Performance Comparison of Imputation Methods for Mixed Data Missing at Random with Small and Large Sample Data Set with Different Variability

ABSTRACT

One of the concerns in the field of statistics is the presence of missing data, which leads to bias in parameter estimation and inaccurate results. However, the multiple imputation procedure is a remedy for handling missing data. This study looked at the best multiple imputation methods used to handle mixed variable datasets with different sample sizes and variability along with different levels of missingness. The study employed the predictive mean matching, classification and regression trees, and the random forest imputation methods. For each dataset, the multiple regression parameter estimates for the complete datasets were compared to the multiple regression parameter estimates found with the imputed dataset. The results showed that the random forest imputation method was the best for mostly a sample of 500 irrespective of the variability. The classification and regression tree imputation methods worked best mostly on sample of 30 irrespective of the variability.

Key words: predictive mean matching, classification and regression tree, random forest, multiple imputation chained equation.

1. INTRODUCTION

Missing data is considered as an unstored data value for a variable in observation of interest [1]. As complete data sets are needed to help firms and institutions to produce more accurate and precise results, the presence of missing data rather leads to inaccurate results, bias in parameter estimation and reduction in statistical power. Missing data invariably give rise to reduced sample size and thus, leads to a less precise confidence interval and reduced power in the tests of significance. All these pitfalls lead to incorrect conclusions and invalid recommendations.

The study assesses the best multiple imputations by chain equation (MICE) procedure for handling missing data for large and small mixed data sets with different variability and with different percentage levels of missingness. One of the fundamental assumptions made was that the missing data were missing at random.

2. TYPES OF MISSING DATA

While the reason for missing data is difficult to establish in a survey with some reasons being the unwillingness on the part of respondents to answer private questions or the forgetfulness to answer certain questions, it is still imperative to carefully examine the pattern of missingness in data to set out the appropriate mechanism to handle such missing data.

According to Rubin [5]; Little and Rubin [28]; Diggle et al. [29]; Diggle and Kenward [30], there are three types of missing data: missing completely at random (MCAR), missing at random (MAR), and missing not at random (MNAR).

When the missingness of data is a result of observed and unobserved (missing) data, then the data is missing completely at random (MCAR) [3]. In this case, the probability of missingness is independent of the observed and unobserved data [4]. With the missing Y value (Y_{miss}*) and* observed Y value (Y_{obs}) the probability of missing Y value is given as $P\ (R|\emptyset)$

where R is an indicator function with 0 representing a missing value and 1 as an observed value; and $\emptyset$ describes the relation between the data and R. Data that is completely missing at random is considered a simple random sample. To test for MCAR assumption, we separate the data into two categories and then, test the difference between the two groups using a two-sample *t*-test. If there is a significant difference between the two groups, the MCAR assumption is satisfied.

When missing data is due to observed data but not unobserved data, then the data is missing at random (MAR) [5, 6]. The missing data is conditional on the observed variable. We can denote this as $P\ (Y_{miss}\ |Y,\ X) = P\ (Y_{miss}\ |X)$ where Y is considered as a missing value, but X is always observed. If a data is considered MAR, then some complete case analyses are valid under weaker assumption than MCAR. When missing data is ignorable (any information about the missing data is not included when dealing with the missing data) and the missing data does not need to be modeled in the analysis of the dataset, then the MAR assumption is satisfied. However, when the missing data is non-ignorable, then the modeling of the missing data leads to accurate parameter estimation. As of now, the MAR mechanism cannot be tested. When dealing with data that are completely missing at random, biased parameter estimates are produced and there is also a loss of statistical power.

When missing data is due to unobserved data but not observed data, then the data is missing not at random (MNAR). The probability of missingness is associated with the missing value itself [4]. The MNAR produces small and biased parameter estimates. Data which is MNAR is non-ignorable since information of the missing data is required and most models are also not precise with this form of missingness. The probability distribution of MNAR is given as $P\ (R\ |\ Y_{obs};\ Y_{mis};\ \emptyset)$, where the missing Y value is Y_{miss}; the observed Y value is Y_{obs} ; R represents the missing data indicator and $\emptyset$ describes the relation between the data and R. MNAR data cannot be tested.

3. METHODS OF IMPUTATION

Several methods have been proposed on how to handle missing data and can be broken down into two categories: traditional and modern methods. The traditional methods are comprised of the deletion methods (such as pairwise and listwise deletion) and the single imputation methods (such as arithmetic mean imputation, regression imputation, and stochastic regression imputation). The modern methods of handling missing data are further broken into two approaches: joint modeling method and multiple imputation of chained equations (MICE).

3.1. Traditional Methods

The two most common traditional methods of handling missing data are the listwise deletion and the pairwise deletion. With the listwise deletion, also known as the complete case analysis (CCA), when at least one value is missing from the entire observation, then the entire observation is dropped from the analysis [7] which is the main shortfall. With this method, there is an assumption that a random sample chosen from the originally targeted sample is collected to represent the complete case, [7] which is not the case in real data since there is often a reason why a data value might be missing.

The pairwise deletion method involves the removing cases on an analysis-by-analysis basis which minimizes the loss that results from the listwise deletion [8]. In pairwise deletion, variables with missing information are deleted in a specific analysis. Else, variables with complete information have their cases included in the analysis. According to Graham (2009), biased parameter estimates are produced because of the diverse sample sizes used in the pairwise deletion method. One of the main shortfalls of the two deletion methods is that the data are missing completely at random. However, the MCAR data can lead to reduced sample size, loss of statistical power, and then generate biased parameter estimates [9] and thus, the deletion methods are not ideal in most situations.

The single imputation methods are another traditional way of handling missing data. With the arithmetic mean imputation, all cases of missing values for a particular variable are replaced with the computed arithmetic mean for that particular variable. Since the mean is biased towards outliers, the arithmetic mean method can affect the parameter estimate and variability of the data.

With the regression imputation method, a regression model predicts the missing value, and the estimated response value replaces the missing data. The regression imputation method produces biased parameter estimates even though it is a better method as compared to the arithmetic mean method.

With the stochastic regression imputation, which is a way to improve regression imputation, accounts from the variability in the predicted incomplete values. This method adds a random error to the predicted value from the regression and able to reproduce the appropriate correlation between the missing value and observed terms. The shortfall of the stochastic regression imputation is that the complexity that arises from the several missing data in multivariate data since each missing data require a unique regression equation. With the response pattern imputation, this method can generate relatively accurate parameter estimates with MCAR data and bias estimates when dealing with MAR data [11].

The most obvious drawback of single imputation is the main assumption of considering the true value as the imputed value. This drawback underestimates of the variance, thus affects statistical tests and confidence interval [27].

3.2. Modern Methods

The shortcomings associated with the traditional methods of handling missing data led to the adoption and implementation of modern methods to handle missing data with high accuracy.

3.2.1. Joint Modeling

The joint modeling (JM) method of handling missing data is most appropriately used when dealing with time-to-event data (data which occur when attention is fixated on the time elapsing prior to experiencing an event) and longitudinal data since the JM gives an efficient estimate of the treatment effect hence decreases the bias in the treatment effect [12]. The time-to-event component and longitudinal component serve as the two components of the joint modeling method. JM comprises of a linear model with a random effect [12].

The model is built on a multivariate distribution. Mostly, the JM model is based commonly on the multivariate normal distribution, which is used to draw missing data simultaneously from all incomplete variables [14]. With the JM method, the missing data are partitioned into groups of identical patterns and the joint model, which is common to all the observations are used to impute the missing entries with each group of the identical missing data pattern. For more information on JM, see [35, 36].

3.2.2. Multiple Imputation of Chained Equation

Multiple imputation of chained equations (MICE), also known as fully conditional specification (FCS) is used for the computation of multiple imputations instead of a single imputation. The multiple imputation method resolves the impreciseness and uncertainties in single imputation. When the cause for the missing value is unknown, then the multiple imputation method aims to provide valid inference [27]. MICE is required when a multivariate distribution is inappropriate, unknown, or both unlike the JM method that requires the assumption of a known multivariate distribution [16]. Unlike the JM, the MICE method imputes variables one-by-one from series of the univariate conditional distribution. One main advantage of the MICE approach is that the method is flexible to the type of data. It can impute data for binary, categorical, and quantitative variables including data sets with mixed type of data.

