of eddies of different sizes. As a consequence, the velocity at any point varies both in magnitude and direction with respect to time. Such a diffused flow is characterized as turbulent flow. At Reynolds number 4,000, the nature of flow in circular pipe is always assumed to be turbulent.

For turbulent flow, a constant source of energy supply is required because turbulence dissipates energy rapidly as the kinetic energy is converted into internal energy by viscous shear stress. Turbulent fluctuations indicate the energy losses for the velocity and pressure distributions in turbulent flows. Reynolds, O. [11] had the first methodical investigation on turbulent flow. Reynolds [11], is one of the renowned researchers who studied turbulent flow.
In particular, a turbulent flow exhibits all of the features, e.g. disorganized, chaotic, irregular behavior. In brief, turbulent flow exhibits irregular temporal behavior at any selected spatial location. Throughout this work, decay of energy of Magneto-hydrodynamic Turbulent Flow for four- point correlations has been considered. Finally, the result has been established how energy decays due to effect of Prandtl Number**.** Kraichnan's [19] established logically different ideas from previous efforts for direct interaction approximation. Using Dessilar's energy decay law Bkar ,pk *et al*. [22] studied the decay of energy of MHD turbulence for four-point correlation and Bkar ,pk *et al*.[23] generalized it for dust particle system. He also obtained [24] energy decay law for rotating dust particles. Bkar, pk *et al*.[25] also studied effects of first-order reactant on MHD turbulence at four-point correlation. Bkar,pk et al,[26] obtained 4-Point Correlations of Dusty Fluid MHD Turbulent Flow in a 1st order Chemical-Reaction. Further he studied [27]. Taylor, 1921 [15] developed the impression of the Lagrangian correlation coefficient. Tailor, G. I. [13, 14] and Von Karman, T. [17,18] described turbulence in terms of collisions between discrete entities and then set up the thought of velocity correlation at two or more points. Taylor, G. I. derived the "energy spectrum" method to explain the probability density function for energy in the turbulent flow field. The study of turbulence had been generalized by Boussinesq [1] and Reynolds [11]. Reynolds, O. in 1883 [11] first found the remarkable difference between laminar and turbulent ~~flow's~~flows. Based on the problems of practical importance, Prandtl [10] established "mixing length" theory such as pipe flows over borders of exact shapes. In

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1938 Taylor, G. I. [16] discussed the non-linearity of the dynamical equations and found the probability distribution of the difference between the velocity components at two points~~-~~. He further [13] established the design that the velocity of the fluid of turbulent motion is a random continuous function ~~.~~of position and time. Kolmogoroff's [6,7] contributed to understand the physics of turbulence~~-~~. Hopf, E. [4,5] also constructed theory of the characteristic functional to turbulence. Some characteristics of turbulent motion are completed by Kampe de Feriet. J. [3].Applying Fourier transformations they [4, 5, and 3] established the three dimensional energy spectrum functions.~~—~~. Monuar Hossain,*et al,* [28] obtained homogeneous fluid turbulence before the final period of decay for four-point correlation in a rotating system for first-order reactant. Azad *et al*.[29] obtained the effect of chemical reaction on statistical theory of dusty fluid MHD turbulent flow for certain variables at three- point distribution functions .Bkar, pk *et al*.[30] also obtained the effect of first order chemical reaction for ~~coriolis~~Coriolis force and dust particles for small Reynolds number in the atmosphere over territory. Azad *et al.*,[31] established effect of chemical reaction on statistical theory of dusty fluid MHD turbulent flow for certain variables at three- point distribution functions. Shimin Yu et al.,[32] studied the effect of Prandtl number on mixed convective heat transfer from a porous cylinder in the steady flow regime. Using Deissler's decay law [20, 21] and Abdul Malek Ph.D Thesis [33]. Now I am going to study the effect of Prandtl number on Deissler's decay law at ~~four point~~four-point correlations. In this context, a few concepts and mathematical tools for the foundation of MHD turbulence have been discussed. This report shows some aspects of fluid dynamics that are relevant to the Deissler's energy decay law

Four-point Correlation and Spectral Equations

We take the momentum equation of MHD turbulence at the point *p* and the induction equation of magnetic field fluctuation four point correlation and equations at p', p'' and p''' as

$$\frac{\partial u_l}{\partial t} + u_k \frac{\partial u_l}{\partial x_k} - h_k \frac{\partial h_l}{\partial x_k} = -\frac{\partial \omega}{\partial x_l} + \nu \frac{\partial^2 u_l}{\partial x_k \partial x_k} \qquad (1)$$

$$\frac{\partial h'_i}{\partial t} + u'_k \frac{\partial h'_i}{\partial x'_k} - h'_k \frac{\partial u'_i}{\partial x'_k} = \frac{\nu}{p_M} \frac{\partial^2 h'_i}{\partial x'_k \partial x'_k} \qquad (2)$$

$$\frac{\partial h''_j}{\partial t} + u''_k \frac{\partial h''_j}{\partial x''_k} - h''_k \frac{\partial u''_j}{\partial x''_k} = \frac{\nu}{p_M} \frac{\partial^2 h''_j}{\partial x''_k \partial x''_k} \qquad (3)$$

$$\frac{\partial h'''_m}{\partial t} + u'''_k \frac{\partial h'''_m}{\partial x'''_k} - h'''_k \frac{\partial u'''_m}{\partial x'''_k} = \frac{\nu}{p_M} \frac{\partial^2 h'''_m}{\partial x'''_k} \qquad (4)$$

where $\omega = \frac{P}{\rho} + \frac{1}{2}\overline{|h|}^2$ is the total MHD pressure $\rho(x,t)$ =is the hydrodynamic pressure, ρ is the fluid density, $P_M = \frac{\nu}{\lambda}$ is the Magnetic Prandtl number, ν is the kinematics viscosity, λ is the magnetic diffusivity, $h_i(x,t)$ is the magnetic field fluctuation, $u_k(x,t)$ is the turbulent velocity , t is the time, x_k is the space co-ordinate and repeated subscripts are summed from 1 to 3 .

Multiplying equation (1) by $h_i' h_j'' h_m'''$ (2) by $u_l h_j'' h_m'''$ (3) by $u_l h_i' h_m'''$ (4) by $u_l h_i' h_j''$ and adding the four equations, we than taking the space or time averages and they are denoted by $\left(\overline{\ldots\ldots}\right)$ or $\langle\ldots\ldots\ldots\rangle$. We get

$$\frac{\partial}{\partial t}(\overline{u_l h_i' h_j'' h_m'''}) + \frac{\partial}{\partial x_k}(\overline{u_l u_k h_i' h_j'' h_m'''}) - \frac{\partial}{\partial x_k}(\overline{h_k h_l h_i' h_j'' h_m'''}) +$$

$$\frac{\partial}{\partial x_k'}(\overline{u_l u_k h_i' h_j'' h_m'''}) - \frac{\partial}{\partial x_k'}(\overline{u_l u_i' h_k' h_j'' h_m'''}) + \frac{\partial}{\partial x_k''}(\overline{u_l u_k'' h_i' h_j'' h_m'''}) -$$

$$\frac{\partial}{\partial x_k''}(\overline{u_l u_j'' h_i' h_k'' h_m'''}) + \frac{\partial}{\partial x_k'''}(\overline{u_l u_k'' h_i' h_j'' h_m'''}) - \frac{\partial}{\partial x_k'''}(\overline{u_l u_j'' h_i' h_j'' h_m'''}) =$$

$$-\frac{\partial}{\partial x_l}(\overline{w h_i' h_j'' h_m'''}) + \frac{\partial^2}{\partial x_k \partial x_k}(\overline{u_l h_i' h_j'' h_m'''}) + \frac{\nu}{p_M}[\frac{\partial^2}{\partial x_k' \partial x_k'}(\overline{u_l h_i' h_j'' h_m'''}) +$$

$$\frac{\partial^2}{\partial x_k'' \partial x_k''}(\overline{u_l h_i' h_j'' h_m'''}) + \frac{\partial^2}{\partial x_k''' \partial x_k'''}(\overline{u_l h_i' h_j'' h_m'''})] \qquad (5)$$

Using the transformations

$$\frac{\partial}{\partial x_k''} = \frac{\partial}{\partial r_k'}, \ \frac{\partial}{\partial x_k'} = \frac{\partial}{\partial r_k'}, \ \frac{\partial}{\partial x_k} = -(\frac{\partial}{\partial r_k} + \frac{\partial}{\partial r_k'} + \frac{\partial}{\partial r_k''})$$

into equations (5) we get,

$$\frac{\partial(\overline{u_l h_i' h_j' h_m'''})}{\partial t} + (1+P_M)\frac{\partial^2}{\partial r_k' \partial r_k'}(\overline{u_l h_i' h_j'' h_m'''}) + (1+P_M)\frac{\partial^2}{\partial r_k'' \partial r_k''}(\overline{u_l h_i' h_j'' h_m'''}) + 2P_M \frac{\partial^2}{\partial r_k \partial r_k'}(\overline{u_l h_i' h_j'' h_m'''})$$

$$+ 2P_M \frac{\partial^2}{\partial r_k' \partial r_k''}(\overline{u_l h_i' h_j'' h_m'''}) + 2P_M \frac{\partial^2}{\partial r_k \partial r_k''}(\overline{u_l h_i' h_j'' h_m'''})] = \frac{\partial}{\partial r_k}(\overline{u_l u_k h_i' h_j'' h_m'''}) + \frac{\partial}{\partial r_k'}(\overline{u_l u_k h_i' h_j'' h_m'''}) +$$

$$\frac{\partial}{\partial r_k''}(\overline{u_l u_k h_i' h_j'' h_m'''}) - \frac{\partial}{\partial r_k}(\overline{h_l h_k h_i' h_j'' h_m'''}) - \frac{\partial}{\partial r_k'}(\overline{h_l h_k h_i' h_j'' h_m'''}) - \frac{\partial}{\partial r_k''}(\overline{h_l h_k h_i' h_j'' h_m'''}) - \frac{\partial}{\partial r_k}(\overline{u_l u_k' h_i' h_j'' h_m'''}) +$$

$$\frac{\partial}{\partial r_k}(\overline{u_l u_i' h_k' h_j'' h_m'''}) - \frac{\partial}{\partial r_k'}(\overline{u_l u_k'' h_i' h_j'' h_m'''}) + \frac{\partial}{\partial r_k'}(\overline{u_l u_j'' h_i' h_k'' h_m'''}) - \frac{\partial}{\partial r_k''}(\overline{u_l u_k''' h_i' h_j'' h_m'''}) + \frac{\partial}{\partial r_k''}(\overline{u_l u_m''' h_i' h_j'' h_m'''}) +$$

$$\frac{\partial}{\partial r_l}(\overline{w h_i' h_j'' h_m'''}) + \frac{\partial}{\partial r_l'}(\overline{w h_i' h_j'' h_m'''}) + \frac{\partial}{\partial r_l''}(\overline{w h_i' h_j'' h_m'''})) \qquad (6)$$

In order to write the equation (6) to spectral form, we can define the following ~~nine dimensional~~nine-dimensional Fourier transforms

$$\left\langle u_l h_i'(\hat{r}) h_j''(\hat{r}') h_m'''(\hat{r}'')\right\rangle$$

$$= \int_{-\infty}^{\infty}\int_{\infty}^{\infty}\int_{\infty}^{\infty}\left\langle \phi_l \gamma_i'(\hat{k})\gamma_j''(\hat{k}')\gamma_m'''(\hat{k}'')\right\rangle \exp[i(\hat{k}.\hat{r}+\hat{k}'.\hat{r}'+\hat{k}''.\hat{r}'')d\hat{k}d\hat{k}'d\hat{k}'' \qquad (7)$$

$$\left\langle u_l u_k' h_i'(\hat{r}) h_j''(\hat{r}') h_m'''(\hat{r}'')\right\rangle$$

$$= \int_{-\infty}^{\infty}\int_{\infty}^{\infty}\int_{\infty}^{\infty}\left\langle \phi_l \phi_k'(\hat{k})\gamma_i'(\hat{k})\gamma_j''(\hat{k}')\gamma_m'''(\hat{k}'')\right\rangle \exp[i(\hat{k}.\hat{r}+\hat{k}'.\hat{r}'+\hat{k}''.\hat{r}'')d\hat{k}d\hat{k}'d\hat{k}'' \qquad (8)$$

$$\left\langle u_l u_i' h_i'(\hat{r}) h_j''(\hat{r}') h_m'''(\hat{r}'')\right\rangle$$

$$= \int_{-\infty}^{\infty}\int_{\infty}^{\infty}\int_{\infty}^{\infty}\left\langle \phi_l \phi_i'(\hat{k})\gamma_i'(\hat{k})\gamma_j''(\hat{k}')\gamma_m'''(\hat{k}'')\right\rangle \exp[i(\hat{k}.\hat{r}+\hat{k}'.\hat{r}'+\hat{k}''.\hat{r}'')d\hat{k}d\hat{k}'d\hat{k}'' \qquad (9)$$

$$\left\langle u_l u_k'' h_i'(\hat{r}) h_j''(\hat{r}') h_m'''(\hat{r}'')\right\rangle$$

$$= \int_{-\infty}^{\infty}\int_{\infty}^{\infty}\int_{\infty}^{\infty}\left\langle \phi_l \phi_k''(\hat{k}')\gamma_i'(\hat{k})\gamma_j''(\hat{k}')\gamma_m'''(\hat{k}'')\right\rangle \exp[i(\hat{k}.\hat{r}+\hat{k}'.\hat{r}'+\hat{k}''.\hat{r}'')d\hat{k}d\hat{k}'d\hat{k}'' \qquad (10)$$

$$\left\langle u_l u_j'' h_i'(\hat{r}) h_k''(\hat{r}') h_m'''(\hat{r}'')\right\rangle$$