The first stage of the multiple imputation chained equation (MICE) process, also termed the imputation stage, involves creating a complete data set by substituting the missing values with estimated values using a multiple imputation (MI) method based on the type of variable(s). The second stage, called the analysis stage, involves analyzing the complete data in the first stage with a statistical method of interest. The pooling stage, which is the final stage, generates single point estimates for the missing observations by merging the analyzed results in the second stage.

4. MI METHODS FOR MIXED DATA

Data containing both quantitative and categorical variable (mixed data) that has missing values can be imputed using several different methods. The methods focused in this paper includes classification and regression trees, predictive mean matching, and random forest.

4.1. Classification and Regression Trees

Classification and regression trees (CART), similarly identified as decision trees, are used to impute missing values. For the classification tree, the predicted response is the class that contains the data while in the regression tree, the predicted response is a real number. The implementation of the imputation method in CART is done by first using the observed data to fit the classification and regression tree. Then the prediction of the terminal node of the fitted tree where each missing observation finally ends up is determined. Finally, the observed value which is derived from a random draw for the elements in the node is regarded as the imputation [23].

4.2. Predictive Mean Matching

The predicted mean matching (PMM) method takes values from observed data to impute missing values which preserves the distribution of the observed data in the missing, thus enables the PMM method to generate realistic values [2]. With PMM, corresponding values from the complete case that are most similar to the missing values replace these missing values [18]. Even when the structural part of the imputation is incorrect, the PMM preserves the non-linear relation which serves as an advantage for using the PMM method [16]. When the assumption is normality is breached, the PMM is considered more suitable than regression even though the PMM is alike to the regression approach [24]. The imputed values are mostly realistic and a good representation of the possible missing value. On the other hand, the PMM method does not work properly on small sample sizes because the PMM does not emphasize on the between imputation variability with small number of predictors [16].

4.3. Random Forest Approaches

The random forest is considered as the collection of several decision trees fit with training data. The random forest is used to impute missing values for continuous variables by drawing randomly from an independent normal distribution, centered on means predicted by the random forest. On the other hand, for categorical variables, the random forest predicts missing values trained on observed values.

4.3.1. Proximity Imputation

With the proximity imputation method, the random forest model can be fit after some method of imputing the missing values has been implemented and this process is termed the pre-imputing of data. The median of the non-missing value is imputed for the quantitative missing values whereas the most occurring non-missing value is imputed for categorical missing values [25]. This is termed as strawman imputation. An $n \times n$ proximity matrix (a square matrix that contains the distance taken pairwise between the elements of the matrix) used to detect structures in the data and symmetry is generated. For each element, i and j that share a common terminal, the (i, j) entry denotes the fraction of tress. One's expectation is to have the same terminal nodes having similar observations and different terminal nodes having dissimilar observations. The original missing values in the data set are imputed using the proximity matrix [25].

For mixed data, the quantitative variable is imputed using the weighted averages of the non-missing observations, with the weights serving as the proximities while the categorical variable is imputed using the category with the largest mean proximity [25]. A new random forest is generated, and the process is iterated a few times [33].

4.3.2. On-the-fly Imputation

Contrary to the proximity imputation, data is imputed simultaneously while growing the forest when employing the on-the-fly imputation (OTF) [25]. One of the shortcomings of the proximity imputation which includes variable importance (a measure of how much including or removing a variable affect the prediction accuracy) and bias estimates is addressed using one-the-fly imputation. With OTF, observed data is used to calculate the split statistics and imputed values reset to missing after each split. When data is missing, a random value from the in-bag observed data is used to impute the value. If the terminal node is reached, the out-of-bag (OOB) observed terminal node data from all the trees is used to impute the missing values. For quantitative values, the mean observed value is used while the highest observed value is used for categorical values. There is a random selection of the variable used to split each node. There is an iteration of the process where in the first iteration, the estimates used are OOB. Then in-bag estimates are used for additional iteration since there is the non-existence of the OOB estimates [25].

4.3.3. missForest and mForest Imputation

The missForest is usually employed to predict missing values using a random forest trained on the observed values of a data matrix. Apart from its use in imputing mixed data, the missForest can also be used to impute complex interaction and non-linear relations [34]. Compared to the other imputation methods, there are prediction problems associated with the missForest algorithm. First imputing data by regressing each variable against the other variables helps in the prediction of the missing data of the response variable [25]. There could be slowness in computation depending on the amount of data. Considering the case of n variables, each iteration will be well fit if there are n forests. The mForest is usually employed when handling large n values that is a computationally faster form of missForest. With this method, n variables are assigned to groups hence resulting in less forest being fit.

Multivariate splitting is used to grow each forest. There is the exclusion of missing values in the response and the split-rule is averaged over observed responses [25]. Final missing response values are imputed using the prediction method. With less computation, some studies have concluded that the performance of the mForest and the missForest are at par.

5. METHODOLOGY

This section describes the data source, generation of the 6 complete datasets, analysis of the data, and the imputation implementation in the study.

5.1 DATA SOURCE AND DESCRIPTION

The data generated for this study is modeled after the 1985 Auto Import Database. This data measures the price of an automobile based on the width of an automobile, engine size, aspiration and drivetrain (denoted as drive wheels). The data can be freely accessed on the UCI Machine Learning Repository at: https://archive.ics.uci.edu/ml/datasets/Automobile.

For this study, the response variable is the *price* of an automobile while the predictor variables were *width* of an automobile, *engine size, aspiration* and *drive wheels*. *Width* of the automobile and *engine size* are quantitative while *aspiration* and *drive wheels* are categorical. The *aspiration* is a binary variable with categorized as *4wd* and *fwd* while drive wheel is a binary variable categorized as *std* and *turbo*. The entire data set contains 795 observations. The regression model was found to be:

$$\hat{Y} = -68978.03 - 2178.85X_1 + 2208.55X_2 + 1098.56X_3 + 79.85X_4$$

where $\hat{Y}$ is the estimated price of an automobile, X_1 represents aspiration, X_2 represents drive wheels, X_3 represents, and X_4 represents engine size.

5.1.1 Evaluating the Model

One of the vital tests conducted during model selection is the test of the significance of the predictors in the model

$$H_0: \beta_1 = \beta_2 = \ldots = \beta_4 = 0$$

$$H_1: \text{At least one } \beta_j \text{ does not equal 0 for } j=1...,4.$$

The Global *F*-test resulted in a p-value of approximately 0 indicating that at least one predictor is significant in the model.

Since the data set is large (795 observations), the central limit theorem satisfies the assumption of normality. For more information on the central limit theorem, see [3] and [6]. Figure 1 shows the residual plot for the main model and indicates assumption of constant error variance was met.

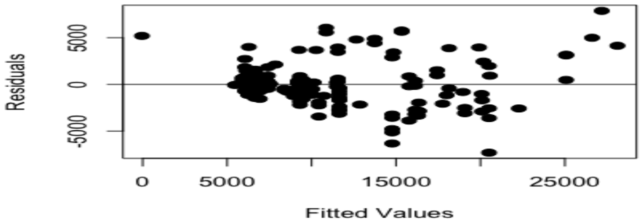


Figure 1: Residual plot for main model

All the 4 predictors have variance inflation factor (VIF) values less than 10 as shown in Table 2, which signifies that there is no serious issue of multicollinearity in the best regression model.

Table 2: VIF values for the best regression model with 4 predictors

Variable	Aspiration	Drive Wheels	Width	Engine Size
VIF Value	1.255447	1.753623	3.041604	3.089541

The ratio of the PRESS statistic and SSE produces a value of 1.114822 (close to 1) which indicates that the regression model has a good predictive ability.