$$= \int_{-\infty}^{\infty}\int_{\infty}^{\infty}\int_{\infty}^{\infty}\left\langle \phi_l \phi_j''(\hat{k}')\gamma_i'(\hat{k})\gamma_j''(\hat{k}')\gamma_m'''(\hat{k}'')\right\rangle \exp[i(\hat{k}.\hat{r}+\hat{k}'.\hat{r}'+\hat{k}''.\hat{r}'')d\hat{k}d\hat{k}'d\hat{k}'' \qquad (11)$$

$$\left\langle u_l u_k h_i'(\hat{r}) h_j''(\hat{r}') h_m'''(\hat{r}'')\right\rangle$$

$$= \int_{-\infty}^{\infty}\int_{\infty}^{\infty}\int_{\infty}^{\infty}\left\langle \phi_l \phi_k \gamma_i'(\hat{k})\gamma_j''(\hat{k}')\gamma_m'''(\hat{k}'')\right\rangle \exp[i(\hat{k}.\hat{r}+\hat{k}'.\hat{r}'+\hat{k}''.\hat{r}'')d\hat{k}d\hat{k}'d\hat{k}'' \qquad (12)$$

$$\left\langle u_l u_i'' h_i'(\hat{r}) h_j''(\hat{r}') h_m'''(\hat{r}'')\right\rangle$$

$$= \int_{-\infty}^{\infty}\int_{\infty}^{\infty}\int_{\infty}^{\infty}\left\langle \phi_l \phi_i''(\hat{k}')\gamma_i'(\hat{k})\gamma_j''(\hat{k}')\gamma_m'''(\hat{k}'')\right\rangle \exp[i(\hat{k}.\hat{r}+\hat{k}'.\hat{r}'+\hat{k}''.\hat{r}'')d\hat{k}d\hat{k}'d\hat{k}'' \qquad (13)$$

$$\left\langle w h_i'(\hat{r}) h_j''(\hat{r}') h_m'''(\hat{r}'')\right\rangle$$

$$= \int_{-\infty}^{\infty}\int_{\infty}^{\infty}\int_{\infty}^{\infty}\left\langle \delta \gamma_i'(\hat{k})\gamma_j''(\hat{k}')\gamma_m'''(\hat{k}'')\right\rangle \exp[i(\hat{k}.\hat{r}+\hat{k}'.\hat{r}'+\hat{k}''.\hat{r}'')d\hat{k}d\hat{k}'d\hat{k}'' \qquad (14)$$

Interchange of points p' *and* p'', p' and p''' the subscripts i and k; i and j results in the relations

$$\overline{u_l u_k''' h_i' h_j'' h_m'''} = \overline{u_l u_k' h_i' h_j'' h_m'''};\ \overline{u_l u_k''' h_i' h_j'' h_m'''} = \overline{u_l u_k' h_i' h_j'' h_m'''};$$

$$\overline{u_l u_m''' h_i' h_j'' h_m'''} = \overline{u_l u_i' h_i' h_k' h_j'' h_m'''};\ \overline{u_l u_j'' h_i' h_k'' h_m'''} = \overline{u_l u_i' h_i' h_k' h_j'' h_m'''};$$

By use of these facts and equations (7) to (14), we can write equation (6) in the form

$$\frac{\partial}{\partial t}\overline{(\phi_l\gamma'_i\gamma''_j\gamma'''_m)} + \frac{\nu}{p_M}[(1+P_M)K^2+(1+p_M)K'^2+(1+p_M)K''^2+2p_M KK'+2p_M KK''+2p_M KK'']$$

$$\overline{(\phi_l\gamma'_i\gamma''_j\gamma'''_m)} = i(K_k+K'_k+K''_k)\overline{(\phi_l\phi_k\gamma'_i\gamma''_j\gamma'''_m)} - i(K_k+K'_k+K''_k)\overline{(\gamma_l\gamma_k\gamma'_i\gamma''_j\gamma'''_m)} - i(K_k+K'_k+K''_k)$$

$$\overline{(\phi_l\phi'_k\gamma'_i\gamma''_j\gamma'''_m)} + i(K_k+K'_k+K''_k)\overline{(\phi_l\phi'_i\gamma'_k\gamma''_j\gamma'''_m)} + i(K_k+K'_k+K''_k)\overline{(\delta\gamma'_i\gamma''_j\gamma'''_m)} \qquad (15)$$

The tensor equation (15) can be converted to the scalar equation by contraction of the indices *i* and *j*;

$$\frac{\partial}{\partial t}\overline{(\phi_l\gamma'_i\gamma''_i\gamma'''_m)} + \frac{\nu}{p_M}[(1+P_M)K^2+(1+p_M)K'^2+(1+p_M)K''^2+2p_M KK'+2p_M KK''+2p_M KK'']$$

$$\overline{(\phi_l\gamma'_i\gamma''_i\gamma'''_m)} = i(K_k+K'_k+K''_k)\overline{(\phi_l\phi_k\gamma'_i\gamma''_i\gamma'''_m)} -$$

$$i(K_k+K'_k+K''_k)\overline{(\gamma_l\gamma_k\gamma'_i\gamma''_i\gamma'''_m)} - i(K_k+K'_k+K''_k)\overline{(\phi_l\phi'_k\gamma'_i\gamma''_i\gamma'''_m)} +$$

$$i(K_k+K'_k+K''_k)\overline{(\phi_l\phi'_i\gamma'_k\gamma''_i\gamma'''_m)} + i(K_k+K'_k+K''_k)\overline{(\delta\gamma'_i\gamma''_i\gamma'''_m)} \qquad (16)$$

If we take the derivative with respect to x_l of the momentum equation (1) at p, we have,

$$-\frac{\partial^2 w}{\partial x_l\partial x_l} = \frac{\partial^2}{\partial x_l\partial x_l}\left(u_l u_k - h_l h_k\right) \qquad (17)$$

Multiplying equation (17) by $h'_i h''_j h'''_m$, taking time averages and writing the equation in terms of the independent variables $\vec{r}$, $\vec{r'}$, $\vec{r''}$ we have,

$$-[\frac{\partial^2}{\partial r_l\partial r_l}+\frac{\partial^2}{\partial r'_l\partial r'_l}+\frac{\partial^2}{\partial r''_l\partial r''_l}+2\frac{\partial^2}{\partial r_l\partial r'_l}+2\frac{\partial^2}{\partial r'_l\partial r''_l}+2\frac{\partial^2}{\partial r_l\partial r''_l}]\overline{(wh'_i h''_j h'''_m)} =$$

$$[\frac{\partial^2}{\partial r_l\partial r_k}+\frac{\partial^2}{\partial r_l\partial r'_k}+\frac{\partial^2}{\partial r'_l\partial r_k}+\frac{\partial^2}{\partial r'_l\partial r'_k}+\frac{\partial^2}{\partial r'_l\partial r''_k}+\frac{\partial^2}{\partial r'_l\partial r''_k}+\frac{\partial^2}{\partial r_k\partial r''_l}+\frac{\partial^2}{\partial r'_k\partial r''_l}+\frac{\partial^2}{\partial r''_l\partial r''_k}](\overline{u_l u_k h'_i h''_j h'''_m} -$$

$$\overline{h_l h_k h'_i h''_j h'''_m}) \qquad (18)$$

$$-(\overline{\delta\gamma_i'\gamma_j''\gamma_m'''}) = \frac{(K_l K_k + K_l K_k' + K_l K_k'' + K_l' K_k + K_l' K_k' + K_l' K_k'' + K_l'' K_k + K_l'' K_k + K_l'' K_k' + K_l'' K_k'')}{K_l K_l + K_l' K_l' + K_l'' K_l'' + 2K_l K_l' + 2K_l' K_l'' + 2K_l K_l''}$$

$$\overline{(\phi_l \phi_k \gamma_i' \gamma_j'' \gamma_m'''} - \overline{\gamma_l \gamma_k \gamma_i' \gamma_j'' \gamma_m'''}) \qquad (19)$$

Equation (19) can be used to eliminate $\left(\overline{\delta\gamma_i'\gamma_j''\gamma_m'''}\right)$ from equation (16) if we take contraction.

Three-point Correlation and Spectral Equations

The spectral equations corresponding to the three-point correlation equations by contraction of the indices *i* and j are

$$\frac{\partial}{\partial t}\overline{(\phi_l \beta_i' \beta_i'')} + \frac{\nu}{p_M}[(1+P_M)(K^2 + K'^2) + 2p_M KK'] \; \overline{(\phi_l \beta_i' \beta_i'')} = \mathrm{i}(K_k + K_k') \; \overline{(\phi_l \phi_k \beta_i' \beta_i'')} -$$

$$\mathrm{i}(K_k + K_k') \; \overline{(\beta_l \beta_k \beta_i' \beta_i'')} - \mathrm{i}(K_k + K_k') \; \overline{(\phi_l \phi_k' \beta_i' \beta_i'')} + \mathrm{i}(K_k + K_k') \; \overline{(\phi_l \phi_i' \beta_i' \beta_i'')} \quad + \mathrm{i}(k_l + k_l')\overline{\gamma \beta_i' \beta_i''}$$

and

$$-(\gamma \; \overline{\beta_i' \beta_j''}) = \frac{(K_l K_k + K_l' K_k + K_l k_k' + K_l' K_k')}{(K_l^2 + K_l'^2 + 2K_l K_l')} \; \overline{(\phi_l \phi_k \beta_i' \beta_i''} - \overline{\beta_l \beta_k \beta_i' \beta_j''})$$

Here the spectral tensors are defined by

$$\left\langle u_l h_i'(\hat{r}) h_j''(\hat{r}')\right\rangle$$
$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \left\langle \phi_l \beta_i'(\hat{k}) \beta_j''(\hat{k}')\right\rangle \exp[i(\hat{k}.\hat{r} + \hat{k}'.\hat{r}') d\hat{k} d\hat{k}'$$

$$\left\langle u_l u_k'(\hat{r}) h_i'(\hat{r}) h_j''(\hat{r}')\right\rangle$$
$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \left\langle \phi_l \phi_k'(\hat{k}) \beta_i'(\hat{k}) \beta_j''(\hat{k}')\right\rangle \exp[i(\hat{k}.\hat{r} + \hat{k}'.\hat{r}') d\hat{k} d\hat{k}'$$

$$\left\langle u_l u_i'(\hat{r}) h_i'(\hat{r}) h_j''(\hat{r}')\right\rangle$$
$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \left\langle \phi_l \phi_i'(\hat{k}) \beta_i'(\hat{k}) \beta_j''(\hat{k}')\right\rangle \exp[i(\hat{k}.\hat{r} + \hat{k}'.\hat{r}') d\hat{k} d\hat{k}'$$

$$\left\langle u_l h_i'(\hat{r}) h_j''(\hat{r}')\right\rangle = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left\langle \phi_l \beta_i'(\hat{k})\beta_j''(\hat{k}')\right\rangle \exp[i(\hat{k}.\hat{r}+\hat{k}'.\hat{r}')d\hat{k}d\hat{k}'$$

$$\left\langle u_l h_k(\hat{r}) h_i'(\hat{r}) h_j''(\hat{r}')\right\rangle = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left\langle \phi_l \beta_k(\hat{k})\beta_i'(\hat{k})\beta_j''(\hat{k}')\right\rangle \exp[i(\hat{k}.\hat{r}+\hat{k}'.\hat{r}')d\hat{k}d\hat{k}'$$

$$\left\langle w h_i'(\hat{r}) h_j''(\hat{r}')\right\rangle = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left\langle \gamma \beta_i'(\hat{k})\beta_j''(\hat{k}')\right\rangle \exp[i(\hat{k}.\hat{r}+\hat{k}'.\hat{r}')d\hat{k}d\hat{k}'$$

Solution Neglecting Quintuple Correlations

Neglecting all the terms on the right side of equation (16), the equation can be integrated between t_1 and t to give

$$\left\langle \phi_l \gamma_i' \gamma_j'' \gamma_m'''\right\rangle = \left\langle \phi_l \gamma_i' \gamma_j'' \gamma_m'''\right\rangle_1 \exp\left[\begin{matrix}\left\{\frac{-\nu}{p_M}(1+p_M)(k^2+k'^2+k''^2+2kk'+2k'k''+2kk')\right\} \\ (t-t_1)\end{matrix}\right] \quad (20)$$

where $\left\langle \phi_l \gamma_i' \gamma_j'' \gamma_m'''\right\rangle_1$ is the value of $\left\langle \phi_l \gamma_i' \gamma_j'' \gamma_m'''\right\rangle$ at $t= t_1$ that is stationary value for small values of *k*, k' and k'' when the quintuple correlations are negligible.

$$\frac{\partial}{\partial t}\overline{(k_k \phi_l \beta_i' \beta_i'')} + \frac{\nu}{p_M}[(1+P_M)(K^2+K'^2)+2p_M KK']\ \overline{(k_k \phi_l \beta_i' \beta_i'')} =$$

$$[a]_1 \int_{-\infty}^{\infty} \exp[-\frac{\nu}{p_M}(t-t_1)\{(1+p_M)(k^2+k'^2+k''^2)+2p_M(kk'+k'k''+k''k)\}]dk'' +$$

$$[b]_1 \int_{-\infty}^{\infty} \exp[-\frac{\nu}{p_M}(t-t_1)\{(1+p_M)(k^2+k'^2+k''^2)+2p_M kk'-2p_M k''k)\}]dk'' +$$

$$[c]_1 \int_{-\infty}^{\infty} \exp[-\frac{\nu}{p_M}(t-t_1)\{(1+p_M)(k^2+k'^2+k''^2)+2p_M kk'-2p_M k''k')\}]dk'' \quad (21)$$