Based on the internally studentized residual, only a few observations had the $|r_i|$ greater than 2.5, hence there are only a few outliers in the response. Also, a few observations were flagged as outliers in X using the leverage value (h_{ii}) as shown in Figure 2 where many of the observations fell above the threshold of 0.012 computed as $(2*p)/n$, where p = 5 and n $=795.$ The Difference in Fits (DFFITS) and Cook's distance were used to check for influential outliers. In figure 3, we noticed lots of observations falling above or below the threshold of +/- 0.158 computed as *+/- (2*sqrt(p/n))),* where $p = 5$ and $n = 795$. In Figure 4, the threshold for influential observation is 0.8710369 (computed as *qf (0.5, p, n-p)*, and there were no influential observations detected. No action was carried out to eliminate potentially influential observation since the reduced model produced strong results.

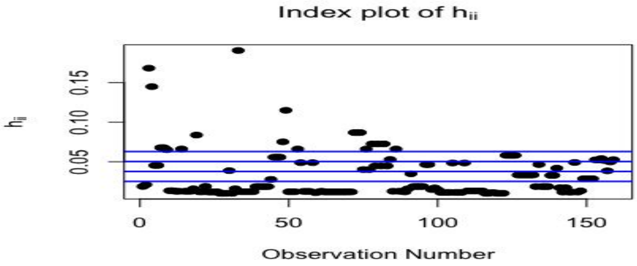
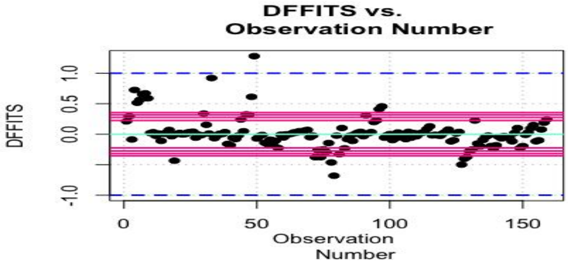


Figure 2: Index plot of h_i



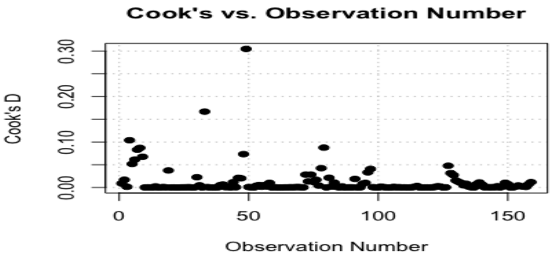


Fig.3: Influential observation by dffits rule

Fig.4: Influential observation by cook's rule

5.2. GENERATION OF DATA

An R package, SimMultiCorrData, was used to generate the dataset with a specified correlation matrix simultaneously. The continuous variables (width, engine size, and price) were generated with Headrick's fifth order power method transformation using the mean and variance from auto import dataset for each variable while preserving the correlation structure. This method matches the six standardized cumulants (mean, variance, skewness, standardized kurtosis, and standardized fifth and sixth cumulants). We assumed the skewness, standardized kurtosis, and the standardized fifth and sixth cumulants were zero [37]. The categorical variables were simulated by discretizing the standard normal variables at quantiles. These quantiles were found by looking at the inverse standard normal based on the probabilities of success for each variable (aspiration and drive wheels) as described in section 5.1. [37].

A pseudo complete dataset of sample sizes of 30 and 500 was generated. For each sample size, a dataset with small, regular, and large variabilities was also generated. We define the regular variability as the same variability from the auto import dataset for each variable. The small variability was obtained by halving the regular while the large variability was obtained by doubling the regular variability from the auto import dataset. A total of 6 complete datasets were produced for each of the 2 sample sizes of 30 and 500 with each having small, regular, and large variabilities.

5.3. MODEL BUILDING FOR THE 6 COMPLETE DATASETS

Using price as the response variable with aspiration, drive wheels, width and engine size as the predictor variables, regression models were fitted for 9 complete datasets.

5.3.1. Model building for 30 observations.

Tables 3, 4, and 5 show the estimated parameters from the regression model along with the *t* test statistic and corresponding p-value for with small, regular, and large variability. All the predictors were needed in the model in the presence of other predictors except for *drive wheels* at the 5% level of significance. Based on Global *F*-test for the three distinct models as shown in table 6, the set of predictor variables were significant in predicting the *price*, hence we left *drive wheels* in the model for comparison of the other sample sizes used. The assumption of constant variance is met based on the random patterns in the residual plots for the sample size of 30 with small, regular and large variability, which is shown in figures 5, 6 and 7, respectively. All the VIF values for the predictors in the three models as indicated in table 7, 8 and 9 were less than 10, hence there is no serious multicollinearity problems.

Table 3: The estimated regression coefficients and p-values for data size of 30 with small variability.

Regression Coefficient	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$	$\hat{\beta}_4$
Estimate	-82853.22	-3789.19	1832.32	1195.46	144.56
t (P-value)	-2.395 (0.02)	-2.980 (0.00)	1.516(0.14)	2.105(0.04)	3.960(0.00)

Table 4: The estimated regression coefficients and p-values for data size of 30 with regular variability.

Regression Coefficient	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$	$\hat{\beta}_4$
Estimate	-54833.35	-3789.19	1832.32	845.32	102.22
t (P-value)	-2.232 (0.03)	-2.980 (0.01)	1.516 (0.14)	2.105 (0.04)	3.960 (0.00)

Table 5: The estimated regression coefficients and p-values for data size of 30 with large variability.

Regression Coefficient	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$	$\hat{\beta}_4$
Estimate	-35020.31	-3789.19	1832.32	597.73	72.28
t (P-value)	-2.002 (0.06)	-2.980 (0.01)	1.516 (0.14)	2.105 (0.04)	3.960 (0.00)

Table 6: Global *F*-test for data size of 30

Variability Type	Small	Regular	Large
F (P-value)	38.5 (0.0000)	38.5 (0.0000)	38.5 (0.0000)

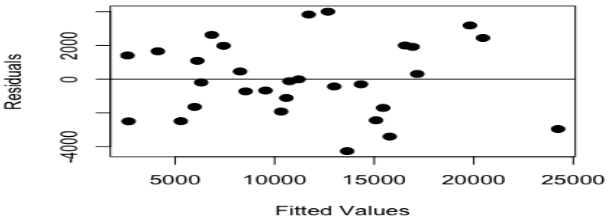


Fig. 5: Residual plot for model of size 30 with small variability

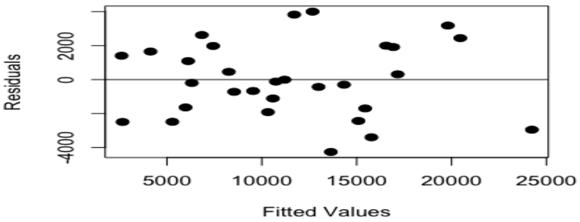


Fig. 6: Residual plot for model of size 30 with regular variability

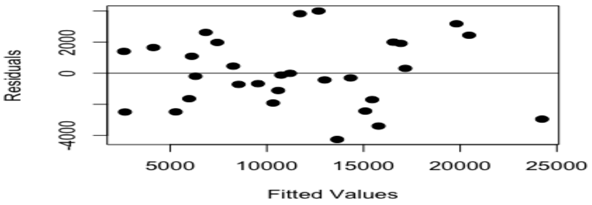


Fig. 7: Residual plot for model of size 30 with large variability

Table 7: VIF for data size of 30 with small variability

Variable	Aspiration	Drive Wheels	Width	Engine Size
VIF Value	1.203736	1.738988	3.171044	3.203679

Table 8: VIF for data size of 30 with regular variability

Variable	Aspiration	Drive Wheels	Width	Engine Size
VIF Value	1.203736	1.738988	3.171044	3.203679

Table 9 VIF for data size of 30 with large variability

Variable	Aspiration	Drive Wheels	Width	Engine Size
VIF Value	1.203736	1.738988	3.171044	3.203679

5.3.2. Model building for 500 observations

Tables 10, 11, and 12 show the estimated parameters from the regression model along with the *t* test statistic and corresponding p-value for with small, regular, and large variability. All the predictors were needed in the model in the presence of other predictors at the 5% level of significance. Based on Global F-test for the three distinct models as shown in table 13, the set of predictor variables were significant in predicting the *price*. The assumption of constant variance is met based on the random patterns in the residual plots for the sample size of 500 with small, regular and large variability, which is shown in figures 8,9 and 10. All the VIF values for the predictors in the three models as indicated in table 14, 15 and 16 were less than 10, hence there is no serious multicollinearity problems.