At t_1, $\gamma^{,s}$ have been assumed independent of; that assumption is not, made for other times. This is one of several assumptions made concerning the initial conditions, although continuity equation satisfied the conditions. The complete specification of initial turbulence is difficult; the assumptions for the initial conditions made here in are partially on the basis of simplicity. Substituting $dk'' = dk_1'' dk_2'' dk_3'' and$ integrating with respect to k_1'', k_2'', k_3'' and we get,

$$\frac{\partial}{\partial t}\overline{(k_k \phi_l \beta_i' \beta_i'')} + \frac{\nu}{p_M}[(1+P_M)(K^2+K'^2)+2p_M KK'] \ \overline{(k_k \phi_l \beta_i' \beta_i'')} =$$

$$\left(\frac{\pi p_M}{\nu(t-t_1)(1+p_M)}\right)^{\frac{3}{2}}[a]_1 \, exp\,[\frac{\nu(t-t_1)(1+p_M)}{p_M}\{\frac{(1+2P_M)(k^2+k'^2)}{(1+p_M)^2}+\frac{2p_M kk'}{(1+p_M)^2}\}]+$$

$$\left(\frac{\pi p_M}{\nu(t-t_1)(1+p_M)}\right)^{\frac{3}{2}}[b]_1 \, exp[\frac{\nu(t-t_1)(1+p_M)}{p_M}\{\frac{(1+2P_M)(k^2)}{(1+p_M)^2}+\frac{2p_M kk'}{(1+p_M)}+k'^2\}]$$

$$+\left(\frac{\pi p_M}{\nu(t-t_1)(1+p_M)}\right)^{\frac{3}{2}}[c]_1 \, exp[-\frac{\nu(t-t_1)(1+p_M)}{p_M}\{k^2+\frac{(1+2P_M)(k'^2)}{(1+p_M)^2}+\frac{2p_M kk'}{(1+p_M)}\}] \quad (22)$$

Which result in

$$\frac{\partial H}{\partial t}+\frac{2\nu k^2}{p_M}H = G$$

where

$$G= k^2\int_{-\infty}^{\infty} 2\pi.i[\langle k_k \phi_l \beta_i' \beta_i''(\hat{k},\hat{k}')\rangle - \langle k_k \phi_l \beta_i' \beta_i''(-\hat{k},-\hat{k}')\rangle]_0 .$$

$$\exp[-\frac{\nu}{p_M}(t-t_0)\{(1+p_M)(k^2+k'^2)+2p_M kk'\}]dk' + k^2\int_{-\infty}^{\infty}\frac{2p_M.\pi^{\frac{5}{2}}}{\nu}i\left[b(\hat{k}.\hat{k}')-b(-\hat{k}.-\hat{k}')\right]_1 .$$

$$\{-\ \omega^{-1}\exp\left[(-\omega^2)\left\{\frac{(1+2p_M)(k^2)}{(1+p_M)^2}+\frac{2p_M kk'}{(1+p_M)}+k'^2\right\}\right]$$

$$+k.exp[(-\omega^2)\left((1+p_M)(k^2+k'^2)+2p_M kk'\right)]\int_0^{\frac{\omega k}{2}}\exp(x^2)dx\}dk' +$$

$$k^2\int_{-\infty}^{\infty}\frac{2p_M.\pi^{\frac{5}{2}}}{\nu}i\left[c(\hat{k}.\hat{k}')-c(-\hat{k}.-\hat{k}')\right]_1 .$$

$$\{-\ \omega^{-1}\exp\left[(-\omega^2)\left\{k^2+\frac{(1+2p_M)(k'^2)}{(1+p_M)^2}+\frac{2p_M kk'}{(1+p_M)}\right\}\right]+$$

$$k'\ exp\ [-\omega^2(\ (1+p_M)(k^2+k'^2)+2p_M kk'\ )].\ \int_0^{\frac{\omega k'}{2}}\exp(x^2)dx\}dk' \qquad (23)$$

Here H is the magnetic energy spectrum function, which represents contributions from various wave numbers (or eddy sizes) to the energy and G is the energy transfer function, which is responsible for the transfer of energy between wave numbers, equation (23) which depends on the initial conditions.

$$(2\pi)^2[\langle k_k\phi_l\beta_i'\beta_i''(\hat{k},\hat{k}')\rangle-\langle k_k\phi_l\beta_i'\beta_i''(-\hat{k},-\hat{k}')\rangle]_0=-\xi_0(k^2k'^4-k^4k'^2) \qquad (24)$$

where ξ_0 is a constant depending on the initial conditions. For the other bracketed quantities above equation is

$$\frac{4p_M.\pi^{\frac{7}{2}}}{\nu}i\left[b(\hat{k}.\hat{k}')-b(-\hat{k}.-\hat{k}')\right]_1=\frac{4p_M.\pi^{\frac{7}{2}}}{\nu}i\left[c(\hat{k}.\hat{k}')-c(-\hat{k}.-\hat{k}')\right]_1$$
$$=-2\xi_1(k^4k'^6-k^6k'^4) \qquad (25)$$

Remembering that $d\hat{k}'=-2\pi.\hat{k}'^2d(\cos\theta)$ and $kk'=kk'\cos\theta,\theta$ is the angle between $\hat{k}$ and $\hat{k}'$ and carrying out the integration with respect to θ , we get,

$$G=-\int_0^{\infty}[\frac{\xi_0(k^2k'^4-k^4k'^2)kk'}{\nu(t-t_0)}\{\quad .\quad \exp[-\frac{\nu}{p_M}(t-t_0)\{(1+p_M)(k^2+k'^2)-2p_M kk'\}]-\quad .$$

$$\exp[-\frac{\nu}{p_M}(t-t_0)\{(1+p_M)(k^2+k'^2)+2p_M kk'\}]\}$$

$$+\frac{\xi_1(k^4k'^6-k^6k'^4)kk'}{\nu(t-t_0)}\ (\ \omega^{-1}\exp[-\omega^2\left(\frac{(1+2p_M)k^2}{(1+p_M)^2}-\frac{2P_M kk'}{1+p_M}+k'^2\right)]-$$

$$\omega^{-1}\exp[-\omega^2\left(\frac{(1+2p_M)k^2}{(1+p_M)^2}+\frac{2P_M kk'}{1+p_M}+k'^2\right)]+\omega^{-1}$$

$$\exp[-\omega^2\left(k^2-\frac{2P_M kk'}{1+p_M}+\frac{(1+2p_M)k'^2}{(1+p_M)^2}\right)]$$

$$-\ \omega^{-1}\exp[-\omega^2\left(k^2+\frac{2P_M kk'}{1+p_M}+\frac{(1+2p_M)k'^2}{(1+p_M)^2}\right)]$$

$$+\{k\ exp[-\omega^2(\ (1+p_M)(k^2+k'^2)-2p_M kk'\ )]$$

$$-k.exp[-\omega^2(\ (1+p_M)(k^2+k'^2)+2p_M kk'\ )]\}\int_0^{\frac{\omega k}{2}}\exp(x^2)dx+\{\ k'\ \exp\ [-\omega^2(\ (1+p_M)(k^2+k'^2)-2p_M kk'$$

$$)]$$

$$-k'\exp[-\omega^2(\ (1+p_M)(k^2+k'^2)+2p_M kk'\ )]\}\int_0^{\frac{\omega k'}{2}}\exp(x^2)dx)]\ dk' \qquad (26)$$

where $\omega=\left(\frac{\nu(t-t_1)(1+p_M)}{p_M}\right)^{\frac{1}{2}}$.