Table 10: The estimated regression coefficients and p-values for data size of 500 with small variability.

Regression Coefficient	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$	$\hat{\beta}_4$
Estimate	-103800	-2730	1940	1548	113.7
t (P-value)	-12.814(0.00)	-8.572(0.00)	6.322(0.00)	11.729(0.00)	13.160(0.00)

Table 11: The estimated regression coefficients and p-values for data size of 500 with regular variability.

Regression Coefficient	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$	$\hat{\beta}_4$
Estimate	-70116.578	-2730.389	1940.197	1094.866	80.427
t (P-value)	-12.154 (0.00)	-8.572 (0.00)	6.322 (0.00)	11.729 (0.00)	13.160(0.00)

Table 12: The estimated regression coefficients and p-values for data size of 500 with large variability.

Regression Coefficient	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$	$\hat{\beta}_4$
Estimate	-46269.082	-2730.389	1940.197	774.187	56.870
t (P-value)	-11.215 (0.00)	-8.572 (0.00)	6.322 (0.00)	11.729 (0.00)	13.160(0.00)

Table 13: Global F-test for data size of 500

Variability Type	Small	Regular	Large
F (P-value)	641.7 (0.0000)	641.7 (0.0000)	641.7 (0.0000)

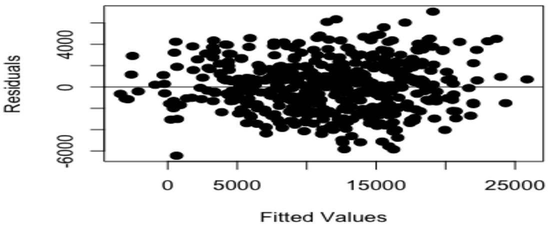


Fig. 8: Residual plot for model of size 500 with small variability

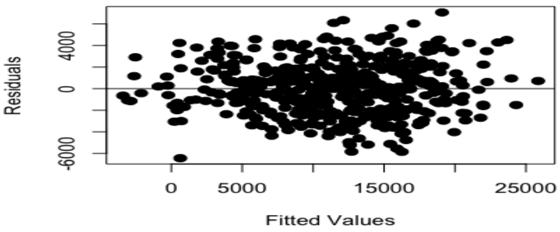


Fig. 9: Residual plot for model of size 500 with regular variability

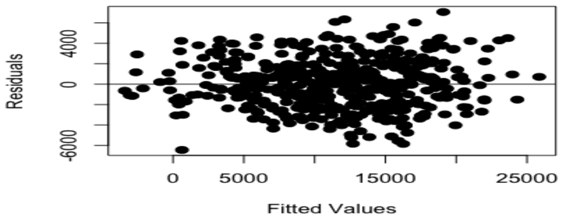


Fig. 10: Residual plot for model of size 150 with large variability

Table 14: VIF for data size of 500 with small variability

Variable	Aspiration	Drive Wheels	Width	Engine Size
VIF Value	1.247094	1.707726	2.929908	3.070909

Table 15: VIF for data size of 500 with regular variability

Variable	Aspiration	Drive Wheels	Width	Engine Size
VIF Value	1.247094	1.707726	2.929908	3.070909

Table 16: VIF for data size of 500 with large variability

Variable	Aspiration	Drive Wheels	Width	Engine Size
VIF Value	1.247094	1.707726	2.929908	3.070909

5.4. RELATIVE EFFICIENCY

Given the number of imputations, *m*, and the fraction of missingness (FMI), the relative efficiency (RE) determines the best imputation procedure that produces the most precise results based on the measure of the differences in accuracy [31]. The RE is defined as

$$RE = \frac{1}{1 + \frac{\lambda}{m}}$$

where λ is the fraction of missingness. For each imputation value, as the fraction of missingness increase, the RE tends to decrease accordingly as shown in table 17. For the purpose of this paper, we used an *m* value of 50 because the large value of *m* tends to yield more precise standard error and p-values [31,32].

Table 17: Relative efficiency for different levels of FMI and m

m/FMI	10%	20%	30%	40 %	50%
10	0.9901	0.9804	0.9709	0.9615	0.9524
20	0.9950	0.9901	0.9852	0.9804	0.9756
30	0.9967	0.9934	0.9901	0.9868	0.9836
40	0.9975	0.9950	0.9926	0.9901	0.9877
50	0.9980	0.9960	0.9940	0.9921	0.9901

5.5. IMPUTATION IMPLEMENTATION

For each of the 6 complete data sets of sample sizes 30 and 500 and variabilities of small, regular, and large, the first level of missingness was achieved by removing 10% of the observations from the predictor variables using the R function *prodNA*. The next 20% level of missingness was achieved by removing 10% level of missingness from the initial 10% removed. The next 30% level of missingness was also achieved by removing 10% level of missingness from the previous 20% and this continued in that sequence till 50% level of missing was attained. This produced a total of 30 missing datasets. Each of the three imputation methods for mixed dataset namely, the predictive mean matching (PMM), classification and regression tree (CART) and the random forest (RF) imputation methods were applied on the 30 missing datasets. For each imputation method, $m = 50$ imputed data sets were created. We then fit a regression model (as described in 5.1) for each of the 50 imputed datasets. The regression coefficient estimates ($\hat{\beta}_0$ to $\hat{\beta}_4$) from the 50 imputed data sets were then pooled together and stored. This is repeated for 1000 iterations and the average of each of the 1000 regression coefficients for each variable were computed and compared to the coefficients of the complete data set found in 5.1.

5.6. ANALYSIS OF INTEREST

The best imputation method for imputing the missing data for a specified percentage of missingness is the one that produces the average regression coefficient from the imputed data, which is closest to the corresponding regression coefficient from the complete data. To evaluate this comparison, we compute the percentage deviation index (PDI) which is calculated as:

$$PDI = \frac{Mean\ of\ Estimated\ Regression\ \ Coefficient - Original\ Regression\ Coefficient\ \cancel{-\ Average\ of\ Estimated\ Re}}{Original\ Regression\ \ Coefficient} * 100.$$

For each of the complete datasets, the best imputation method is the one with the PDI closest to zero. The R^2 value measures the prediction accuracy for a regression model and was computed for each of the 30 datasets.

6. RESULTS

This section of the study evaluates the analysis on the 30 multiple imputed datasets compared to the 6 complete data sets using the methods described in section 5.6.

6.1 Analysis for Sample size of 30 with small variability

We see from the PMM method in table 18, the closeness in value of the estimated mean regression coefficient to the mean regression coefficient from the complete dataset at the respective level of missingness.

For the CART method, the closeness in value of the estimated mean regression coefficient to the mean regression coefficient from the complete dataset at the different level of missingness
is shown in table 20.

Considering the imputed dataset for the RF method, the closeness in value of the estimated mean regression coefficient to the mean regression coefficient from the complete dataset at the diverse level of missingness is shown in table 22.

As indicated in tables 19, 21 and 23, the PDI of the CART method is closest to zero among the three imputation methods which implied that the PMM is the best imputation method when considering this type of data.

At 10% level of missingness, the R^2 values for the PMM, CART and RF methods are closest in value to the R^2 value of the complete dataset.

Table 18. Estimated mean of regression coefficients for each percentage of missingness for a sample size of 30 with small variability from the PMM method.

FMI/ Estimated Parameter/ R^2 value	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$	$\hat{\beta}_4$	R^2 value
10%	-113860.7	-2580.082	2135.510	1696.407	112.207	0.8323
20%	-156668.1	-1611.734	2334.245	2389.058	77.746	0.8450
30%	-133906.7	-1747.793	2339.214	2008.822	101.513	0.7879
40%	-145104.5	-1998.508	1652.049	2193.880	104.674	0.7488
50%	-108826.2	-3152.522	1689.462	1620.482	128.020	0.6980
Actual Parameter from complete data set	-82853.22	-3789.19	1832.32	1195.46	144.56	0.838

Table 19. PDI for the estimated regression coefficients for sample size of 30 with small variability from the PMM model.

FMI/ Estimated Parameter	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$	$\hat{\beta}_4$	Mean
10%	0.3742	-0.3191	0.1655	0.4190	-0.2238	0.0832
20%	0.8909	-0.5746	0.2739	0.9984	-0.4622	0.2253
30%	0.6162	-0.5387	0.2766	0.6804	-0.2978	0.1473
40%	0.7513	-0.4726	-0.0984	0.8352	-0.2759	0.1479
50%	0.3135	-0.1680	-0.0780	0.3555	-0.1144	0.0617
Mean	0.5892	-0.4146	0.1079	0.6577	-0.2748	0.1331

Table 20. Estimated mean of regression coefficients for each percentage of missingness for a sample size of 30 with small variability from the CART method.

FMI/ Estimated Parameter/ R^2 value	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$	$\hat{\beta}_4$	R^2 value
10%	-99337.21	-2300.507	2277.565	1475.387	109.7196	0.8061
20%	-101935.9	-1880.861	3214.774	1539.970	79.2240	0.7684
30%	-93821.36	-1435.086	3755.843	1427.984	61.9492	0.6861
40%	-54746.93	-1745.642	3510.737	799.4550	85.2806	0.6148
50%	-44128.60	-2197.659	2989.593	658.9630	85.8674	0.5236
Actual Parameter from complete data set	-82853.22	-3789.19	1832.32	1195.46	144.56	0.838

Table 21. PDI for the estimated regression coefficients for sample size of 30 with small variability from the CART model

FMI/ Estimated Parameter	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$	$\hat{\beta}_4$	Mean
10%	0.1989	-0.3929	0.2430	0.2341	-0.2410	0.0844
20%	0.2303	-0.5036	0.75448	0.2881	-0.4519	0.0635
30%	0.1324	-0.6213	1.04977	0.1945	-0.5714	0.0368
40%	-0.3392	-0.5393	0.9160	-0.3312	-0.4100	-0.0141
50%	-0.4674	-0.4200	0.6315	-0.4487	-0.4060	-0.222

Mean	-0.0489	-0.4954	0.7189	-0.0126	-0.4161	-0.0508

Table 22. Estimated mean of regression coefficients for each percentage of missingness for a sample size of 30 with small variability from the RF method.

FMI/ Estimated Parameter/ R^2 value	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$	$\hat{\beta}_4$	R^2 value
10%	-96548.73	-4196.755	1540.982	1404.631	154.8746	0.8254
20%	-91545.67	-310.2089	196.2865	1328.216	138.9847	0.7661
30%	-110879.9	-1896.349	2704.616	1641.696	104.2328	0.6897
40%	-102228.0	-2204.240	3366.193	1513.572	97.1247	0.6059
50%	-100531.4	-802.0412	4489.046	1513.686	52.7117	0.5316
Actual Parameter from complete data set	-82853.22	-3789.19	1832.32	1195.46	144.56	0.838

Table 23. PDI for the estimated regression coefficients for sample size of 30 with small variability from the RF model

FMI/ Estimated Parameter	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$	$\hat{\beta}_4$	Mean
10%	0.1652	0.1075	-0.1589	0.1749	0.0713	0.0720
20%	0.1049	-0.9181	-0.8928	0.1110	-0.0385	-0.327
30%	0.3382	-0.4995	0.4760	0.3732	-0.2789	0.0818
40%	0.2338	-0.4182	0.8371	0.2661	-0.3281	0.118
50%	0.2133	-0.7883	1.4499	0.2661	-0.6353	0.0101
Mean	0.2111	-0.5033	0.3422	0.2383	-0.2419	0.00928

6.2 Analysis for Sample size of 30 with regular variability

We see from the PMM method in table 24, the closeness in value of the estimated mean regression coefficient to the mean regression coefficient from the complete dataset at the respective level of missingness.

For the CART method, the closeness in value of the estimated mean regression coefficient to the mean regression coefficient from the complete dataset at the different level of missingness as indicated in table 26.

Considering the imputed dataset for RF method, the closeness in value of the estimated mean regression coefficient to the mean regression coefficient from the complete dataset at the diverse level of missingness is shown in table 28.

As indicated in tables 25, 27 and 29, the PDI of the CART method is closest to zero among the three imputation methods which implied that the CART is the best imputation method when considering this type of data.

At 10% level of missingness, the R^2 values for the PMM, CART and RF methods are closest in value to the R^2 value of the complete dataset.

Table 24. Estimated mean of regression coefficients for each percentage of missingness for a sample size of 30 with regular variability from the PMM method.

FMI/ Estimated Parameter/ R^2 value	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$	$\hat{\beta}_4$	R^2 value
10%	-65531.45	-3038.283	2443.910	1025.445	78.6124	0.8102
20%	-79177.26	-2968.759	2297.999	1222.981	85.9305	0.7941
30%	-95356.30	-1878.115	2204.837	1488.819	65.9907	0.7214
40%	-84585.33	-3184.238	2647.687	1339.289	65.1588	0.6191
50%	-83597.78	-3226.548	2690.413	1326.301	63.4450	0.5991
Actual Parameter from	-54833.35	-3789.19	1832.32	845.32	102.22	0.838

complete data set						

Table 25. PDI for the estimated regression coefficients for sample size of 30 with regular variability from the PMM model.

FMI/ Estimated Parameter	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$	$\hat{\beta}_4$	Mean
10%	0.1951	-0.1981	0.3337	0.2130	-0.2309	0.0626
20%	0.4439	-0.2165	0.2541	0.4467	-0.1593	0.154
30%	0.7390	-0.5043	0.2033	0.76124	-0.3544	0.169
40%	0.5425	-0.1596	0.4449	0.5843	-0.3625	0.210
50%	0.5245	-0.1484	0.4683	0.5689	-0.3793	0.207
Mean	0.4890	-0.2454	0.3409	0.5148	-0.2973	0.160

Table 26. Estimated mean of regression coefficients for each percentage of missingness for a sample size of 30 with regular variability from the CART method.

FMI/ Estimated Parameter/ R^2 value	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$	$\hat{\beta}_4$	R^2 value
10%	-50560.78	-3942.212	2059.869	763.1116	113.1268	0.8231
20%	-81491.53	-2217.607	844.8390	1269.244	94.4400	0.7310
30%	-71488.29	-1164.968	2168.806	1102.110	73.4001	0.5554
40%	-65110.92	-1713.514	3629.544	998.3172	56.6923	0.4941
50%	-48822.49	-1604.709	4610.970	744.3413	45.9696	0.4610
Actual Parameter from complete data set	-54833.35	-3789.19	1832.32	845.32	102.22	0.838

Table 27. PDI for the estimated regression coefficients for sample size of 30 with regular variability from the CART model.

FMI/ Estimated Parameter	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$	$\hat{\beta}_4$	Mean
10%	-0.0779	0.0403	0.1241	-0.0972	0.1066	0.0192
20%	0.4861	-0.4147	-0.5389	0.5014	-0.0761	-0.008.4
30%	0.3037	-0.6925	0.1836	0.3037	-0.2819	-0.03.67
40%	0.1874	-0.5477	0.980	0.1809	-0.4453	0.0712
50%	-0.1096	-0.5765	1.5164	-0.1194	-0.5502	0.0321
Mean	0.1579	-0.4382	0.4532	0.1539	-0.2494	0.0155

Table 28. Estimated mean of regression coefficients for each percentage of missingness for a sample size of 30 with regular variability from the RF method.