Integrating equation (26) with respect to k' .We has

$$G=G_\beta+G_\gamma \qquad (27)$$

where, $G_\beta=-\frac{\pi^{\frac{1}{2}}\xi_0 p_M^{\frac{5}{2}}}{\nu^{\frac{3}{2}}(t-t_0)^{\frac{3}{2}}(1+p_M)^{\frac{5}{2}}}\exp\left\{-\frac{\nu(t-t_0)(1+2p_M)k^2}{p_M(1+p_M)}\right\}$

$$.\left[\frac{15p_M k^4}{4\nu^2(t-t_0)^2(1+p_M)}+\left\{\frac{5p_M{}^2}{(1+p_M)^2\nu(t-t_0)}-\frac{3}{2\nu(t-t_0)}\right\}k^6+\frac{p_M}{1+p_M}\left\{\frac{p_M{}^2}{(1+p_M)^2}-1\right\}k^8\right] \qquad (28)$$

and, $G_\gamma=G_{\gamma_1}+G_{\gamma_2}+G_{\gamma_3}+G_{\gamma_4}$

The quantity G_β represents the transfer function arising owing to consideration of magnetic field at three point correlation equation; G_γ arises from consideration of the four –point equation. Integration over all wave number shows that

$$\int_0^\infty G.d\vec{k}=0 \qquad (29)$$

Indicating that the expression for G satisfies the conditions of continuity and homogeneity, physically, it was to be expected, since G is a measure of transfer of energy and the numbers must be zero. Hence

$$H=\exp\left[-\frac{2\nu k^2(t-t_0)}{p_M}\right]\int G\exp\left[-\frac{2\nu k^2(t-t_0)}{p_M}\right]dt+J(k)\exp\left[-\frac{2\nu k^2(t-t_0)}{p_M}\right]$$

where $J(k)=\frac{N_0k^2}{\pi}$ is a constant of integration and can be obtained as by Corrsin [2]

$$H=\frac{N_0k^2}{\pi}\exp\left[\frac{-2\nu k^2(t-t_0)}{p_M}\right]+\exp\left[\frac{-2\nu k^2(t-t_0)}{p_M}\right]\int[G_\beta+(G_{\gamma_1}+G_{\gamma_2}+G_{\gamma_3}+G_{\gamma_4})\ ]\exp\left[\frac{-2\nu k^2(t-t_0)}{p_M}\right]\text{dt}$$

where, $G=G_\beta+G_{\gamma_1}+G_{\gamma_2}+G_{\gamma_3}+G_{\gamma_4}$

Then after integration equation

$$H=\frac{N_0k^2}{\pi}\exp\left[-\frac{2\nu k^2(t-t_0)}{p_M}\right]+H_\beta+[H_{\gamma_1}+H_{\gamma_2}+H_{\gamma_3}+H_{\gamma_4}\ ] \qquad (30)$$

The terms $H_\beta, H_{\gamma_1}, H_{\gamma_2}, H_{\gamma_3}$ and H_{γ_4} can be expressed as follows:

$$H_\beta=\frac{\xi_0\pi^{\frac{1}{2}}p_M{}^{\frac{5}{2}}}{8\nu^{\frac{3}{2}}(1+p_M)^{\frac{7}{2}}}\exp\left(\frac{-\nu(t-t_0)(1+2p_M)}{p_M(1+p_M)}\right)k^2$$

$$[\frac{3p_Mk^4}{2\nu^2(t-t_0)^{5/2}}+\left(\frac{(7p^2{}_M-6p_M)}{3\nu(1+p_M)(t-t_0)^{3/2}}\right)k^6-\left(\frac{(3p^2{}_M-2p_M+3)}{3(1+p^2{}_M)(t-t_0)^{1/2}}\right)k^8$$

$$+\left(\frac{8\nu^{1/2}(3p^2{}_M-2p_M+3)}{3(1+p_M)^{5/2}p^{1/2}{}_M}\right)k^9F(\omega)],$$

$$F(\omega)=\exp(-\omega^2)\int_0^\omega\exp(x^2)dx,\omega=[\frac{\nu(t-t_0)}{p_M(1+p_M)}]^{1/2}k,$$

Here H_1 and H_2 magnetic energy spectrum arising from consideration of the three and four –point correlation equations respectively. The total magnetic turbulent energy is

$$\frac{\langle h_i h_i' \rangle}{2} = \int_0^\infty H dk \tag{31}$$

here

$$\int_0^\infty H_1 dk = \frac{N_0 p^{\frac{3}{2}}{}_M \nu^{-\frac{3}{2}} (t-t_0)^{-\frac{3}{2}}}{8\sqrt{2\pi}} + \xi_0 Q \nu^{-6} (t-t_0)^{-5},$$

$$\int_0^\infty H_2 dk = \xi_1 [R\nu^{-\frac{17}{2}} (t-t_1)^{-\frac{15}{2}} + S\nu^{-\frac{19}{2}} (t-t_1)^{-\frac{17}{2}}],$$

$$L_1 = Q_2 + Q_4 + Q_6 + Q_7, L_2 = Q_1 + Q_3 + Q_5$$

By using above values,equation (31) we get

$$\frac{\langle h_i h_i' \rangle}{2} = \frac{N_0 p^{\frac{3}{2}}{}_M \nu^{\frac{-3}{2}} (t-t_0)^{\frac{-3}{2}}}{8\sqrt{2\pi}} + \xi_0 . Q . \nu^{-6} . (t-t_0)^{-5}$$

$$+ [\xi_1 L_1 \nu^{-\frac{17}{2}} (t-t_1)^{-\frac{15}{2}} + \xi_1 L_2 \nu^{-\frac{19}{2}} (t-t_1)^{-\frac{17}{2}}] \tag{32}$$

$$\langle h^2 \rangle = A(t-t_0)^{-3/2} + B(t-t_0)^{-5} + C(t-t_1)^{-15/2} + D(t-t_1)^{-17/2}, \tag{33}$$

This is the energy decay law of MHD turbulence for four point correlations.where,

$\langle h^2 \rangle = \langle h_i h_i' \rangle$, A= $\frac{N_0 p^{\frac{3}{2}}{}_M \nu^{-\frac{3}{2}}}{4\sqrt{2\pi}}$, *B*=2 $\xi_0 Q \nu^{-6}$, $C = 2\xi_1 L_1 \nu^{-\frac{17}{2}}$ and $D = 2\xi_1 L_2 \nu^{-\frac{19}{2}}$.