FMI/ Estimated Parameter/ R^2 value	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$	$\hat{\beta}_4$	R^2 value
10%	-64149.01	-4196.755	1540.982	993.2241	109.5129	0.8253
20%	-61169.23	-3102.089	1962.865	939.1905	98.2770	0.7661
30%	-75693.21	-1896.349	2704.616	1160.854	73.7037	0.6897
40%	-69751.49	-2204.240	3366.193	1070.257	68.67754	0.6059
50%	-69673.64	-829.6779	4482.022	1071.819	37.3873	0.5316
Actual Parameter from complete data set	-54833.35	-3789.19	1832.32	845.32	102.22	0.838

Table 29. PDI for the estimated regression coefficients for sample size of 30 with regular variability from the RF model.

FMI/ Estimated Parameter	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$	$\hat{\beta}_4$	Mean
10%	0.1698	0.1075	-0.1589	0.1749	0.0713	0.0730

20%	0.1155	-0.1813	0.0712	0.1110	-0.0385	0.0156
30%	0.3804	-0.4995	0.4760	0.3732	-0.2789	0.0902
40%	0.2720	-0.4182	0.8371	0.2660	-0.3281	0.126
50%	0.2706	-0.7810	1.4460	0.2679	-0.6342	0.114
Mean	0.2417	-0.3545	0.5343	0.2386	-0.2417	0.0837

6.3. Analysis for Sample size of 30 with large variability

We see from table 30, the closeness in value of the estimated mean regression coefficient to the mean regression coefficient from the complete dataset at the respective level of missingness for the PMM method.

For the CART method, the closeness in value of the estimated mean regression coefficient to the mean regression coefficient from the complete dataset at the different level of missingness is shown in table 32.

Considering the imputed dataset for RF method, the closeness in value of the estimated mean regression coefficient to the mean regression coefficient from the complete dataset at the diverse level of missingness is shown in table 34.

As indicated in tables 31,33 and 35, the PDI of the CART method is closest to zero among the three imputation methods which implied that the CART is the best imputation method when considering this type of data.

At 10% level of missingness, the R^2 values for the PMM, CART and RF methods are closest in value to the R^2 value of the complete dataset.

Table 30. Estimated mean of regression coefficients for each percentage of missingness for a sample size of 30 with large variability from the PMM method.

FMI/ Estimated Parameter/ R^2 value	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$	$\hat{\beta}_4$	R^2 value
10%	-22117.53	-4860.589	2570.559	390.3774	76.7728	0.8572
20%	-33252.45	-4341.491	2186.129	582.6866	60.7146	0.7681
30%	-45248.03	-3807.438	1299.328	785.1202	55.9785	0.7536
40%	-46400.55	-3520.290	1376.959	809.2406	49.5510	0.6960
50%	-41112.64	-5470.598	1209.332e	758.7642	51.5799	0.6.663
Actual Parameter from complete data set	-35020.31	-3789.19	1832.32	597.73	72.28	0.838

Table 31. PDI for the estimated regression coefficients for sample size of 30 with large variability from the PMM model.

FMI/ Estimated Parameter	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$	$\hat{\beta}_4$	Mean
10%	-0.3684	0.2827	0.4028	-0.3469	0.0621	0.0065
20%	-0.0504	0.1457	0.1930	-0.0251	-0.1600	0.0206
30%	0.2920	0.0048	-0.2908	0.3135	-0.2255	0.0188
40%	0.3249	-0.0709	-0.2485	0.3538	-0.3144	0.0089
50%	0.1739	0.4437	-0.3399	0.2694	-0.2863	0.0521
Mean	0.0744	0.1612	-0.0566	0.1129	-0.1848	0.0214

Table 32. Estimated mean of regression coefficients for each percentage of missingness for a sample size of 30 with large variability from the CART method

FMI/ Estimated Parameter/ R^2 value	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$	$\hat{\beta}_4$	R^2 value
10%	-31946.36	-3942.212	2059.86	539.6014	79.9927	0.8231
20%	-53975.19	-2245.837	759.7	901.7992	67.3925	0.7319

30%	-56282.21	-2852.966	-1100.05	972.6104	79.5794	0.6652
40%	-54289.70	-3553.769	387.86	932.2304	69.5446	0.6848
50%	-38753.6	-4100.374	1433.102	667.9877	76.5142	0.6181
Actual Parameter from complete data set	-35020.31	-3789.19	1832.32	597.73	72.28	0.838

Table 33. PDI for the estimated regression coefficients for sample size of 30 with large variability from the CART model

FMI/ Estimated Parameter	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$	$\hat{\beta}_4$	Mean
10%	-0.0877	0.0403	0.1241	-0.0972	0.1067	0.0173
20%	0.5412	-0.4073	-0.5853	0.50870	-0.0676	-0.00206
30%	0.6071	-0.2470	-1.6003	0.6271	0.1009	-0.102
40%	0.5502	-0.0621	-0.7883	0.5596	-0.0378	0.0443
50%	0.1066	0.0821	-0.2178	0.1175	0.0585	0.0294
Mean	0.3434	-0.1188	-0.6135	0.3431	0.0321	-0.00271

Table 34. Estimated mean of regression coefficients for each percentage of missingness for a sample size of 30 with large variability from the RF method

FMI/ Estimated Parameter/ R^2 value	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$	$\hat{\beta}_4$	R^2 value
10%	-41238.96	-4196.755	1540.982	702.3155	77.4373	0.8253
20%	-39689.85	-3102.089	1962.865	664.1080	69.4923	0.7661
30%	-50812.45	-1896.349	2704.616	820.8479	52.1164	0.6896
40%	-46787.16	-2204.240	3366.193	756.7860	48.5623	0.6059
50%	-47821.45	-827.1917	4467.267	759.0041	26.4521	0.5316
Actual Parameter from complete data set	-35020.31	-3789.19	1832.32	597.73	72.28	0.838

Table 35. PDI for the estimated regression coefficients for sample size of 30 with large variability from the RF model

FMI/ Estimated Parameter	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$	$\hat{\beta}_4$	Mean
10%	0.1775	0.1075	-0.1589	0.1749	0.0713	0.0745
20%	0.1333	-0.1813	0.0712	0.1110	-0.0385	0.0191
30%	0.4509	-0.4995	0.4760	0.3732	-0.2789	0.104
40%	0.3360	-0.4182	0.8371	0.2661	-0.3281	0.139
50%	0.3655	-0.7816	1.4380	0.2698	-0.6340	0.132
Mean	0.2926	-0.3546	0.5326	0.2390	-0.2416	0.0936

6.4 Analysis for Sample size of 500 with small variability

We see from table 36, the closeness in value of the estimated mean regression coefficient to the mean regression coefficient from the complete dataset at the respective level of missingness for the PMM method.

For the CART method, the closeness in value of the estimated mean regression coefficient to the mean regression coefficient from the complete dataset at the different level of missingness is indicated in table 38.

Considering the imputed dataset for RF method, the closeness in value of the estimated mean regression coefficient to the mean regression coefficient from the complete dataset at the diverse level of missingness is shown in table 40.

As indicated in tables 37, 39 and 41, the PDI of the RF method is closest to zero among the three imputation methods which implied that the RF is the best imputation method when considering this type of data.

At 10% level of missingness, the R^2 values for the PMM, CART and RF methods are closest in value to the R^2 value of the complete dataset.

Table 36. Estimated mean of regression coefficients for each percentage of missingness for a sample size of 500 with small variability from the PMM method.

FMI/ Estimated Parameter/ R^2 value	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$	$\hat{\beta}_4$	R^2 value
10%	-103893.00	-2462.6620	1957.6970	1553.9660	108.1973	0.7916
20%	-107215.70	-2251.3880	1961.5730	1617.7560	99.1899	0.7551
30%	-105920.50	-2213.5760	1820.1220	1610.0520	94.0290	0.7032
40%	-100686.90	-2082.3870	1833.8440	1531.9900	91.5546	0.6480
50%	-90633.97	-1991.9760	1914.2280	1378.6420	89.5345	0.5882
Actual Parameter from complete data set	-103800.00	-2730.0000	1940.00	1548.00	113.7000	0.8370

Table 37. PDI for the estimated regression coefficients for sample size of 500 with small variability from the PMM model.

FMI/ Estimated Parameter	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$	$\hat{\beta}_4$	Mean
10%	0.0009	-0.0979	0.0091	0.0039	-0.0484	-0.0265
20%	0.0329	-0.1753	0.0111	0.0451	-0.1276	-0.0428
30%	0.0204	-0.1892	-0.0618	0.0401	-0.1730	-0.0727
40%	-0.0300	-0.2372	-0.0547	-0.0103	-0.1948	-0.1050
50%	-0.1268	-0.2703	-0.0133	-0.1094	-0.2125	-0.1460
Mean	-0.0205	-0.1940	-0.0219	-0.0061	-0.1513	-0.0788

Table 38. Estimated mean of regression coefficients for each percentage of missingness for a sample size of 500 with small variability from the CART method.

FMI/ Estimated Parameter/ R^2 value	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$	$\hat{\beta}_4$	R^2 value
10%	-105194.70	-2428.8030	2006.8170	1575.6590	106.2510	0.7949
20%	-112654.4	-2355.077	2034.343	1712.408	92.2291	0.7413
30%	-109857.3	-2012.258	2049.193	1671.442	87.8637	0.6935
40%	-100686.10	-2362.6750	1989.8700	1545.3710	83.7254	0.6301
50%	-100038.40	-2089.1280	1900.2290	1546.0220	76.5229	0.5626
Actual Parameter from complete data set	-103800.00	-2730.0000	1940.0000	1548.000	113.7000	0.8370

Table 39. PDI for the estimated regression coefficients for sample size of 500 with small variability from the CART model.

FMI/ Estimated Parameter	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$	$\hat{\beta}_4$	Mean
10%	0.0134	-0.1103	0.0344	0.0179	-0.0655	-0.0220
20%	0.0853	-0.1373	0.0486	0.1062	-0.1888	-0.0172
30%	0.0584	-0.2629	0.0563	0.0797	-0.2272	-0.0592
40%	-0.0300	-0.1346	0.0257	-0.0017	-0.2636	-0.0808
50%	-0.0362	-0.2348	-0.0205	-0.0013	-0.3270	-0.1240
Mean	0.0182	-0.1760	0.0289	0.0402	-0.2144	-0.0606

Table 40. Estimated mean of regression coefficients for each percentage of missingness for a sample size of 500 with small variability from the RF method.

FMI/ Estimated Parameter/ R^2 value	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$	$\hat{\beta}_4$	R^2 value
10%	-95357.64	-2688.8900	1888.4290	1400.0080	123.9874	0.7937
20%	-101278.70	-2452.4290	1967.3700	1500.1890	114.9021	0.7444
30%	-99413.51	-2315.8420	2106.6770	1473.0740	110.5008	0.6914
40%	-100110.30	-2205.1490	2158.1920	1493.0530	103.3730	0.6175
50%	-95610.29	-2124.4950	2400.9050	1430.2050	95.3939	0.5485
Actual Parameter from complete data set	-103800.00	-2730.0000	1940.0000	1548.0000	113.7000	0.8370

Table 41. PDI for the estimated regression coefficients for sample size of 500 with small variability from the RF model.

FMI/ Estimated Parameter	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$	$\hat{\beta}_4$	Mean
10%	-0.0813	-0.0151	-0.0266	-0.0956	0.0905	-0.0256
20%	-0.0243	-0.1017	0.0141	-0.0309	0.0106	-0.0264
30%	-0.0423	-0.1517	0.0859	-0.0484	-0.0281	-0.0369
40%	-0.0355	-0.1923	0.1125	-0.0355	-0.0908	-0.0483
50%	-0.0789	-0.2218	0.2376	-0.0761	-0.1610	-0.0600
Mean	-0.0525	-0.1365	0.0847	-0.0573	-0.0358	-0.0395

6.5. Analysis for Sample size of 500 with regular variability

We see from table 42, the closeness in value of the estimated mean regression coefficient to the mean regression coefficient from the complete dataset at the respective level of missingness for the PMM method.

For the CART method, the closeness in value of the estimated mean regression coefficient to the mean regression coefficient from the complete dataset at the different level of missingness is shown in table 44.

Considering the imputed dataset for RF method, the closeness in value of the estimated mean regression coefficient to the mean regression coefficient from the complete dataset at the diverse level of missingness is indicated in table 46.

As indicated in tables 43, 45 and 47, the PDI of the RF method is closest to zero among the three imputation methods which implied that the RF is the best imputation method when considering this type of data.

At 10% level of missingness, the R^2 values for the PMM, CART and RF methods are closest in value to the R^2 value of the complete dataset.

Table 42. Estimated mean of regression coefficients for each percentage of missingness for a sample size of 500 with regular variability from the PMM method.

FMI/ Estimated Parameter/ R^2 value	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$	$\hat{\beta}_4$	R^2 value
10%	-70244.06	-2461.573	1958.300	1098.599	76.5293	0.7915
20%	-72592.93	-2253.0830	1964.1520	1142.7950	70.1314	0.7549
30%	-71740.61	-2212.2250	1819.5860	1139.1700	66.4620	0.7033
40%	-67961.05	-2085.6120	1831.4270	1081.8520	64.8342	0.6480
50%	-61096.22	-1986.0450	1914.1730	975.9748	63.3122	0.5883
Actual Parameter from complete data set	-70116.58	-2730.3890	1940.1970	1094.866	80.4270	0.8370

Table 43. PDI for the estimated regression coefficients for sample size of 500 with regular variability from the PMM model.

FMI/ Estimated Parameter	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$	$\hat{\beta}_4$	Mean
10%	0.0018	-0.0985	0.0093	0.0034	-0.0485	-0.0265
20%	0.0353	-0.1748	0.0123	0.0438	-0.1280	-0.0423
30%	0.0232	-0.1898	-0.0622	0.0405	-0.1736	-0.0724
40%	-0.0307	-0.2361	-0.0561	-0.0119	-0.1939	-0.1060
50%	-0.1286	-0.2726	-0.0134	-0.1086	-0.2128	-0.1470
Mean	-0.0198	-0.1944	-0.0220	-0.0066	-0.1514	-0.0788

Table 44. Estimated mean of regression coefficients for each percentage of missingness for a sample size of 500 with regular variability from the CART method.

FMI/ Estimated Parameter/ R^2 value	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$	$\hat{\beta}_4$	R^2 value
10%	-71206.46	-2428.8020	2006.8170	1114.1590	75.1308	0.7949
20%	-76576.54	-2352.2960	2035.2870	1211.5850	65.1795	0.7415
30%	-71194.95	-2477.070	2069.316	1129.807	65.0326	0.7032
40%	-69535.82	-2182.6100	1876.5640	1105.6910	63.9851	0.6350
50%	-60128.91	-2302.2250	1909.2660	959.1289	66.6857	0.5858
Actual Parameter from complete data set	-70116.58	-2730.3890	1940.1970	1094.8660	80.4270	0.8370

Table 45. PDI for the estimated regression coefficients for sample size of 500 with regular variability from the CART model.

FMI/ Estimated Parameter	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$	$\hat{\beta}_4$	Mean
10%	0.0155	-0.1105	0.0343	0.0176	-0.0659	-0.0218
20%	0.0921	-0.1385	0.0490	0.1066	-0.1896	-0.0161
30%	0.0154	-0.0928	0.0665	0.0319	-0.1914	-0.0341
40%	-0.0083	-0.2006	-0.0328	0.0099	-0.2044	-0.0872
50%	-0.1424	-0.1568	-0.0159	-0.1240	-0.1709	-0.1220
Mean	-0.0055	-0.1398	0.0202	0.0084	-0.1644	-0.0562

Table 46. Estimated mean of regression coefficients for each percentage of missingness for a sample size of 500 with regular variability from the RF method.