If L_1 =0 and L_2 =0 that is *C*=0 and *D*=0 in equation (33) than we get,

$$\langle h^2 \rangle = A_1 (t-t_0)^{-3/2} + B_1 (t-t_0)^{-5} \tag{34}$$

This is the energy decay of MHD turbulence in three- point correlations which was obtained earlier by Sarker and Kishore [12]

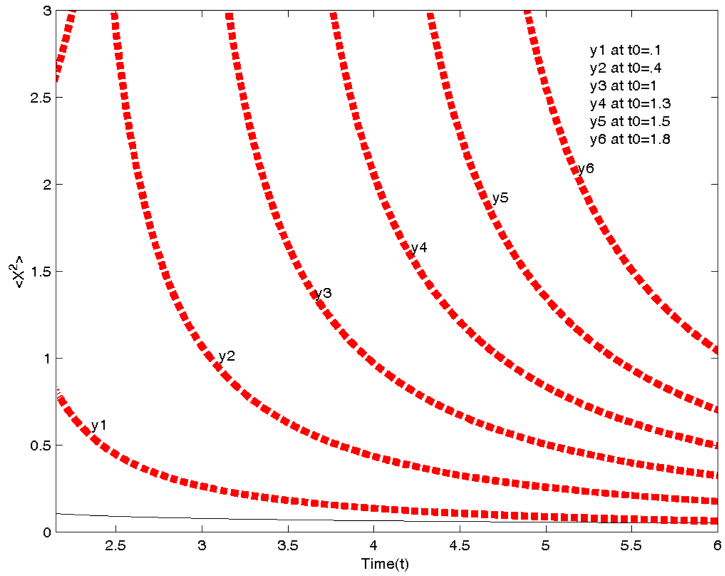
Table:

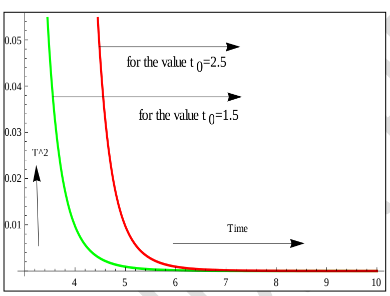
Table-1:The value of the constants and parameter used in equation (33)

Fluid	P_M	ν	N_0	ξ_0	ξ_1	A	B	C	D
Mercury	0.015	0.10	.1	.01	.02	.00058	4.18×10^{-7}	3.69×10^{-13}	5.87
	0.015	0.08	.1	.01	.02	.00081	1.6×10^{-6}	-1.01×10^{-12}	20.03
Mix Gas	0.2	80	.1	.01	.02	1.15×10^{-6}	5.75×10^{-18}	3.78×10^{-16}	9.95×10^{-13}
	0.2	200	.1	.01	.02	3.15×10^{-7}	2.36×10^{-20}	6.12×10^{-18}	6.44×10^{-15}
Hyd Gas	.04	100	.1	.01	.02	2.5×10^{-6}	6.8×10^{-17}	2.7×10^{-14}	9.79×10^{-13}
	0.4	300	.1	.01	.02	4.86×10^{-7}	9.4×10^{-20}	1.9×10^{-16}	2.3×10^{-15}
Hel Gas	0.7	120	.1	.01	.02	4.6×10^{-6}	4.8×10^{-16}	7.4×10^{-13}	9.4×10^{-23}
	0.7	400	.1	.01	.02	7.6×10^{-7}	3.4×10^{-19}	3.3×10^{-15}	1.2×10^{-15}

The graphical representations and explanations

Figure 1:Sketch of equation(33)





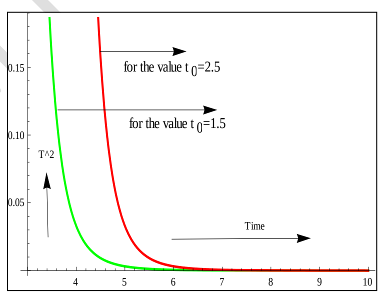
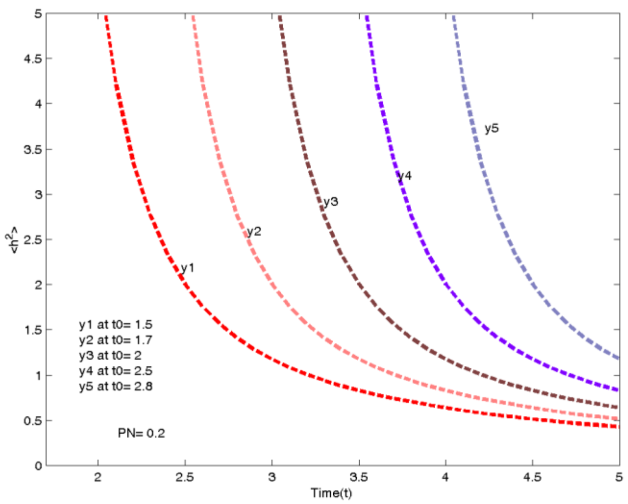


Figure1 (a): Sketch of equation(33)   Figure1 (b): Sketch of equation(33)

Figure 1(a), Figure 1(b) represents the energy decay curve for four-point correlations of equation (33). When the Prandtl no. is small as of mercury P_M =0.015 and It is observed that the energy decreases more rapidly as viscosity decreases.

Figure 2: Sketch of equation(33)



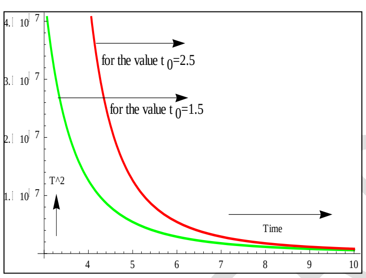


Figure-(2a): Sketch of equation(33)

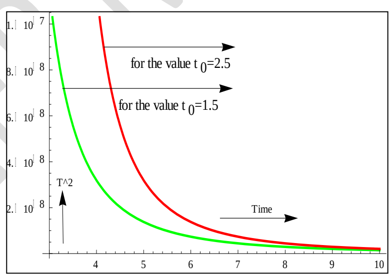
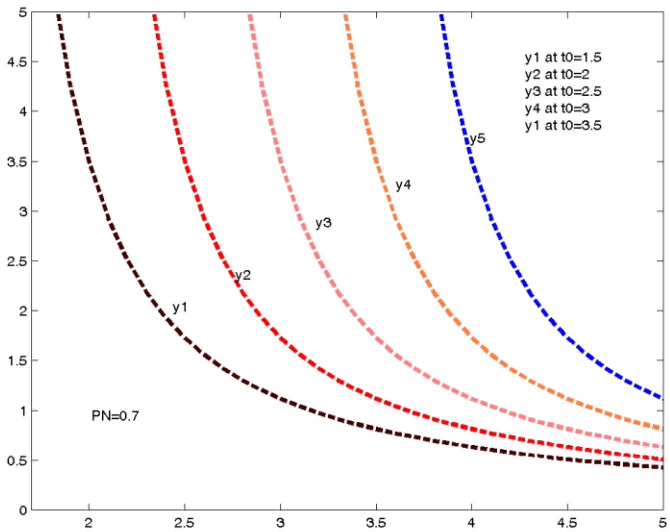


Figure-(2b): Sketch of equation(33)

Figure-(2a) and Figure-(2b) are the energy curve of equation (33) when the Prandtl no. is as of mixture of gas for P_M =0.2 and ν =80 in fig. (2a) and ν =200 in fig. (2b). In this case, energy decreases rapidly as viscosity decreases.