FMI/ Estimated Parameter/ R^2 value	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$	$\hat{\beta}_4$	R^2 value
10%	-64111.75	-2688.9250	1888.6320	989.7214	87.6866	0.7936
20%	-65815.03	-2592.9720	1915.2900	1021.3500	83.4956	0.7434
30%	-64042.88	-2451.8920	2002.1750	994.2339	81.1726	0.6831
40%	-62631.29	-2319.6130	2081.6430	971.0694	79.0294	0.6260
50%	-59706.33	-2407.291	2174.290	932.3030	75.3107	0.5602
Actual Parameter from complete data set	-70116.58	-2730.3890	1940.1970	1094.8660	80.4270	0.8370

Table 47. PDI for the estimated regression coefficients for sample size of 500 with regular variability from the RF model.

FMI/ Estimated Parameter	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$	$\hat{\beta}_4$	Mean
10%	-0.0856	-0.0152	-0.0266	-0.0960	0.0903	-0.0266
20%	-0.0613	-0.0503	-0.0128	-0.0671	0.0382	-0.0307
30%	-0.0866	-0.1020	0.0319	-0.0919	0.0093	-0.0479
40%	-0.1068	-0.1504	0.0729	-0.1131	-0.0174	-0.0629

50%	-0.1485	-0.1183	0.1207	-0.1485	-0.0636	-0.0716
Mean	-0.0978	-0.0873	0.0372	-0.1033	0.0113	-0.0480

6.6. Analysis for Sample size of 500 with large variability

We see from the PMM method in table 48, the closeness in value of the estimated mean regression coefficient to the mean regression coefficient from the complete dataset at the respective level of missingness.

For the CART method, the closeness in value of the estimated mean regression coefficient to the mean regression coefficient from the complete dataset at the different level of missingness in indicated in table 50.

Considering the imputed dataset for RF method, the closeness in value of the estimated mean regression coefficient to the mean regression coefficient from the complete dataset at the diverse level of missingness is shown in table 53.

As indicated in tables 49, 51 and 53, the PDI of the CART method is closest to zero among the three imputation methods which implied that the CART is the best imputation method when considering this type of data.

At 10% level of missingness, the R^2 values for the PMM, CART and RF methods are closest in value to the R^2 value of the complete dataset.

Table 48. Estimated mean of regression coefficients for each percentage of missingness for a sample size of 500 with large variability from the PMM method

FMI/ Estimated Parameter/ R^2 value	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$	$\hat{\beta}_4$	R^2 value
10%	-48385.21	-2420.7740	1931.9760	811.5718	51.4061	0.8006
20%	-46815.54	-2366.1490	2017.0950	788.2721	49.4639	0.7672
30%	-46809.00	-2166.7340	2113.5000	787.2274	46.3549	0.7187
40%	-45842.28	-2162.6730	2031.9550	782.0615	42.5703	0.6789
50%	-43988.68	-1922.4720	2324.3070	748.3461	38.5242	0.6095
Actual Parameter from complete data set	-46269.08	-2730.3890	1940.1970	774.1870	56.8700	0.8370

Table 49. PDI for the estimated regression coefficients for sample size of 500 with large variability from the PMM model

FMI/ Estimated Parameter	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$	$\hat{\beta}_4$	Mean
10%	0.0457	-0.1134	-0.0042	0.0483	-0.0961	-0.0239
20%	0.0118	-0.1334	0.0396	0.0182	-0.1302	-0.0388
30%	0.0117	-0.2064	0.0893	0.0168	-0.1849	-0.0547
40%	-0.0092	-0.2079	0.0473	0.0102	-0.2514	-0.0822
50%	-0.0493	-0.2959	0.1980	-0.0334	-0.3226	-0.1010
Mean	0.0021	-0.1914	0.0740	0.0120	-0.1970	-0.0601

Table 50. Estimated mean of regression coefficients for each percentage of missingness for a sample size of 500 with large variability from the CART method.

FMI/ Estimated Parameter/ R^2 value	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$	$\hat{\beta}_4$	R^2 value
10%	-47173.18	-2428.8040	2006.8170	787.8292	53.1255	0.7949
20%	-50997.24	-2354.0570	2034.3330	856.4394	46.0923	0.7414
30%	-49159.38	-2336.4330	2155.6480	829.4511	43.7683	0.7028
40%	-45739.94	-2252.973	2157.513	775.1587	44.3250	0.6638
50%	-41715.28	-2265.8550	2118.6030	718.0899	42.8254	0.5901

Actual Parameter from complete data set	-46269.08	-2730.3890	1940.1970	774.1870	56.8700	0.8370

Table 51. PDI for the estimated regression coefficients for sample size of 500 with large variability from the CART model.

FMI/ Estimated Parameter	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$	$\hat{\beta}_4$	Mean
10%	0.0195	-0.1105	0.0343	0.0176	-0.0658	-0.0210
20%	0.1022	-0.1378	0.0485	0.1062	-0.1895	-0.0141
30%	0.0625	-0.1443	0.1110	0.0714	-0.2304	-0.0260
40%	-0.0114	-0.1749	0.1120	0.0013	-0.2206	-0.0587
50%	-0.0984	-0.1701	0.0920	-0.0725	-0.2470	-0.0992
Mean	0.0149	-0.1475	0.0796	0.0248	-0.1907	-0.0438

Table 52. Estimated mean of regression coefficients for each percentage of missingness for a sample size of 500 with large variability from the RF method.

FMI/ Estimated Parameter/ R^2 value	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$	$\hat{\beta}_4$	R^2 value
10%	-42023.05	-2690.2220	1887.9330	699.7680	61.9947	0.7937
20%	-41432.82	-2547.8700	2189.3830	681.0770	1.0374	0.7392
30%	-42045.18	-2608.2730	2249.2190	701.5647	55.1047	0.6605
40%	-38799.95	-2608.5330	2156.3560	655.2298	54.4928	0.6083
50%	-38809.45	-2493.2390	2045.1260	661.4966	51.5938	0.5430
Actual Parameter from complete data set	-46269.08	-2730.3890	1940.1970	774.1870	56.8700	0.8370

Table 53. PDI for the estimated regression coefficients for sample size of 500 with large variability from the RF model.

FMI/ Estimated Parameter	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$	$\hat{\beta}_4$	Mean
10%	-0.0918	-0.0147	-0.0269	-0.0961	0.0901	-0.0279
20%	-0.1045	-0.0668	0.1284	-0.1203	-0.9818	-0.2290
30%	-0.0913	-0.0447	0.1593	-0.0938	-0.0310	-0.0203
40%	-0.1614	-0.0446	0.1114	-0.1537	-0.0418	-0.0580
50%	-0.1612	-0.0869	0.0541	-0.1456	-0.0928	-0.0865
Mean	-0.1220	-0.0516	0.0853	-0.1219	-0.2115	-0.0843

7. CONCLUSION

A performance analysis on the 30 mixed datasets based on the PDI's of the three different imputation methods showed that the CART method was the best imputation method for dataset with sample size of 30 with small, regular and large variabilities as well as datasets with sample size of 500 with large variability. On the other hand, the RF method was the best imputation method for datasets with sample size of 500 with small and regular variabilities.

Even though, the PMM method is considered as the default imputation method in the R package, the RF methods worked best mostly on a sample size of 500 datasets irrespective of the variability. The classification and regression tree imputation methods worked best mostly on sample size of 30 irrespective of the variability.

For future works, studies should look at the best imputation methods for mixed dataset with a different statistic for measuring categorial variables (such as, the point biserial) and also look at the variability in the response variable. One could also look at different sample sizes as well.

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