Figure 3: Sketch of equation(33)



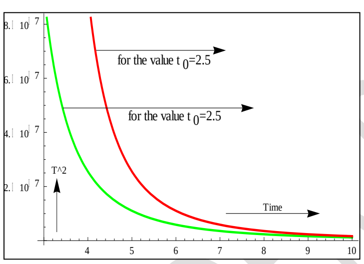


Figure-(3a): Sketch of equation(33)

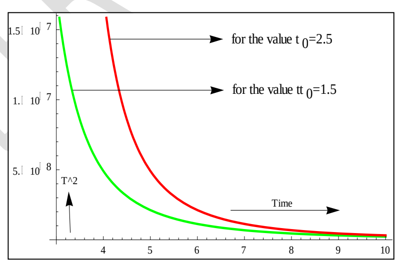
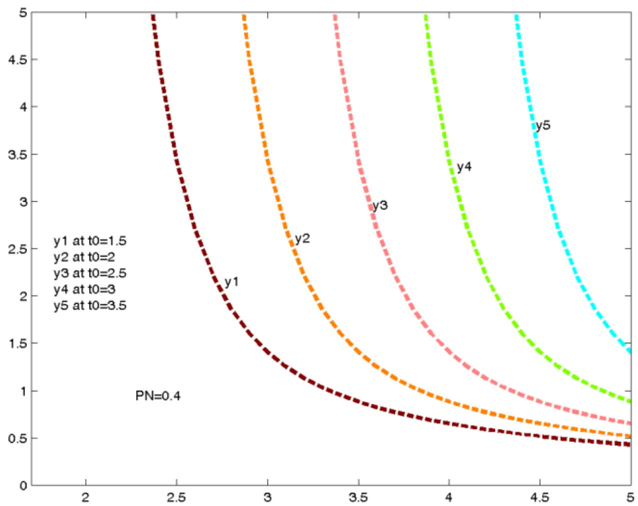


Figure-(3b): Sketch of equation(33)

Figure-(3a), Figure-(3b) indicate the curve of energy equation (33). When the Prandtl no. is as of Hydrogen gas, P_M =0.4 and ν =100 and ν =300. Result: Energy decreases as well as viscosity ~~decries.~~decreases

Figure 4: Sketch of equation(33)



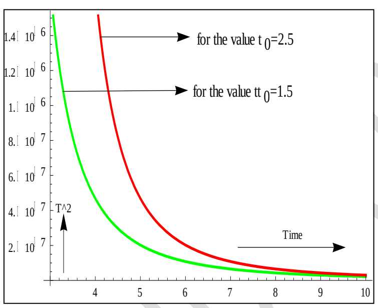


Figure-(4a): Sketch of equation(33)

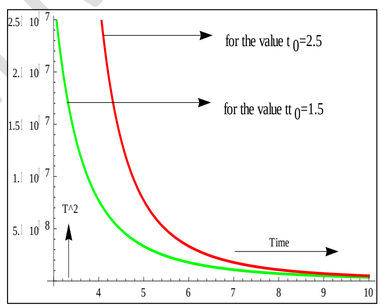


Figure-(4b): Sketch of equation(33)

Figure-(4a) and Figure-(4b) are the energy curve of equation (33). When the Prandtl No. is as of Helium gas P_r=0.7 and ν =120 and ν =400. Energy decreases rapidly as viscosity decreases from 400 to 120.

Comparing fig (1a)-(4b): we see that Energy changes rapidly as Prandtl no. changes. Figure (1)-(4): $y1$, $y2$, $y3$, $y4$, $y5$and $y6$ are represented the energy decay curves of MHD turbulence for four-point correlations of equation (33) at several times From figure 1 and Figure 4, we see that, in four- point correlations system energy die out faster than the three- point correlations system in MHD turbulent flow.

Comparison between four -point and three point correlations of equation :

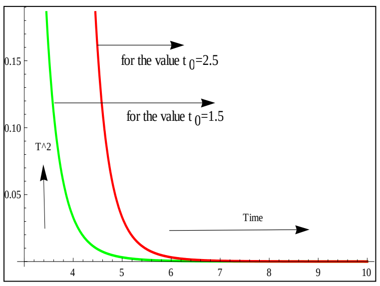


Fig (5a): Energy curves of equation. (33)

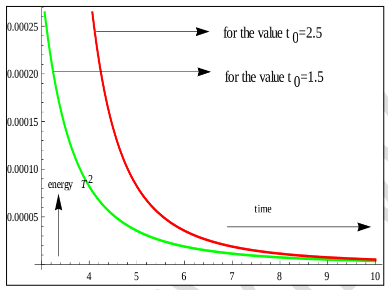


Fig (5b): Energy curves of equation. (34)

Fig-(5a) and Fig (5b) represents the energy decay curve for four-point and three-point correlations of equation . When the Prandtl no. is small as of mercury P_M =0.015. It is clear that, in four-point correlations energy decreases more rapidly than three point correlations.

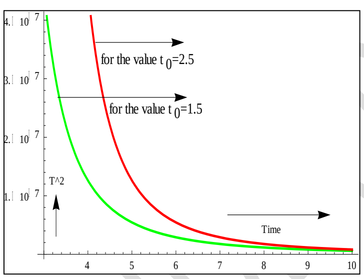


Fig (6a): Energy curves of equation. (33)

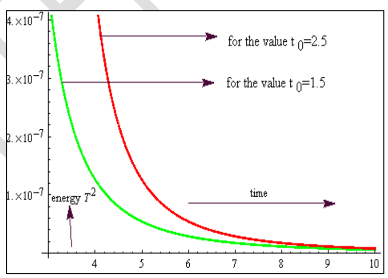
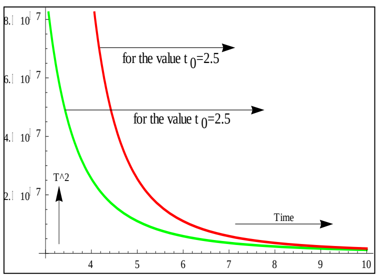


Fig (6b): Energy curves of equation. (34)

When the Prandtl no. is as of mixture of gas P_M =0.2 i.e. for large Prandtl no. we conclude that, energy at four- point correlations and three -point correlations has no change significantly.



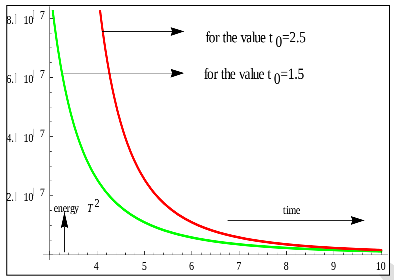


Fig (7a) Energy curves of equ.(33)

Fig (7b) Energy curves of equ.(34)

Fig-(7a) and Fig-(7b) indicate the energy curve equation (33) and (4.5.1). When the Prandtl no. as of Hydrogen gas P_M =0.4 .

We observed that there is no change in energy for four point and three point correlations as for same viscosity

Conclusion

- For mercury, I observed that the energy decreases more rapidly as viscosity decreases.
- In Helium gas for P_r=0.7 and ν =120 and ν =400. Energy decreases rapidly as viscosity decreases .
- It is observed that the decay law for four-point correlations systems energy decreases rapidly more and more by exponential manner than the decreases of three point correlation systems.
- I observed that there is no change in energy for four point and three point correlations as for same viscosity.
- If the time increases than energy decay also increases.
- We finally conclude that from all above the figures that energy decreases as viscosity and Prandtl number decrease.

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